Homework No. 4 Solution

Due 10:10 am, May 1, 2007

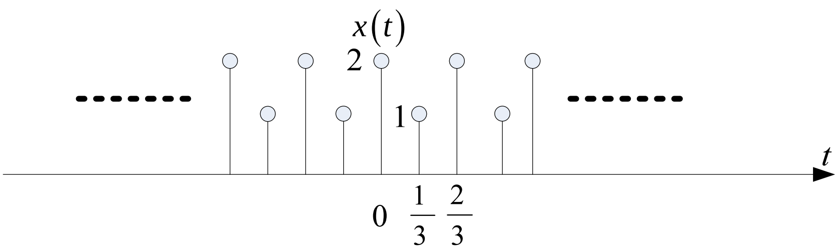
3.50 (a)

$$x(t) = \sin(3\pi t) + \cos(4\pi t)$$
$$= \frac{1}{2j}e^{j(3)\pi t} - \frac{1}{2j}e^{j(-3)\pi t} + \frac{1}{2}e^{j(4)\pi t} + \frac{1}{2}e^{j(-4)\pi t}$$

by inspection

$$X[k] = \begin{cases} \frac{1}{2} & k = \pm 4 \\ \frac{1}{2j} & k = 3 \\ \frac{-1}{2j} & k = -3 \\ 0 & \text{otherwise} \end{cases}$$

(b)



$$T = \frac{2}{3}, \quad \omega_0 = 3\pi$$
$$X[k] = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}dt$$
$$= \frac{3}{2}\int_0^{\frac{2}{3}}\left[2\delta(t) + \delta\left(t - \frac{1}{3}\right)\right]e^{-jk3\pi t}dt$$

化簡到最後，如果只剩下exponential,
則將之拆解成cos和sin !!!!

$$= \frac{3}{2}\left[2 + e^{-jk\pi}\right]$$
$$= 3 + \frac{3}{2}e^{-jk\pi}$$
$$= 3 + \frac{3}{2}\cos(k\pi)$$

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3.51 (a)

$$x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{j2\pi kt}$$
$$= je^{j(1)2\pi t} - je^{j(-1)2\pi t} + je^{j(3)2\pi t} + je^{j(-3)2\pi t}$$
$$= -2\sin(2\pi t) + 2\cos(6\pi t)$$

(e)

$$X[k] = e^{-j2\pi k} \qquad -4 \le k < 4$$

$$\begin{aligned}
x(t) &= \sum_{k=-4}^{4} e^{-j2\pi k} \cdot e^{j2\pi kt} \\
&= \sum_{k=-4}^{4} e^{j2\pi k(t-1)} \\
&= \frac{1-e^{9\cdot j2\pi(t-1)}}{1-e^{j2\pi(t-1)}} \\
&= \frac{e^{j9\pi(t-1)}}{e^{j\pi(t-1)}} \frac{e^{-j9\pi(t-1)}-e^{j9\pi(t-1)}}{e^{-j\pi(t-1)}-e^{j\pi(t-1)}} \\
&= 1\cdot\frac{-2j\sin(9\pi(t-1))}{-2j\sin(\pi(t-1))} \\
&= \frac{\sin(9\pi(t-1))}{\sin(\pi(t-1))} = \frac{\sin(9\pi t)}{\sin(\pi t)}
\end{aligned}$$

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3.54 (a)

$$\begin{aligned}
X(\omega) &= \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \\
&= \int_{3}^{\infty} e^{-t(2+j\omega)} dt \\
&= \frac{e^{-3(2+j\omega)}}{2+j\omega}
\end{aligned}$$

(b)

$$\begin{aligned}
X(\omega) &= \int_{-\infty}^{\infty} e^{-4|t|} e^{-j\omega t} dt \\
&= \int_{0}^{\infty} e^{-4t} e^{-j\omega t} dt + \int_{-\infty}^{0} e^{4t} e^{-j\omega t} dt \\
&= \frac{8}{16+\omega^2}
\end{aligned}$$

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3.67 (a)

$$X(\omega) = \frac{1}{1+j\omega}$$

$$Y(\omega) = \frac{1}{2+j\omega} + \frac{1}{3+j\omega} = \frac{5+2j\omega}{(2+j\omega)(3+j\omega)}$$

$$\begin{aligned}
H(\omega) &= \frac{Y(\omega)}{X(\omega)} = \frac{5+7j\omega+2(j\omega)^2}{(2+j\omega)(3+j\omega)} \\
&= 2 - \frac{1}{2+j\omega} - \frac{2}{3+j\omega}
\end{aligned}$$

$$h(t) = 2\delta(t) - \left(e^{-2t} + 2e^{-3t}\right)u(t)$$

(c)

$$X(\omega)=\frac{1}{2+j\omega}$$
$$Y(\omega)=\frac{2}{(2+j\omega)^2}$$
$$H(\omega)=\frac{2}{2+j\omega}$$
$$h(t)=2e^{-2t}u(t)$$

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3.77 (a)

$$\int_{-\infty}^{\infty}x(t)\,dt=X(0)=1$$

(b)

$$\int_{-\infty}^{\infty}|x(t)|^2\,dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2\,d\omega=\frac{16}{3\pi}$$

(c)

$$\int_{-\infty}^{\infty}x(t)e^{j3t}dt=X(-3)=2$$

(d)

Assume that $X_e(\omega)$ is a real and even function, and $X(\omega)=X_e(\omega+1)$. Since $X_e(\omega)$ is real and even, so is $x_e(t)$. Then we have $x(t)=x_e(t)e^{-jt}=|x_e(t)|e^{-jt}$, which means $\arg\{x(t)\}=-t$.

even function的phase shift為零

(e)

$$\begin{aligned}x(0)&=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(\omega)e^{j\omega(0)}dt\\&=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(\omega)\,dt\\&=\frac{4}{\pi}\end{aligned}$$

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