Homework No. 6

Due 12:00, May 22, 2007

1. Use the defining equation for the DTFS coefficients to evaluate the DTFS representation of the following signals:

   (1) $x[n] = 2\sin\left(\frac{14\pi}{19}n\right) + \cos\left(\frac{10\pi}{19}n\right) + 1$ (5%)

   (2) $x[n] = \sum_{m=-\infty}^{\infty}(-1)^m\left(\delta[n-2m] + \delta[n+3m]\right)$ (5%)

2. Use the definition of the DTFS to determine the time-domain signals represented by the following DTFS coefficients: (7%)

$$X[k] = a_k = \cos\left(\frac{10\pi}{19}k\right) + j2\sin\left(\frac{4\pi}{19}k\right)$$

3. Use the defining equation for the DTFT to evaluate the frequency-domain representations of the following signals:

   (1) $x[n] = \left(\frac{3}{4}\right)^n u[n-4]$ (5%)

   (2) $x[n] = \frac{1}{2}\delta[4-4n]$ (5%)

4. Use the equation describing the DTFT representation to determine the time-domain signals corresponding to the following DTFTs:

   (1) $X(\Omega) = \cos(\Omega) + j\sin(\Omega)$ (5%)

   (2) $X(\Omega) = \sin\left(\frac{\Omega}{2}\right) + \cos(\Omega)$ (5%)

5. You are given $x[n] = n(3/4)^{|n|} \xleftrightarrow{DTFT} X(\Omega)$. Without evaluating $X(\Omega)$, find $y[n]$ if

   (1) $Y(\Omega) = \text{Re}\{X(\Omega)\}$ (5%)

   (2) $Y(\Omega) = \frac{d}{d\Omega}X(\Omega)$ (5%)

   (3) $Y(\Omega) = X(\Omega) \circledast X(\Omega - \pi/2)$ (5%)

   (4) $Y(\Omega) = \frac{d}{d\Omega}\left\{e^{-j4\Omega}\left[X\left(e^{-j\left(\Omega+\frac{\pi}{4}\right)}\right) + X\left(e^{-j\left(\Omega-\frac{\pi}{4}\right)}\right)\right]\right\}$ (5%)

6. Use the duality property to evaluate the DTFS of $\frac{\sin(11\pi n/20)}{\sin(\pi n/20)}$. (8%)

7. A linear time-invariant system is described by the input-output relation

$$y[n] = x[n] + 2x[n-1] + x[n-2]$$

(1) Determine $h[n]$, the impulse response of the system and $H(\Omega)$, the frequency response of the system. (10%)

(2) Now consider a new system whose frequency response is $H_1(\Omega) = H(\Omega + \pi)$. Determine $h_1[n]$, the impulse response of the new system. (5%)

8. Use the tables of transforms and properties to find the DTFTs and the inverse DTFT of the following signals:

(1) $x[n] = (n-2)\left(u[n+4] - u[n-5]\right)$. (5%)

(2) $x[n] = \left(\frac{1}{3}\right)^n u[n+2]$. (5%)

(3) $X(\Omega) = \left[e^{-j2\Omega}\frac{\sin(15\Omega/2)}{\sin(\Omega/2)}\right] \circledast \left[\frac{\sin(7\Omega/2)}{\sin(\Omega/2)}\right]$. (5%)

(4) $X(\Omega) = \cos(4\Omega) \circledast \left[\frac{\sin(3\Omega/2)}{\sin(\Omega/2)}\right]$. (5%)