Homework No. 5 Solution

Due 10:10 am, May 17, 2005

3.58 (a)

$$\begin{aligned} x(t) &= \sin(2\pi t)e^{-t}u(t) \\ &= \frac{1}{2j}e^{j2\pi t}e^{-t}u(t) - \frac{1}{2j}e^{-j2\pi t}e^{-t}u(t) \end{aligned}$$

$$\begin{aligned} e^{-t}u(t) &\overset{FT}{\longleftrightarrow} \frac{1}{1+j\omega} \\ e^{j2\pi t}s(t) &\overset{FT}{\longleftrightarrow} S(j(\omega-2\pi)) \\ X(j\omega) &= \frac{1}{2j}\left[\frac{1}{1+j(\omega-2\pi)} - \frac{1}{1+j(\omega+2\pi)}\right] \end{aligned}$$

(b)

$$\begin{aligned} e^{-3|t|} &\overset{FT}{\longleftrightarrow} \frac{6}{9+\omega^2} \\ s(t-1) &\overset{FT}{\longleftrightarrow} e^{-j\omega}S(j\omega) \\ tw(t) &\overset{FT}{\longleftrightarrow} j\frac{d}{d\omega}W(j\omega) \\ X(j\omega) &= j\frac{d}{d\omega}\left[e^{-j\omega}\frac{6}{9+\omega^2}\right] \\ &= \frac{6e^{-j\omega}}{9+\omega^2} - \frac{12j\omega^{-j\omega}}{(9+\omega^2)^2} \end{aligned}$$

(c)

$$\begin{aligned} \frac{\sin(Wt)}{\pi t} &\overset{FT}{\longleftrightarrow} \begin{cases} 1 & \omega \le W \\ 0, & \text{otherwise} \end{cases} \\ s_1(t)s_2(t) &\overset{FT}{\longleftrightarrow} \frac{1}{2\pi}S_1(j\omega) * S_2(j\omega) \end{aligned}$$

$$X(j\omega) = \begin{cases} 5 - \frac{|\omega|}{\pi} & \pi < |\omega| \le 5\pi \\ 4 & |\omega| \le \pi \\ 0 & \text{otherwise} \end{cases}$$

(d)

$$\begin{aligned} x(t) &= \frac{d}{dt} te^{-2t}\sin(t)u(t) \\ &= \frac{d}{dt} te^{-2t}u(t)\frac{e^{jt}-e^{-jt}}{2j} \end{aligned}$$

$$te^{-2t}u(t) \quad \stackrel{FT}{\longleftrightarrow} \quad \frac{1}{(2+j\omega)^2}$$

$$\begin{aligned} e^{jt}s(t) \quad &\stackrel{FT}{\longleftrightarrow} \quad S(j(\omega-1)) \\ \frac{d}{dt}s(t) \quad &\stackrel{FT}{\longleftrightarrow} \quad j\omega S(j\omega) \\ X(j\omega) \quad &= \quad j\omega\frac{1}{2j}\left[\frac{1}{(2+j(\omega-1))^2} - \frac{1}{(2+j(\omega+1))^2}\right] \end{aligned}$$

3.59 (a)

$$\begin{aligned} \frac{1}{(1+j\omega)^2} \quad &\stackrel{FT}{\longleftrightarrow} \quad te^{-t}u(t) \\ j\omega S(j\omega) \quad &\stackrel{FT}{\longleftrightarrow} \quad \frac{d}{dt}s(t) \end{aligned}$$

$$\begin{aligned} x(t) &= \frac{d}{dt}[te^{-t}u(t)] \\ &= (1-t)e^{-t}u(t) \end{aligned}$$

(b)

$$\begin{aligned} \frac{2\sin(\omega)}{\omega} \quad &\stackrel{FT}{\longleftrightarrow} \quad \text{rect}(t) = \begin{cases} 1 & |t| \le 1 \\ 0, & \text{otherwise} \end{cases} \\ S(j2\omega) \quad &\stackrel{FT}{\longleftrightarrow} \quad \frac{1}{2}s(\frac{t}{2}) \\ S(j(\omega-2)) \quad &\stackrel{FT}{\longleftrightarrow} \quad e^{j2t}s(t) \end{aligned}$$

$$\begin{aligned} x(t) &= \text{rect}(\frac{t}{2})e^{j2t} - \text{rect}(\frac{t}{2})e^{-j2t} \\ &= 2j\text{rect}(\frac{t}{2})\sin(2t) \end{aligned}$$

(c)

$$
\begin{aligned}
\frac{1}{j\omega} + \pi\delta(j\omega) &\overset{FT}{\longleftrightarrow} u(t) \\
\frac{1}{2+j\omega} &\overset{FT}{\longleftrightarrow} e^{-2t}u(t) \\
2\pi\delta(\omega) &\overset{FT}{\longleftrightarrow} 1 \\
X(j\omega) &= -0.5\frac{1}{(j\omega+2)} + 0.5\frac{1}{j\omega} + 0.5\pi\delta(\omega) - 1.5\pi\delta(\omega) \\
X(j\omega) &\overset{FT}{\longleftrightarrow} x(t) = -0.5e^{-2t}u(t) + 0.5u(t) - \frac{3}{4}
\end{aligned}
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(d)

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\because \frac{2\sin(\omega)}{\omega} \overset{\mathcal{F}}{\longleftrightarrow} \text{rect}(t) = \begin{cases} 1, & |t| \le 1 \\ 0, & \text{otherwise} \end{cases}
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$$
\text{let } S(j\omega) = 2 \cdot \frac{2\sin(2\omega)}{2\omega} \overset{\mathcal{F}}{\longleftrightarrow} s(t) = \text{rect}\left(\frac{t}{2}\right) = \begin{cases} 1, & |t| \le 2 \\ 0, & \text{otherwise} \end{cases}
$$

$$
S_1(j\omega) = 2\sin(4\omega) \cdot S(j\omega) \overset{\mathcal{F}}{\longleftrightarrow} s_1(t) = -js(t+4) + js(t-4)
$$

$$
X(j\omega) = \frac{d}{d\omega} S_1(j\omega) \overset{\mathcal{F}}{\longleftrightarrow} x(t) = -jts_1(t)
$$

$$
x(t) = -t\text{rect}\left(\frac{t+4}{2}\right) + t\text{rect}\left(\frac{t-4}{2}\right)
$$

3.68 (a)

$$
\begin{aligned}
j\omega Y(j\omega) + 3Y(j\omega) &= X(j\omega) \\
H(j\omega) &= \frac{Y(j\omega)}{X(j\omega)} \\
&= \frac{1}{j\omega + 3} \\
h(t) &= e^{-3t}u(t)
\end{aligned}
$$

(b)

$$
\begin{aligned}
(j\omega)^2 Y(j\omega) + 5j\omega Y(j\omega) + 6Y(j\omega) &= -j\omega X(j\omega) \\
H(j\omega) &= \frac{-j\omega}{(j\omega)^2 + 5j\omega + 6} \\
&= -\frac{3}{3+j\omega} + \frac{2}{2+j\omega} \\
h(t) &= (-3e^{-3t} + 2e^{-2t})u(t)
\end{aligned}
$$

3.75 (b)

$$X[k] = \frac{\sin(k\frac{\pi}{8})}{\pi k} \quad \xleftrightarrow{FS;\,\pi} \quad x(t) = \begin{cases} 1 & |t| \leq \frac{\pi}{8\omega_o} \\ 0, & \frac{\pi}{8\omega_o} < |t| \leq \frac{2\pi}{\omega_o} \end{cases}$$

$$\begin{aligned} \pi^2 \sum_{k=-\infty}^{\infty} \frac{\sin^2(k\pi/8)}{\pi^2 k^2} &= \frac{\pi^2}{T} \int_{-0.5T}^{0.5T} |x(t)|^2 \, dt \\ &= \frac{\pi\omega_o}{2} \int_{-\frac{\pi}{8\omega_o}}^{\frac{\pi}{8\omega_o}} |1|^2 \, dt \\ &= \frac{\omega_o 2\pi^2}{2(8)\omega_o} \\ &= \frac{\pi^2}{8} \end{aligned}$$

(c)

$$X(j\omega) = \frac{2(2)}{\omega^2 + 2^2} \quad \xleftrightarrow{FT} \quad x(t) = e^{-2|t|}$$

$$\begin{aligned} \frac{1}{2} \int_{-\infty}^{\infty} \left(\frac{4}{\omega^2 + 2^2} \right)^2 d\omega &= \pi \int_{-\infty}^{\infty} |x(t)|^2 \, dt \\ &= 2\pi \int_0^{\infty} e^{-4t} \, dt \\ &= \frac{\pi}{2} \end{aligned}$$