Homework5 Solution

1.(15%)

$$X[k]=\left(-\frac{1}{3}\right)^{|k|},\omega_0=\pi$$

$$x(t)=\sum_{k=0}^{\infty}\left(-\frac{1}{3}\right)^{k}e^{jk\pi t}+\sum_{k=1}^{\infty}\left(-\frac{1}{3}\right)^{-k}e^{-jk\pi t}=\frac{1}{1+\frac{1}{3}e^{j\pi t}}+\frac{-\frac{1}{3}e^{-j\pi t}}{1+\frac{1}{3}e^{-j\pi t}}$$

$$=\frac{4}{5+3cos\,(\pi t)}$$

2.(30%)

$$x(t)=\sum_{n=-\infty}^{\infty}C_n e^{jn\left(\frac{\pi}{2}\right)t}\quad,\qquad \omega_0=\frac{2\pi}{4}=\frac{\pi}{2}$$

$$C_n=\frac{1}{4}\int_{-2}^{2}x(t)e^{-j\frac{n\pi}{2}t}dt$$

$$=\frac{1}{4}\int_{-2}^{2}2t^2e^{-j\frac{n\pi}{2}t}dt$$

$$=\frac{1}{2}\int_{-2}^{2}t^2\cos\left(\frac{n\pi}{2}t\right)dt-\frac{j}{2}\int_{-2}^{2}t^2\sin\left(\frac{n\pi}{2}t\right)dt$$

$$=\frac{2}{2}\int_{0}^{2}t^2\cos\left(\frac{n\pi}{2}t\right)dt$$

$$=\frac{2t^2}{n\pi}\sin\left(\frac{n\pi}{2}t\right)+2t\left(\frac{2}{n\pi}\right)^2\cos\left(\frac{n\pi}{2}t\right)-2\left(\frac{2}{n\pi}\right)^3\sin\left(\frac{n\pi}{2}t\right)\Bigg|_0^2$$

$$=\frac{16}{(n\pi)^2}(-1)^n,n\neq 0$$

$$C_0=\frac{1}{4}\int_{-2}^{2}x(t)dt=\frac{1}{4}\int_{-2}^{2}2t^2\,dt=\frac{8}{3}$$

注意：因為我們希望能看到dc的值，所以要特別算Co

$$\therefore x(t)=\frac{8}{3}+\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\frac{16}{n^2\pi^2}(-1)^n e^{j\frac{n\pi}{2}t}$$

3. (15%)

$y(t)= e^{-|t|}\,[\,(1/2)e^{j5t} + (1/2)e^{-j5t}\,]$

$F\{ e^{-|t|} \} = 2/[1+(2\pi f)^2] = X(f)$

$$\therefore F\{ y(t) \} = \frac{1}{2}\left[X\left(f-\frac{5}{2\pi}\right)+X\left(f+\frac{5}{2\pi}\right)\right]$$

$$= \frac{1}{1+\left[2\pi\left(f-\frac{5}{2\pi}\right)\right]^2}+\frac{1}{1+\left[2\pi\left(f+\frac{5}{2\pi}\right)\right]^2}$$

4.(40%)

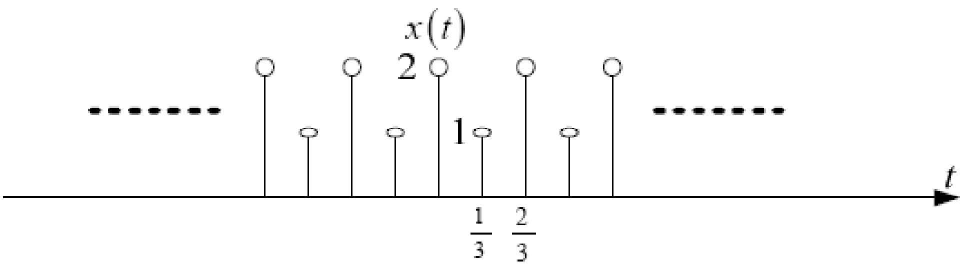
(a)

$$\begin{aligned} x(t) &= \sin(3\pi t)+\cos(4\pi t) \\ &= \frac{1}{2j}e^{j(3)\pi t}-\frac{1}{2j}e^{j(-3)\pi t}+\frac{1}{2}e^{j(4)\pi t}+\frac{1}{2}e^{j(-4)\pi t} \end{aligned}$$

By inspection

$$X[k]=\begin{cases} \frac{1}{2} & k=\pm 4 \\ \frac{1}{2j} & k=3 \\ \frac{-1}{2j} & k=-3 \\ 0 & \text{otherwise} \end{cases}$$

(b)



$$T=\frac{2}{3},\quad \omega_0=3\pi$$

$$X[k]=\frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}dt$$

應為3/2

$$=\frac{2}{3}\int_0^{\frac{2}{3}}\left[2\delta(t)+\delta\left(t-\frac{1}{3}\right)\right]e^{-jk3\pi t}dt$$

$$=3+\frac{2}{3}e^{-jk\pi}$$