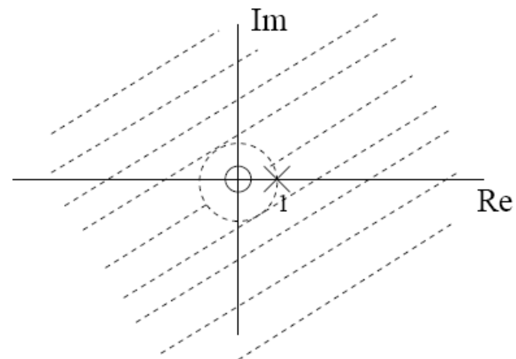
Homework No. 9 Solution

Due 10:10 am, June 20, 2006

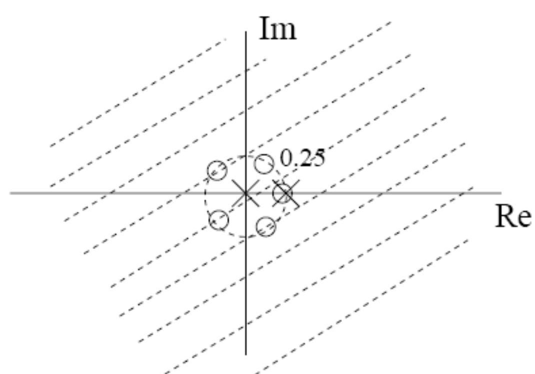
7.17 (c) (10%) $x[n] = u[n]$

$$X(z) = \sum_{n=0}^{\infty} z^{-n} = \frac{1}{1-z^{-1}}, \quad |z| > 1$$

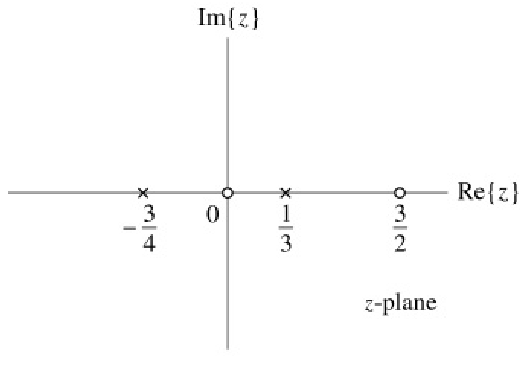


(d) (10%) $x[n] = \left(\frac{1}{4}\right)^n \left(u[n] - u[n-5]\right)$

$$X(z) = \sum_{n=0}^{4} \left(\frac{1}{4}z^{-1}\right)^n = \frac{1-\left(\frac{1}{4}z^{-1}\right)^5}{1-\frac{1}{4}z^{-1}} = \frac{\left[z^5 - \left(\frac{1}{4}\right)^5\right]}{z^4\left(z-\frac{1}{4}\right)}, \quad \text{all } z$$



7.19 (18%)



(1) $|z| > \frac{3}{4}$
$x[n]$ is right-sided.

(2) $\frac{1}{3} < |z| < \frac{3}{4}$
$x[n]$ is two-sided.

(3) $|z| < \frac{1}{3}$
$x[n]$ is left-sided.

7.24 (a) (13%)

$$X(z) = \frac{1 + \frac{7}{6}z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{3}z^{-1}\right)}, \quad |z| > \frac{1}{2}$$

$$X(z) = \frac{2}{1 - \frac{1}{2}z^{-1}} + \frac{-1}{1 + \frac{1}{3}z^{-1}}$$

$$x[n] = \left[2(\frac{1}{2})^n - (-\frac{1}{3})^n\right] u[n]$$

(d) (13%)

$$X(z) = \frac{z^2 - 3z}{z^2 + \frac{3}{2}z - 1}, \quad \frac{1}{2} < |z| < 2$$

$$X(z) = \frac{-1}{1 - \frac{1}{2}z^{-1}} + \frac{2}{1 + 2z^{-1}}$$

$$x[n] = -\left(\frac{1}{2}\right)^n u[n] - 2(-2)^n u[-n-1]$$

7.28 (c) (10%)

$$X(z) = \cos\left(z^{-3}\right), \quad |z| > 0$$

$$\cos(\alpha) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}(\alpha)^{2k}$$

$$
\begin{aligned}
X(z) &= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}(z^{-3})^{2k} \\
&= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} z^{-6k} \\
x[n] &= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \delta[n-6k]
\end{aligned}
$$

(d) (10%) $X(z) = \ln\left(1+z^{-1}\right), \quad |z|>0$

$$
\ln(1+\alpha) = \sum_{k=0}^{\infty} \frac{(-1)^{k-1}}{k}(\alpha)^k
$$

$$
\begin{aligned}
X(z) &= \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k}(z^{-1})^k \\
x[n] &= \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} \delta[n-k]
\end{aligned}
$$

Problem 1. (16%) $H(z) = \dfrac{1-2z^{-1}}{1-\frac{1}{3}z^{-1}}, \quad |z| > \dfrac{1}{3},$

The corresponding inverse systems can be represented by

$$
H_{I1}(z) = \frac{1-\frac{1}{3}z^{-1}}{1-2z^{-1}}, \quad |z|>2
$$

and

$$
H_{I2}(z) = \frac{1-\frac{1}{3}z^{-1}}{1-2z^{-1}}, \quad |z|<2 .
$$

Neither of $H_{I1}(z)$ and $H_{I2}(z)$ is both causal and stable, so there does not exist a both causal and stable inverse system for $H(z)$.