Homework No. 8 Solution

1.

(1) $x(t)=\dfrac{d^2}{dt^2}\left(e^{-3(t-2)}u(t-2)\right)$ (10%)

$$e^{-3t}u(t)\overset{\mathcal{L}}{\longleftrightarrow}\frac{1}{s+3},\ \text{with ROC }\operatorname{Re}\{s\}>-3$$

$$e^{-3(t-2)}u(t-2)\overset{\mathcal{L}}{\longleftrightarrow}\frac{1}{s+3}e^{-2s},\ \text{with ROC }\operatorname{Re}\{s\}>-3$$

$$x(t)=\frac{d^2}{dt^2}\left(e^{-3(t-2)}u(t-2)\right)\overset{\mathcal{L}}{\longleftrightarrow}X(s)=\frac{s^2}{s+3}e^{-2s},\ \text{with ROC }\operatorname{Re}\{s\}>-3$$

(2) $X(s)=s^{-1}\dfrac{d}{ds}\left(\dfrac{e^{-3s}}{s}\right)$ with ROC $\operatorname{Re}\{s\}>0$ (10%)

$$A(s)=\frac{1}{s}\overset{\mathcal{L}}{\longleftrightarrow}a(t)=u(t)$$

right-sided

$$B(s)=e^{-3s}A(s)\overset{\mathcal{L}}{\longleftrightarrow}b(t)=a(t-3)=u(t-3)$$

$$C(s)=\frac{d}{ds}B(s)\overset{\mathcal{L}}{\longleftrightarrow}c(t)=-tb(t)=-tu(t-3)$$

$$X(s)=\frac{1}{s}C(s)\overset{\mathcal{L}}{\longleftrightarrow}\begin{aligned}x(t)&=u(t)*c(t)=\int_{-\infty}^{t}c(\tau)\,d\tau=-\int_{3}^{t}\tau\,d\tau\\&=-\frac{1}{2}\left(t^2-9\right)u(t-3)\quad\left(\because u(t)*c(t),\ 3\le t<\infty\right).\end{aligned}$$

2. $X(s)=\dfrac{-s-4}{s^2+3s+2}=\dfrac{-3}{s+1}+\dfrac{2}{s+2}$

(1) With ROC $\operatorname{Re}\{s\}<-2$

Left-sided: $x(t)=\left(3e^{-t}-2e^{-2t}\right)u(-t)$. (5%)

(2) With ROC $\operatorname{Re}\{s\}>-1$

Right-sided: $x(t)=\left(-3e^{-t}+2e^{-2t}\right)u(t)$. (5%)

(3) With ROC $-2<\operatorname{Re}\{s\}<-1$

Two-sided: $x(t)=3e^{-t}u(-t)+2e^{-2t}u(t)$. (5%)

3. $H(s)=\dfrac{2s^2+2s-2}{s^2-1}=2+\dfrac{1}{s+1}+\dfrac{1}{s-1}$

(a) Causal system : $h(t)=2\delta(t)+\left(e^{-t}+e^{t}\right)u(t)$. (5%)

(b) Stable system: $h(t)=2\delta(t)+e^{-t}u(t)-e^{t}u(-t)$. (5%)

4.

zero at: -5

poles at: $-1\pm 3j$

(1) All poles are in the left half of s-plane, and with ROC: $\mathrm{Re}\{s\}>-1$, the system is both stable and causal. (7%)

(2) All zeros are in the left half of s-plane, so a stable and causal inverse system exists. (8%)

5.

(1) $x(t)=u(t)-u(t-6)\Rightarrow X(s)=\int_{0^-}^{6} e^{-st}dt=\frac{1-e^{-6s}}{s}$. (10%)

(2)

$$\begin{aligned}X(s)&=\int_{0^-}^{1}\frac{1}{2j}\left(e^{j\pi t}-e^{-j\pi t}\right)e^{-st}dt\\&=\frac{1}{2j}\left[\frac{1}{j\pi-s}\left(e^{j\pi-s}-1\right)-\frac{1}{-j\pi-s}\left(e^{-j\pi-s}-1\right)\right]\\&=\frac{1}{2j}\left[\frac{1}{j\pi-s}\left(-e^{-s}-1\right)+\frac{1}{j\pi+s}\left(-e^{-s}-1\right)\right]\\&=\frac{1}{2j}\frac{\left(-e^{-s}-1\right)\left(j\pi+s+j\pi-s\right)}{-\pi^2-s^2}=\frac{\pi\left(1+e^{-s}\right)}{s^2+\pi^2}.\end{aligned}$$

(10%)

6.

(1) $x(t-1)\overset{\mathcal{L}}{\longleftrightarrow}e^{-1s}X(s)=e^{-1s}\frac{2s}{s^2+2}$. (5%)

(2) $x(t)*\frac{d}{dt}x(t)$

$$b(t)=\frac{d}{dt}x(t)\overset{\mathcal{L}}{\longleftrightarrow}B(s)=sX(s)$$

$$y(t)=x(t)*b(t)\overset{\mathcal{L}}{\longleftrightarrow}Y(s)=X(s)B(s)=sX^2(s)=s\left(\frac{2s}{s^2+2}\right)^2.$$

(5%)

(3) $e^{-3t}x(t)\overset{\mathcal{L}}{\longleftrightarrow}X(s+3)=\frac{2(s+3)}{(s+3)^2+2}$. (5%)

(4) $\int_0^t x(3\tau)d\tau\overset{\mathcal{L}}{\longleftrightarrow}Y(s)=\frac{X(s/3)}{3s}=\frac{2}{s^2+18}$. (5%)