Homework 4 Solution

1.(15%)

(i) If $y_h[n]=A(1/2)^n$, then we need to verify

$$A\left(\frac{1}{2}\right)^n - \frac{1}{2}A\left(\frac{1}{2}\right)^{n-1} = 0$$

it's true.

(ii) For n>=0

$$B\left(\frac{1}{3}\right)^n - \frac{1}{2}B\left(\frac{1}{3}\right)^{n-1} = \left(\frac{1}{3}\right)^n$$

There for B=-2.

(iii)

From eq. (1) we know that y[0]=x[0]+(1/2)y[-1]=z[0]=1, now we also have

y[0]=A+B

A=1-B=3.

2. (25%)

$$x(t)=e^{3t}u(t)$$

and

$$y(t)=y_p(t)+y_h(t)$$

$y_h(t)$ is a solution of the homogeneous differential equation

$$\frac{d}{dt}y(t)+2y(t)=0$$

A common method for finding the particular solution for an exponential input signal is to look for a so-called forced response, i.e. a signal of the same form as the input. Since $x(t)=e^{3t}u(t)$ for $t>0$, we hypothesize a solution for $t>0$ of the form

$$y_p(t)=Ye^{3t}(t)$$

Where Y is a number that we must determine.
For t>0 yields

$$3Ye^{3t}+2Ye^{3t} = e^{3t}$$
$$Y=1/5$$
$$y_p(t)=(1/5)e^{3t}(t) \qquad t>0$$
$$y_h(t)=Ae^{st}$$
$$Ase^{st}+2Ae^{et} = Ae^{et}(s+2) = 0$$
$$s=-2$$
$$y(t)=Ae^{-2t}+(1/5)e^{3t},\ t>0$$

and set y(0)=0

$$A=-(1/5)$$

Thus for t>0

$$y(t)=(1/5)(e^{3t}-e^{-2t})$$

or

$$y(t)=(1/5)(e^{3t}-e^{-2t})u(t).$$

3.(40%)

(i)

$$\frac{d^2y(t)}{dt^2}+2\frac{dy(t)}{dt}+y(t)=\frac{dt}{dt}=1$$

$y_p(t)=C=1$

$y(t)=Ae^{-t}+Bte^{-t}+1$

$y'(t)=-Ae^{-t}+Be^{-t}-Bte^{-t}$

$y(0)=-1$

$y'(0)=1$

$\Rightarrow A=-2$, $B=1$

$y(t)=-2e^{-t}-te^{-t}+1$

(ii)

$$\frac{d^2y(t)}{dt^2}+2\frac{dy(t)}{dt}+y(t)=\frac{d(-t)}{dt}=-1$$

$y_p(t)=C=1$

$y(t)=Ae^{-t}+Bte^{-t}-1$

$y'(t)=-Ae^{-t}+Be^{-t}-Bte^{-t}$

$y(0)=-1$

$y'(0)=1$

$\Rightarrow A=0$, $B=1$

$y(t)=te^{-t}-1$

(iii)

$$\frac{d^2y(t)}{dt^2}+2\frac{dy(t)}{dt}+y(t)=\frac{d[\sin(t)+\cos(t)]}{dt}=\cos(t)-\sin(t)$$

$y_p(t)=A\cos(t) + B\sin(t) - 2A\sin(t) + 2B\cos(t) - A\cos(t) - B\sin(t) = \cos(t)-\sin(t)$

$A=B=(1/2)$

$y(t)=$

$y'(t)=-Ce^{-t} + De^{-t} - Dte^{-t} - (1/2)\sin(t) + (1/2)\cos(t)$

$y(0)=-1$

$y'(0)=1$

$\Rightarrow C=-(3/2)$, $D=-1$

$y(t)= -(3/2)e^{-t} - te^{-t} + (1/2)\cos(t) + (1/2)\sin(t)$

4.(20%)

Natural response:

$$r-\frac{1}{2}=0\Rightarrow r=\frac{1}{2}\Rightarrow y^{(h)}[n]=c\left(\frac{1}{2}\right)^{n}$$

$$y[-1]=3=c\left(\frac{1}{2}\right)^{-1}\Rightarrow c=\frac{3}{2}\Rightarrow y^{(n)}[n]=\frac{3}{2}\left(\frac{1}{2}\right)^{n}$$

Forced response:

$$y^{(p)}[n]=k\left(\frac{-1}{2}\right)^{n}u[n]$$

$$k\left(\frac{-1}{2}\right)^{n}-k\frac{1}{2}\left(\frac{-1}{2}\right)^{n-1}=2\left(\frac{-1}{2}\right)^{n}\Rightarrow\left(\frac{-1}{2}\right)k-k\frac{1}{2}=2\left(\frac{-1}{2}\right)\Rightarrow k=1$$

$$\therefore y^{(p)}[n]=\left(\frac{-1}{2}\right)^{n}u[n]$$

$$y^{(f)}[n]=c\left(\frac{1}{2}\right)^{n}+\left(\frac{-1}{2}\right)^{n},\ n\geq 0$$

Translate initial condition

$$y[n]=\frac{1}{2}y[n-1]+2x[n]$$

$$y[0]=\frac{1}{2}y[-1]+2x[0]=\frac{1}{2}0+2=2$$

$$y[0]=2=c+1\Rightarrow c=1$$

$$\therefore y^{(f)}[n]=\left(\frac{1}{2}\right)^{n}+\left(\frac{-1}{2}\right)^{n},\ n\geq 0$$