1.

(1) $N=19,\ \Omega_o = 2\pi/19$

$$x[n] = 2\sin(14n\pi/19) + \cos(10n\pi/19) + 1$$
$$= -je^{j14n\pi/19} + je^{-j14n\pi/19} + 0.5\left(e^{j10n\pi/19} + e^{-j10n\pi/19}\right) + e^{j(0)2n\pi/19}$$

By inspection

$$X[k] = \begin{cases} 0.5 & k=-5 \\ j & k=-7 \\ 1 & k=0 \\ -j & k=7 \\ 0.5 & k=5 \\ 0 & \text{o.w. on } k\in\{-9 \sim 9\} \end{cases}$$

(2) $x[n] = \sum_{m=-\infty}^{\infty}\left[(-1)^m\left(\delta[n-2m]+\delta[n+2m]\right)\right]$

$N=12,\ \Omega_o = 2\pi/6$

$$X[k] = \frac{1}{12}\sum_{n=-5}^{6} x[n]e^{-jkn\pi/6}$$
$$= \frac{1}{6}\cos\left(\frac{2k\pi}{3}\right) - \frac{1}{6}\cos\left(\frac{k\pi}{2}\right) - \frac{1}{6}\cos\left(\frac{k\pi}{3}\right) + \frac{1}{6}$$

2.

$N=19,\ \Omega_o = 2\pi/19$

$$X[k] = 2j\sin(4k\pi/19) + \cos(10k\pi/19)$$
$$= -e^{-j4k\pi/19} + e^{j4k\pi/19} + 0.5\left(e^{j10k\pi/19} + e^{-j10k\pi/19}\right)$$

By inspection

$$x[n] = \begin{cases} 19/2 & n=\pm 5 \\ -19 & n=2 \\ 19 & n=-2 \\ 0 & \text{o.w. on } n\in\{-9 \sim 9\} \end{cases}$$

3.

(1)

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} = \sum_{n=-\infty}^{\infty} \left(\frac{3}{4}\right)^n e^{-j\Omega n} = \frac{\left(\frac{3}{4}e^{-j\Omega}\right)^4}{1-\frac{3}{4}e^{-j\Omega}}$$

$$\left|X(\Omega)\right| = \frac{\left(\frac{3}{4}\right)^n}{\sqrt{\frac{25}{16}-\frac{3}{2}\cos(\Omega)}}; \angle X(\Omega) = -4\Omega + \tan^{-1}\left(\frac{3\sin(\Omega)}{4-3\cos(\Omega)}\right)$$

(2)

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} = \frac{1}{2}e^{-j\Omega}$$

$$\left|X(\Omega)\right| = \frac{1}{2}; \angle X(\Omega) = -\Omega$$

4.

(1)

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\Omega}e^{j\Omega n}d\Omega$$

by orthogonality

$$= \delta[n+1]$$

(2) Since $X(\Omega)$ doesn't periodic in Ω with period 2π, the DTFT is not exist.

However, if you still evaluate it, you should get the following formula.

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\frac{e^{j\frac{\Omega}{2}}-e^{-j\frac{\Omega}{2}}}{2j}+\frac{e^{j\Omega}+e^{-j\Omega}}{2}\right)e^{j\Omega n}d\Omega$$

$$= \frac{1}{2}\delta[n+1]+\frac{1}{2}\delta[n-1]+\frac{\cos(n\pi)}{2\pi j(n+0.5)}-\frac{\cos(n\pi)}{2\pi j(0.5-n)}$$

$$\left(\frac{1}{2\pi}\cdot\frac{1}{2j}\int_{-\pi}^{\pi} e^{j\Omega/2}e^{j\Omega n}d\Omega = \frac{1}{2\pi}\cdot\frac{1}{2j}\left.\frac{e^{j(n+1/2)\Omega}}{j(n+0.5)}\right|_{-\pi}^{\pi} = \frac{1}{2\pi}\cdot\frac{1}{2j}\frac{je^{jn\pi}+je^{-jn\pi}}{j(n+0.5)} = \frac{1}{2\pi j}\cdot\frac{\cos(n\pi)}{n+0.5}\right)$$

5.

(1)

Since $x[n]$ is real and odd, $X(\Omega)$ is purely imaginary, thus $y[n]=0$.

(2)

$$y[n] = -jnx[n] = -jn^2\left(\frac{3}{4}\right)^{|n|}$$

(3)

$$y[n] = 2\pi x_1[n]x[n] = 2\pi n^2\left(\frac{3}{4}\right)^{2|n|} e^{j\frac{\pi}{2}n}$$

(4)

$$y[n] = -j\left[e^{-j\frac{\pi}{4}(n-4)}x[n-4] + e^{j\frac{\pi}{4}(n-4)}x[n-4]\right]$$

6.

$$\frac{\sin(11\pi n/20)}{\sin(\pi n/20)} = \frac{\sin((2\times5+1)\pi n/(2\times10))}{\sin(\pi n/(2\times10))}$$

$$\Omega_0 = \frac{\pi}{10} \Rightarrow N = 2\pi \Big/ \frac{\pi}{10} = 20$$

$$\because \begin{cases} 1, & |n| \le 5 \\ 0, & 5 < |n| \le 10 \end{cases} \xleftrightarrow{F} \frac{1}{20}\cdot\frac{\sin(11\pi k/20)}{\sin(\pi k/20)}$$

$$\therefore \frac{1}{20}\cdot\frac{\sin(11\pi n/20)}{\sin(\pi n/20)} \xleftrightarrow{F} X[k] = \frac{1}{20}\begin{cases} 1, & |k| \le 5 \\ 0, & 5 < |k| \le 10 \end{cases} = X[k+20i]$$

$$\frac{\sin(11\pi n/20)}{\sin(\pi n/20)} \xleftrightarrow{F} X[k] = \begin{cases} 1, & |k| \le 5 \\ 0, & 5 < |k| \le 10 \end{cases} = X[k+20i]$$

where k and i are integers.

7.

(1)

$$y[n] = x[n] \otimes h[n] = x[n] \otimes \left(\delta[n] + 2\delta[n-1] + \delta[n-2]\right)$$

$$h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$$

$$H\left(\Omega\right) = 1 + 2e^{-j\Omega} + e^{-j2\Omega} = 2e^{-j\Omega}\left(0.5e^{j\Omega} + 1 + 0.5e^{-j\Omega}\right) = 2e^{-j\Omega}\left(1 + \cos\left(\Omega\right)\right)$$

(2)

$$\begin{aligned} h_1\left[n\right] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} H_1\left(\Omega\right)e^{j\Omega n}d\Omega \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} H\left(\Omega+\pi\right)e^{j\Omega n}d\Omega \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} H\left(\Omega\right)e^{j(\Omega-\pi)n}d\Omega \\ &= \frac{e^{j\pi n}}{2\pi}\int_{-\pi}^{\pi} H\left(\Omega\right)e^{j\Omega n}d\Omega \\ &= -1^n h[n] \\ &= \delta[n] - 2\delta[n-1] + \delta[n-2] \end{aligned}$$

8.

(1)

$$u[n+4] - u[n-5] \overset{DTFT}{\longleftrightarrow} \frac{\sin(9\Omega/2)}{\sin(\Omega/2)}$$

$$ns[n] \overset{DTFT}{\longleftrightarrow} j\frac{d}{d\Omega}S(\Omega)$$

$$\therefore x[n] = j\frac{d}{d\Omega}\frac{\sin(9\Omega/2)}{\sin(\Omega/2)} - 2\frac{\sin(9\Omega/2)}{\sin(\Omega/2)}$$

(2)

$$x[n] = \left(\frac{1}{3}\right)^n u[n+2] = \left(\frac{1}{3}\right)^{-2}\left(\frac{1}{3}\right)^{n+2} u[n+2]$$

$$\left(\frac{1}{3}\right)^n u[n] \overset{DTFT}{\longleftrightarrow} \frac{1}{1-\frac{1}{3}e^{-j\Omega}}$$

$$s[n+2] \overset{DTFT}{\longleftrightarrow} e^{j2\Omega}S(\Omega)$$

$$\therefore X\left(\Omega\right) = \frac{9e^{j2\Omega}}{1-\frac{1}{3}e^{-j\Omega}}$$

(3)

Let the first part be $A(\Omega)$, and the second be $B(\Omega)$.

$$a[n]=\begin{cases}1 & |n-2|\le 7\\ 0 & o.w.\end{cases}$$

$$b[n]=\begin{cases}1 & |n|\le 3\\ 0 & o.w.\end{cases}$$

$$\therefore x[n]=2\pi a[n]b[n]=\begin{cases}2\pi & |n|\le 3\\ 0 & o.w.\end{cases}$$

(4) This problem should be $X(\Omega)=\cos(4\Omega)\left[\dfrac{\sin(3\Omega/2)}{\sin(\Omega/2)}\right]$.

All of you will get full points in this problem.

$$A(\Omega)=\frac{\sin(3\Omega/2)}{\sin(0.5\Omega)}\xleftrightarrow{DTFT} a[n]=\begin{cases}1 & |n|\le 1\\ 0 & o.w.\end{cases}$$

$$X(\Omega)=\cos(4\Omega)A(\Omega)\xleftrightarrow{DTFT} x[n]=0.5a[n+4]+0.5a[n-4]$$

$$\therefore x[n]=\begin{cases}0.5 & |n+4|\le 1,|n-4|\le 1\\ 0 & o.w.\end{cases}$$