Homework No. 6 Solution

1.

(a) (10%)

$$x(t)=\int_{-\infty}^{t}\frac{\sin(2\pi\tau)}{\pi\tau}d\tau$$

$$\frac{\sin(2\pi t)}{\pi t}\xleftrightarrow{FT}\begin{cases}1 & ,w\le 2\pi\\ 0 & ,otherwise\end{cases}$$

$$\int_{-\infty}^{t}s(\tau)d\tau\xleftrightarrow{FT}\frac{S(jw)}{jw}+\pi S(j0)\delta(w)$$

$$X(jw)=\begin{cases}\pi\delta(w) & ,w=0\\ \dfrac{1}{jw} & ,|w|\le 2\pi, w\ne 0\\ 0 & ,otherwise\end{cases}$$

(b) (10%)

$$x(t)=(\frac{\sin(t)}{\pi t})*\frac{d}{dt}[(\frac{\sin(2t)}{\pi t})]$$

$$x(t)=a(t)*b(t)\xleftrightarrow{FT}X(jw)=A(jw)B(jw)$$

$$\frac{\sin(Wt)}{\pi t}\xleftrightarrow{FT}\begin{cases}1 & ,|w|\le W\\ 0 & ,otherwise\end{cases}$$

$$\frac{d}{dt}s(t)\xleftrightarrow{FT}jwS(jw)$$

$$X(jw)=\begin{cases}jw & ,|w|\le 1\\ 0 & ,otherwise\end{cases}$$

2.

(a) (10%)

$$\frac{1}{(1+jw)^2}\xleftrightarrow{FT}te^{-t}u(t)$$

$$jwS(jw)\xleftrightarrow{FT}\frac{d}{dt}s(t)$$

$$x(t)=\frac{d}{dt}[te^{-t}u(t)]$$
$$=(1-t)e^{-t}u(t)$$

(b)(10%)

$$X(jw)=\frac{2\sin(w)}{w(jw+2)}$$

$$S_1(jw)=\frac{2\sin(w)}{w}\overset{FT}{\longleftrightarrow}s_1(t)=\begin{cases}1 & ,|t|\le 1\\ 0 & ,otherwise\end{cases}$$

$$S_2(jw)=\frac{1}{(jw+2)}\overset{FT}{\longleftrightarrow}s_2(t)=e^{-2t}u(t)$$

$$x(t)=s_1(t)*s_2(t)=\begin{cases}0 & ,t<-1\\ \frac{1}{2}[1-e^{-2(t+1)}] & ,-1\le t<1\\ \frac{e^{-2t}}{2}[e^2-e^{-2}] & ,t\ge 1\end{cases}$$

(c)(10%)

$$\because \frac{2\sin(w)}{w}\overset{FT}{\longleftrightarrow}rect(\frac{t}{2})=\begin{cases}1 & ,|t|\le 1\\ 0 & ,otherwise\end{cases}$$

$$let\ S(jw)=2\cdot\frac{2\sin(2w)}{2w}\overset{FT}{\longleftrightarrow}s(t)=rect(\frac{t}{4})=\begin{cases}1 & ,|t|\le 2\\ 0 & ,otherwise\end{cases}$$

$$S_1(jw)=2\sin(4w)\cdot S(jw)\overset{FT}{\longleftrightarrow}s_1(t)=-js(t+4)+js(t-4)$$

$$X(jw)=\frac{d}{dw}S_1(jw)\overset{FT}{\longleftrightarrow}x(t)=-jts_1(t)$$

$$x(t)=-t\cdot rect(\frac{t+4}{4})+t\cdot rect(\frac{t-4}{4})$$

3.

(a)(10%)

$$jwY(jw)+3Y(jw)=X(jw)$$

$$H(jw)=\frac{Y(jw)}{X(jw)}$$

$$=\frac{1}{jw+3}$$

$$h(t)=e^{-3t}u(t)$$

(b) (10%)

$$\frac{d^3}{dt^3}y(t)-3\frac{d}{dt}y(t)-2y(t)=3\frac{d^2}{dt^2}x(t)+8\frac{d}{dt}x(t)-10x(t)$$

$$\Rightarrow [(jw)^3-3jw-2]Y(jw)=[3(jw)^2+8jw-10]X(jw)$$

$$\Rightarrow H(jw)=\frac{Y(jw)}{X(jw)}=\frac{-3w^2+8jw-10}{-jw^3-3jw-2}$$

$$=\frac{-3w^2+8jw-10}{(jw+1)^2(jw-2)}$$

$$=\frac{A}{(jw+1)^2}+\frac{B}{(jw+1)}+\frac{C}{(jw-2)}$$

$$A=\left\{\frac{A}{(jw+1)^2}(jw+1)^2+\frac{B}{(jw+1)}(jw+1)^2+\frac{C}{(jw-2)}(jw+1)^2\right\}\Big|_{w=j}$$

$$=\left\{(jw+1)^2H(w)\right\}\Big|_{w=j}$$

$$=\left\{(jw+1)^2\times\frac{-3w^2+8jw-10}{(jw+1)^2(jw-2)}\right\}\Big|_{w=j}$$

$$=5$$

$$B=\frac{1}{j}\frac{d}{dw}\left\{\frac{A}{(jw+1)^2}(jw+1)^2+\frac{B}{(jw+1)}(jw+1)^2+\frac{C}{(jw-2)}(jw+1)^2\right\}\Big|_{w=j}$$

$$=\left\{\frac{1}{j}\frac{d}{dw}(jw+1)^2H(w)\right\}\Big|_{w=j}$$

$$=\left\{\frac{1}{j}\frac{d}{dw}(\frac{-3w^2+8jw-10}{(jw-2)})\right\}\Big|_{w=j}$$

$$=1$$

$$C=\left\{(jw-2)H(w)\right\}\Big|_{w=-2j}$$

$$=2$$

$$\Rightarrow H(jw)=\frac{5}{(jw+1)^2}+\frac{1}{(jw+1)}+\frac{2}{(jw-2)}$$

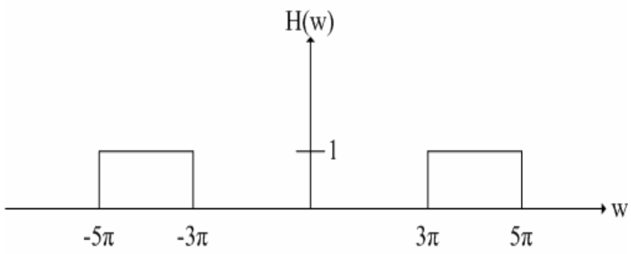
$$\Rightarrow h(t)=5te^{-t}u(t)+e^{-t}u(t)-2e^{2t}u(-t)$$

4.

(a)(5%)

$$h_1(t)=\frac{\sin\pi t}{\pi t}\overset{F}{\longleftrightarrow}H_1(w)=\begin{cases}1 & ,|w|<\pi\\ 0 & ,otherwise\end{cases}$$

$$\Rightarrow H(w)=H_1(w-4\pi)+H_1(w+4\pi)$$

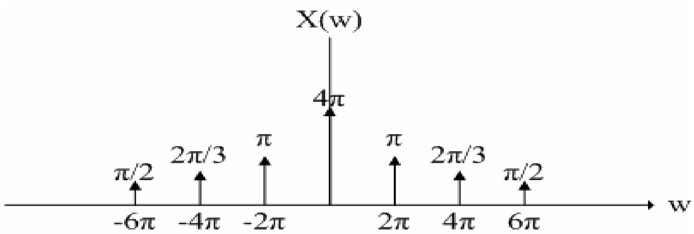


(b) (5%)

$$x(t) = 2 + \sum_{k=1}^{3} \frac{1}{1+k}(e^{jk2\pi t} + e^{-jk2\pi t})$$

$$\Rightarrow X[0] = 2, X[k] = \frac{1}{1+|k|} \ for \ k = \pm 1, \pm 2, \pm 3$$

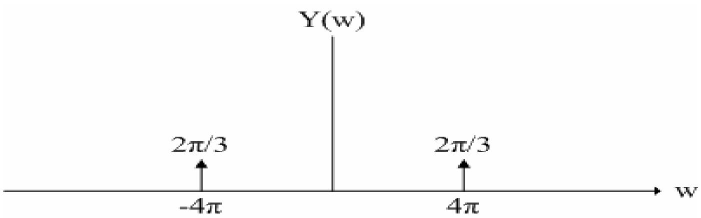
$$\Rightarrow X[w] = 2\pi \sum_{k=-3}^{3} X[k]\delta(w - 2\pi k)$$



(c) (5%)

$$Y(w) = X(w)H(w)$$

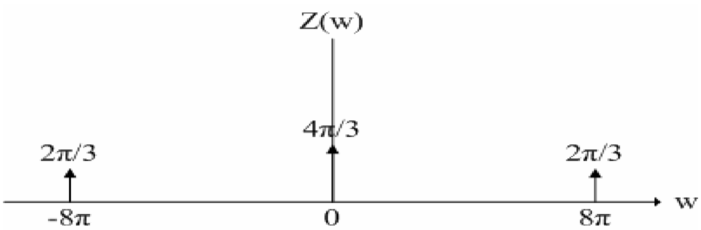
$$= \frac{2\pi}{3}[\delta(w - 4\pi) + \delta(w + 4\pi)]$$



(d) (5%)

$$Z(w) = Y(w - 4\pi) + Y(w + 4\pi)$$

$$\Rightarrow z(t) = \frac{2}{3}(1 + \cos(8\pi t))$$



5.(10%)

$$X[k]=\frac{\sin(k\frac{\pi}{8})}{\pi k}\xleftrightarrow{FS;\pi}x(t)=\begin{cases}1 & ,|t|\le\frac{\pi}{8w_0}\\ 0 & ,\frac{\pi}{8w_0}<|t|\le\frac{2\pi}{w_0}\end{cases}$$

$$\pi^2\sum_{k=-\infty}^{\infty}\frac{\sin^2(k\pi/8)}{\pi^2k^2}=\frac{\pi^2}{T}\int_{-0.5T}^{0.5T}|x(t)|^2dt$$

$$=\frac{\pi w_0}{2}\int_{-\frac{\pi}{8w_0}}^{\frac{\pi}{8w_0}}|1|^2dt$$

$$=\frac{2\pi^2w_0}{16w_0}$$

$$=\frac{\pi^2}{8}$$