Homework No. 5 Solution
Due 11:10am, May 4, 2006

3.50 (c) (10%)

Graph to find $T = 14,\ \omega_o = \frac{\pi}{7}$

$$\begin{aligned}
X[k] &= \frac{1}{14}\int_{-7}^{7} x(t)e^{-jk\frac{\pi}{7}t}\,dt \\
&\text{By the sifting property} \\
&= \frac{1}{14}\left[e^{j(k-1)\frac{6\pi}{7}} + e^{j(k-1)\frac{4\pi}{7}} + e^{j(k-1)\frac{2\pi}{7}} + 1 + e^{j(1-k)\frac{2\pi}{7}} + e^{j(1-k)\frac{4\pi}{7}} + e^{j(1-k)\frac{6\pi}{7}}\right] \\
&= \frac{1}{7}\left[\cos((k-1)\frac{6\pi}{7}) + \cos((k-1)\frac{4\pi}{7}) + \cos((k-1)\frac{2\pi}{7}) + \frac{1}{2}\right]
\end{aligned}$$

(d) (10%)

$$\begin{aligned}
x(t) &= |\sin(\pi t)| \\
T &= 1 \\
\omega_0 &= 2\pi \\
X[k] &= \int_0^1 \sin(\pi t)e^{-jk2\pi t}dt \\
&= \frac{1}{2j}\int_0^1 [e^{j\pi t} - e^{-j\pi t}]e^{-jk2\pi t}dt \\
&= \frac{1}{2j}[\frac{1}{j\pi(1-2k)}(e^{j\pi(1-2k)} - 1) - \frac{1}{j\pi(1+2k)}(e^{-j\pi(1+2k)} - 1)] \\
&= \frac{4k}{\pi(1-4k^2)}
\end{aligned}$$

3.51 (c) (10%)

$X[k] = (-\frac{1}{3})^{|k|}, \quad \omega_o = 1$

$$\begin{aligned} x(t) &= \sum_{m=-\infty}^{\infty} (-\frac{1}{3})^{|k|} e^{jkt} \\ &= \sum_{m=0}^{\infty} (-\frac{1}{3}e^{jt})^k + \sum_{m=1}^{\infty} (-\frac{1}{3}e^{-jt})^k \\ &= \frac{1}{1+\frac{1}{3}e^{jt}} - \frac{\frac{1}{3}e^{-jt}}{1+\frac{1}{3}e^{-jt}} \\ &= \frac{8}{10+6\cos(t)} \end{aligned}$$

(d) (10%)

$\omega_o = \pi$

$$\begin{aligned} x(t) &= \sum_{m=-\infty}^{\infty} X[k] e^{j\pi kt} \\ &= 2e^{-j0.25\pi} e^{j(-4)\pi t} + e^{j0.25\pi} e^{j(-3)\pi t} + e^{-j0.25\pi} e^{j(3)\pi t} \\ &\quad +2e^{j0.25\pi} e^{j(4)\pi t} \\ &= 4\cos(4\pi t + 0.25\pi) + 2\cos(3\pi t - 0.25\pi) \end{aligned}$$

3.54 (c) (10%)

$x(t) = te^{-t}u(t)$

$$\begin{aligned} X(j\omega) &= \int_0^{\infty} te^{-t} e^{-j\omega t}\, dt \\ &= \frac{1}{(1+j\omega)^2} \end{aligned}$$

(d) (10%)

$x(t) = \sum_{m=0}^{\infty} a^m \delta(t-m), \quad |a| < 1$

$$\begin{aligned} X(j\omega) &= \int_0^{\infty} (\sum_{m=0}^{\infty} a^m \delta(t-m)) e^{-j\omega t}\, dt \\ &= \sum_{m=0}^{\infty} (ae^{-j\omega})^m \\ &= \frac{1}{1-ae^{-j\omega}} \end{aligned}$$

3.67 (b) (10%)

$x(t) = e^{-3t}u(t), \quad y(t) = e^{-3(t-2)}u(t-2)$

$$\begin{aligned} X(j\omega) &= \frac{1}{3+j\omega} \\ Y(j\omega) &= e^{-j2\omega}\frac{1}{3+j\omega} \end{aligned}$$

$$\begin{aligned} H(j\omega) &= e^{-j2\omega} \\ h(t) &= \delta(t-2) \end{aligned}$$

(f) (10%)

$$\begin{aligned} X(\omega) &= \frac{1}{2+j\omega} \\ Y(\omega) &= \frac{2}{(2+j\omega)^2}e^{-j2\omega} \\ \Rightarrow H(\omega) &= \frac{2}{2+j\omega}e^{-j2\omega} \\ \Rightarrow h(t) &= 2e^{-2(t-2)}u(t-2) \end{aligned}$$

3.77m (a) (4%)

$\int_{-\infty}^{\infty} x(t)dt = X(\omega)|_{\omega=0} = 1$

(b) (4%)

$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-3}^{1} |X(\omega)|^2 d\omega = \frac{8}{3\pi}$

(c) (4%)

$\int_{-\infty}^{\infty} x(t)e^{-jt}dt = X(\omega)|_{\omega=1} = 2$

(d) (4%)

$X(\omega)$ is real and even shifted by 1 to the left.

$\Rightarrow X(\omega) = X_e(\omega + 1)$

Since $X_e(\omega)$ is real and even, so is $x_e(t)$, thus $x(t) = x_e(t)e^{-j(1)t} = |x_e(t)|e^{-j(1)t}$.

$\Rightarrow \arg\{x(t)\} = -t$

(e) (4%)

$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega(0)}d\omega = \frac{2}{\pi}$