Homework No. 7
Due 11:10 am, May 25, 2006

1. Use the defining equation for the DTFS coefficients to evaluate the DTFS representation of the following signals:

    (1) $x[n] = \cos^2\left(\frac{6\pi}{17}n + \frac{\pi}{3}\right)$. (8%)

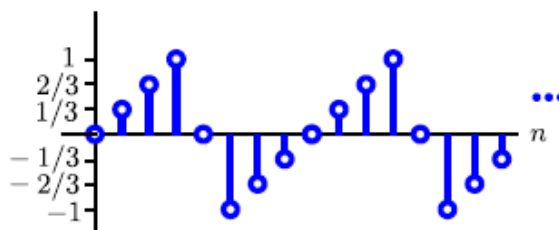   (2) $x[n] = x[n + 8]$ as depicted in Figure 1. (7%)

Figure 1

2. Use the definition of the DTFS to determine the time-domain signals represented by the following DTFS coefficient: (8%)

$$X[k] = a_k = 2\sin\left(\frac{14\pi k}{19}\right) + \cos\left(\frac{10\pi}{19}k\right) + 1$$

3. Use the defining equation for the DTFT to evaluate the frequency-domain representations of the following signals:

    (1) $x[n] = \left(\frac{2}{5}\right)^n u[n + 4]$. (5%)

    (2) $x[n] = \begin{cases} \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\pi}{N}n\right) & , \ |n| \le N \\ 0 & , \ \text{otherwise} \end{cases}$. (7%)

4. Use the equation describing the DTFT representation to determine the time-domain signals corresponding to the following DTFTs:

    (1) $|X(\Omega)| = \begin{cases} 1, & \pi/4 < |\Omega| < 3\pi/4 \\ 0, & \text{otherwise} \end{cases}$, $\arg\{X(\Omega)\} = -4\Omega$. (5%)

    (2) $X(\Omega) = \sin\left(\frac{\Omega}{2}\right) + \cos(\Omega)$. (5%)

5. Use the tables of transforms and properties to find the DTFTs and the inverse DTFT of the following signals:

    (1) $x[n] = (n - 2)\big(u[n + 4] - u[n - 5]\big)$. (5%)

(2) $x[n] = \cos\left(\dfrac{\pi}{4}n\right)\left(\dfrac{1}{2}\right)^n u[n-2]$. (8%)

(3) $X(\Omega) = \left[e^{-j2\Omega}\,\dfrac{\sin(15\Omega/2)}{\sin(\Omega/2)}\right] \circledast \left[\dfrac{\sin(7\Omega/2)}{\sin(\Omega/2)}\right]$. (7%)

6.   Use the duality property to evaluate the DTFS of $\dfrac{\sin(11\pi n/20)}{\sin(\pi n/20)}$. (8%)

7.   You are given $x[n] = n(3/4)^{|n|} \xleftrightarrow{\ DTFT\ } X(\Omega)$. Without evaluating $X(\Omega)$, find $y[n]$ if

(1) $Y(\Omega) = \mathrm{Im}\{X(\Omega)\}$. (5%)

(2) $Y(\Omega) = \dfrac{d}{d\Omega}\left\{e^{-j4\Omega}\left[X\left(\Omega + \dfrac{\pi}{4}\right) + X\left(\Omega - \dfrac{\pi}{4}\right)\right]\right\}$. (7%)

8.

(1)   Determine the frequency response and the impulse response for the system described by the following difference equation: (5%)

$$y[n] + \frac{1}{2}y[n-1] = x[n] - 2x[n-1]$$

(2)   Determine the difference-equation descriptions for the system with the following impulse and frequency responses: (10%)

(a) $h[n] = \delta[n] + 2\left(\dfrac{1}{2}\right)^n u[n] + \left(\dfrac{-1}{2}\right)^n u[n]$.

(b) $H(\Omega) = 1 + \dfrac{e^{-j\Omega}}{\left(1 - \dfrac{1}{2}e^{-j\Omega}\right)\left(1 + \dfrac{1}{4}e^{-j\Omega}\right)}$.