Homework No. 5 Solution
Due 10:10 am, May 3, 2007

3.58 (a)

$$\begin{aligned}
x(t) &= \sin(2\pi t)e^{-t}u(t) \\
&= \frac{1}{2j}e^{j2\pi t}e^{-t}u(t) - \frac{1}{2j}e^{-j2\pi t}e^{-t}u(t)
\end{aligned}$$

$$e^{-t}u(t) \xleftarrow{\ FT\ } \frac{1}{1+j\omega}$$

$$e^{j2\pi t}s(t) \xleftarrow{\ FT\ } S(j(\omega - 2\pi))$$

$$X(j\omega) = \frac{1}{2j}\left[\frac{1}{1+j(\omega - 2\pi)} - \frac{1}{1+j(\omega + 2\pi)}\right]$$

(b)

$$e^{-3|t|} \xleftarrow{\ FT\ } \frac{6}{9+\omega^2}$$

$$s(t-1) \xleftarrow{\ FT\ } e^{-j\omega}S(j\omega)$$

$$tw(t) \xleftarrow{\ FT\ } j\frac{d}{d\omega}W(j\omega)$$

$$X(j\omega) = j\frac{d}{d\omega}\left[e^{-j\omega}\frac{6}{9+\omega^2}\right]$$

$$= \frac{6e^{-j\omega}}{9+\omega^2} - \frac{12\,j\omega e^{-j\omega}}{\left(9+\omega^2\right)^2}$$

(c)

$$\frac{\sin(Wt)}{\pi t} \xleftarrow{\ FT\ } \begin{cases} 1 & |\omega| \le W \\ 0, & \text{otherwise} \end{cases}$$

$$s_1(t)s_2(t) \xleftarrow{\ FT\ } \frac{1}{2\pi}S_1(j\omega) * S_2(j\omega)$$

$$X(j\omega) = \begin{cases} 5 - \frac{|\omega|}{\pi} & \pi < |\omega| \le 5\pi \\ 4 & |\omega| \le \pi \\ 0 & \text{otherwise} \end{cases}$$

(d)

$$\begin{aligned}
x(t) &= \frac{d}{dt}te^{-2t}\sin(t)u(t) \\
&= \frac{d}{dt}te^{-2t}u(t)\frac{e^{jt}-e^{-jt}}{2j}
\end{aligned}$$

$$te^{-2t}u(t) \quad\xleftarrow{\ FT\ }\quad \frac{1}{(2+j\omega)^2}$$

$$e^{jt}s(t) \quad\xleftarrow{\ FT\ }\quad S(j(\omega-1))$$

$$\frac{d}{dt}s(t) \quad\xleftarrow{\ FT\ }\quad j\omega S(j\omega)$$

$$X(j\omega) = j\omega\frac{1}{2j}\left[\frac{1}{(2+j(\omega-1))^2}-\frac{1}{(2+j(\omega+1))^2}\right]$$

3.59 (a)

$$\frac{1}{(1+j\omega)^2} \quad\xleftarrow{\ FT\ }\quad te^{-t}u(t)$$

$$j\omega S(j\omega) \quad\xleftarrow{\ FT\ }\quad \frac{d}{dt}s(t)$$

$$\begin{aligned}
x(t) &= \frac{d}{dt}[te^{-t}u(t)] \\
&= (1-t)e^{-t}u(t)
\end{aligned}$$

(b)

$$\frac{2\sin(\omega)}{\omega} \quad\xleftarrow{\ FT\ }\quad \mathrm{rect}(t)=\begin{cases}1 & |t|\leq 1 \\ 0, & \text{otherwise}\end{cases}$$

$$S(j2\omega) \quad\xleftarrow{\ FT\ }\quad \frac{1}{2}s(\frac{t}{2})$$

$$S(j(\omega-2)) \quad\xleftarrow{\ FT\ }\quad e^{j2t}s(t)$$

$$\begin{aligned}
x(t) &= \mathrm{rect}(\frac{t}{2})e^{j2t}-\mathrm{rect}(\frac{t}{2})e^{-j2t} \\
&= 2j\mathrm{rect}(\frac{t}{2})\sin(2t)
\end{aligned}$$

(c)

$$\frac{1}{j\omega} + \pi\delta(j\omega) \xleftarrow{\quad FT \quad} u(t)$$

$$\frac{1}{2+j\omega} \xleftarrow{\quad FT \quad} e^{-2t}u(t)$$

$$2\pi\delta(\omega) \xleftarrow{\quad FT \quad} 1$$

$$X(j\omega) = -0.5\frac{1}{(j\omega+2)} + 0.5\frac{1}{j\omega} + 0.5\pi\delta(\omega) - 1.5\pi\delta(\omega)$$

$$X(j\omega) \xleftarrow{\quad FT \quad} x(t) = -0.5e^{-2t}u(t) + 0.5u(t) - \frac{3}{4}$$

(d)

$$\because \frac{2\sin(\omega)}{\omega} \xleftrightarrow{\quad F \quad} \mathrm{rect}(t) = \begin{cases} 1, & |t| \le 1 \\ 0, & \text{otherwise} \end{cases}$$

$$\text{let } S(j\omega) = 2 \cdot \frac{2\sin(2\omega)}{2\omega} \xleftrightarrow{\quad F \quad} s(t) = \mathrm{rect}\left(\frac{t}{2}\right) = \begin{cases} 1, & |t| \le 2 \\ 0, & \text{otherwise} \end{cases}$$

$$S_1(j\omega) = 2\sin(4\omega) \cdot S(j\omega) \xleftrightarrow{\quad F \quad} s_1(t) = -js(t+4) + js(t-4)$$

$$X(j\omega) = \frac{d}{d\omega}S_1(j\omega) \xleftrightarrow{\quad F \quad} x(t) = -jts_1(t)$$

$$x(t) = -t\mathrm{rect}\left(\frac{t+4}{2}\right) + t\mathrm{rect}\left(\frac{t-4}{2}\right)$$

3.68 (a)

$$j\omega Y(j\omega) + 3Y(j\omega) = X(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

$$= \frac{1}{j\omega + 3}$$

$$h(t) = e^{-3t}u(t)$$

(b)

$$(j\omega)^2 Y(j\omega) + 5j\omega Y(j\omega) + 6Y(j\omega) = -j\omega X(j\omega)$$

$$H(j\omega) = \frac{-j\omega}{(j\omega)^2 + 5j\omega + 6}$$

$$= -\frac{3}{3+j\omega} + \frac{2}{2+j\omega}$$

$$h(t) = (-3e^{-3t} + 2e^{-2t})u(t)$$

3

3.75 (b)

$$X[k] = \frac{\sin(k\frac{\pi}{8})}{\pi k} \quad \overset{FS;\,\pi}{\longleftrightarrow} \quad x(t) = \begin{cases} 1 & |t| \le \frac{\pi}{8\omega_o} \\ 0, & \frac{\pi}{8\omega_o} < |t| \le \frac{2\pi}{\omega_o} \end{cases}$$

$$\pi^2 \sum_{k=-\infty}^{\infty} \frac{\sin^2(k\pi/8)}{\pi^2 k^2} = \frac{\pi^2}{T} \int_{-0.5T}^{0.5T} |x(t)|^2 \, dt$$

$$= \frac{\pi\omega_o}{2} \int_{-\frac{\pi}{8\omega_o}}^{\frac{\pi}{8\omega_o}} |1|^2 \, dt$$

$$= \frac{\omega_o 2\pi^2}{2(8)\omega_o}$$

$$= \frac{\pi^2}{8}$$

(c)

$$X(j\omega) = \frac{2(2)}{\omega^2 + 2^2} \quad \overset{FT}{\longleftrightarrow} \quad x(t) = e^{-2|t|}$$

$$\frac{1}{2} \int_{-\infty}^{\infty} \left(\frac{4}{\omega^2 + 2^2} \right)^2 d\omega = \pi \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= 2\pi \int_{0}^{\infty} e^{-4t} \, dt$$

$$= \frac{\pi}{2}$$