Homework No. 8 Solution

Due 10:10 am, June 14, 2007

7.17 (d) (10%) $x[n]=\left(\frac{1}{4}\right)^n\left(u[n]-u[n-5]\right)$

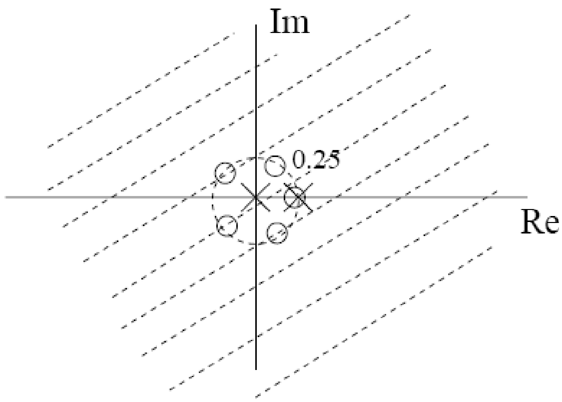
$$
\begin{aligned}
X(z) &= \sum_{n=0}^{4}\left(\frac{1}{4}z^{-1}\right)^n \\
&= \frac{1-\left(\frac{1}{4}z^{-1}\right)^5}{1-\frac{1}{4}z^{-1}} \\
&= \frac{\left[z^5-\left(\frac{1}{4}\right)^5\right]}{z^4\left(z-\frac{1}{4}\right)}, \quad \text{all } z
\end{aligned}
$$

(4%)

4 poles at $z=0$, 1 pole at $z=\frac{1}{4}$

5 poles at $z=\frac{1}{4}e^{jk\frac{2\pi}{5}}$ for $k=0,\ 1,\ 2,\ 3,\ 4$

Note that the zero at $z=\frac{1}{4}$ cancels the pole at $z=\frac{1}{4}$. (4%)



(2%)

(f) (10%) $x[n]=3^n u[-n-1]$

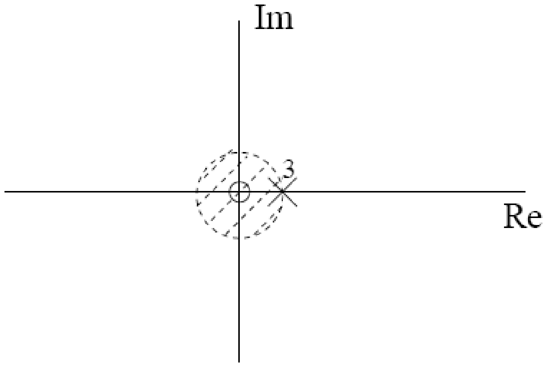
$$
\begin{aligned}
X(z) &= \sum_{n=-\infty}^{-1}\left(3z^{-1}\right)^n \\
&= \sum_{n=1}^{\infty}\left(\frac{1}{3}z\right)^n
\end{aligned}
$$

(4%)

$$
\begin{aligned}
&= \frac{\frac{1}{3}z}{1-\frac{1}{3}z} \\
&= \frac{-1}{1-3z^{-1}}, \quad |z|<3
\end{aligned}
$$

Pole at $z=3$

Zero at $z=0$ (4%)



(2%)

■

7.19 (18%)

$$X(z) = \frac{C(z^4-1)}{z(z-\sqrt{2}e^{j\frac{\pi}{4}})(z-\sqrt{2}e^{-j\frac{\pi}{4}})} \quad (10\%)$$

There are 2 possible ROCs

(1) $|z| > \sqrt{2}$
$x[n]$ is right-sided.

(2) $|z| < \sqrt{2}$
~~$x[n]$ is two-sided.~~ left-sided

(8%)

■

7.24 (a) (13%)

$$X(z) = \frac{1+\frac{7}{6}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)}, \quad |z| > \frac{1}{2}$$

$$X(z) = \frac{2}{1-\frac{1}{2}z^{-1}} + \frac{-1}{1+\frac{1}{3}z^{-1}}$$

$$x[n] = \left[2\left(\frac{1}{2}\right)^n - \left(-\frac{1}{3}\right)^n\right]u[n]$$

(g) (13%)
solution 1:

$$X(z) = \frac{2z^4-2z^3-2z^2}{z^2-1} = 2z^2-2z+\frac{-z^{-1}}{1+z^{-1}}+\frac{-z^{-1}}{1-z^{-1}}$$

$$x[n] = 2\delta[n+2]-2\delta[n+1]-(-1)^{n-1}u[n-1]-u[n-1]$$

solution 2:

$$X(z) = \frac{2z^4-2z^3-2z^2}{z^2-1} = 2z^2+\frac{-z}{1+z^{-1}}+\frac{-z}{1-z^{-1}}$$

$$x[n] = 2\delta[n+2]-(-1)^{n+1}u[n+1]-u[n+1]$$

■

7.28 (c) (10%) $X(z)=\cos(z^{-3}), \quad |z|>0$

$$\cos(\alpha) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}(\alpha)^{2k}$$

(5%)

$$X(z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}(z^{-3})^{2k}$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} z^{-6k}$$

$$x[n] = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}\delta[n-6k]$$

(5%)

(d) (10%)

solution 1:

$$X(z)=\ln(1+z^{-1}), \quad |z|>0$$

$$\ln(1+\alpha)=\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}\alpha^k \quad (5\%)$$

$$X(z) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k}(z^{-1})^k$$

$$x[n] = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k}\delta[n-k]$$

(5%)

solution 2:

$$X(z)=\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}z^{-k}=\sum_{k=-\infty}^{\infty}\frac{(-1)^{k-1}}{k}u[k-1]z^{-k}=\sum_{k=-\infty}^{\infty}x[k]z^{-k}$$

$$\therefore x[n]=\frac{(-1)^{n-1}}{n}u[n-1]$$

■

Problem 1. (16%) $H(z)=\dfrac{1-2z^{-1}}{1-\frac{1}{3}z^{-1}}, \quad |z|>\dfrac{1}{3},$

The possible corresponding inverse systems can be represented by

$$H_{I1}(z)=\frac{1-\frac{1}{3}z^{-1}}{1-2z^{-1}}, \quad |z|>2$$

and

$$H_{I2}(z)=\frac{1-\frac{1}{3}z^{-1}}{1-2z^{-1}}, \quad |z|<2.$$

Neither $H_{I1}(z)$ nor $H_{I2}(z)$ is both causal and stable.

■