Homework No. 4
Due 13:00 , April 9 , 2008

1. Consider the difference equation

$$y[n] - \frac{1}{2}y[n-1] = x[n] \qquad (1)$$

and suppose that

$$x[n] = \left(\frac{1}{3}\right)^n u[n] \qquad (2)$$

Assume that the solution y[n] consists of the sum of a particular solution $y_p[n]$ to the first equation and homogeneous solution $y_h[n]$ satisfying the equation

$$y_h[n] - \frac{1}{2}y_h[n-1] = 0.$$

(i) Verify that the homogeneous solution is given by

$$y_h[n] = A\left(\frac{1}{2}\right)^n$$

(ii) Let us consider obtaining a particular solution

$$y_p[n] - \frac{1}{2}y_p[n-1] = \left(\frac{1}{3}\right)^n u[n]$$

By assuming that $y_p[n]$ is of the form $B\left(\frac{1}{3}\right)^n$ for $n \geq 0$, and substitution this in the above difference equation, determine value of B.

(iii) Suppose that the LTI system described by eq.(1) and initially at rest has as its input the signal specified by eq.(2). Since x[n]=0 for n<0, we have that y[n]=0 for n<0. Also, from part (i) and (ii) we has the form

$$y[n] = A\left(\frac{1}{2}\right)^n + B\left(\frac{1}{3}\right)^n$$

for $n \geq 0$. In order to solve for the unknown constant A, we must specify a value for y[n] for some $n \geq 0$. Use the condition of initial rest and equations above to determine y[0]. From this value determine the const A.

2. Consider a system whose input x(t) and output y(t) satisfy the first-order differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t).$$

The system also satisfies the condition of initial rest, determine the system output y(t) when the input is $x(t)=e^{3t}u(t)$.

3. Determine the homogeneous and particular solutions for the system described by the following differential equation for the given inputs and initial conditions:

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt}$$

$$y(0^-) = -1, \left.\frac{dy(t)}{dt}\right|_{t=0^-} = 1$$

(i) x(t)=t
(ii) x(t)=-t
(iii) x(t)=sin(t)+cos(t)

4. Identify the natural and forced responses for the systems described by the following difference equation with input and initial conditions as specified:

$$y[n] - \frac{1}{2}y[n-1] = 2x[n]$$

$$y[-1] = 3$$

$$x[n] = \left(\frac{-1}{2}\right)^n u[n]$$