Homework No. 4 Solution

Due 10:10 am, May 17, 2005

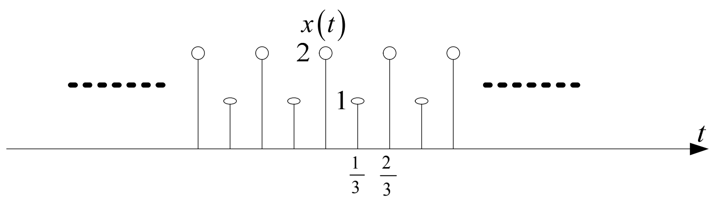
3.50 (a)

$$\begin{aligned} x(t) &= \sin(3\pi t) + \cos(4\pi t) \\ &= \frac{1}{2j}e^{j(3)\pi t} - \frac{1}{2j}e^{j(-3)\pi t} + \frac{1}{2}e^{j(4)\pi t} + \frac{1}{2}e^{j(-4)\pi t} \end{aligned}$$

By inspection

$$X[k] = \begin{cases} \frac{1}{2} & k = \pm 4 \\ \frac{1}{2j} & k = 3 \\ \frac{-1}{2j} & k = -3 \\ 0 & \text{otherwise} \end{cases}$$

(b)



$$T = \frac{2}{3}, \quad \omega_0 = 3\pi$$

$$\begin{aligned} X[k] &= \frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}dt \\ &= \frac{2}{3}\int_0^{\frac{2}{3}}\left[2\delta(t) + \delta\left(t - \frac{1}{3}\right)\right]e^{-jk3\pi t}dt \\ &= 3 + \frac{2}{3}e^{-jk\pi} \end{aligned}$$

3.51 (a)

$$X[k] = j\delta[k-1] - j\delta[k+1] + \delta[k-3] + \delta[k+3], \quad \omega_o = 2\pi$$

$$\begin{aligned} x(t) &= \sum_{m=-\infty}^{\infty} X[k]e^{j2\pi kt} \\ &= je^{j(1)2\pi t} - je^{j(-1)2\pi t} + e^{j(3)2\pi t} + e^{j(-3)\pi t} \\ &= -2\sin(2\pi t) + 2\cos(6\pi t) \end{aligned}$$

(e)

$$X[k] = e^{-j2\pi k} \quad -4 \le k < 4$$

$$\begin{aligned} x(t) &= \sum_{m=-4}^{4} e^{j2\pi k(t-1)} \\ &= \frac{\sin(9\pi t)}{\sin(\pi t)} \end{aligned}$$

3.54 (a)

$$\begin{aligned} X(j\omega) &= \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt \\ &= \int_{3}^{\infty} e^{-2t} e^{-j\omega t}\, dt \\ &= \frac{e^{-3(2+j\omega)}}{2+j\omega} \end{aligned}$$

(b)

$$\begin{aligned} X(j\omega) &= \int_{-\infty}^{\infty} e^{-4|t|} e^{-j\omega t}\, dt \\ &= \int_{0}^{\infty} e^{-4t} e^{-j\omega t}\, dt + \int_{-\infty}^{0} e^{4t} e^{-j\omega t}\, dt \\ &= \frac{8}{16+\omega^2} \end{aligned}$$

3.67 (a)

$$\begin{aligned} X(j\omega) &= \frac{1}{1+j\omega} \\ Y(j\omega) &= \frac{1}{2+j\omega} + \frac{1}{3+j\omega} \\ &= \frac{5+2j\omega}{(2+j\omega)(3+j\omega)} \end{aligned}$$

$$\begin{aligned} H(j\omega) &= \frac{Y(j\omega)}{X(j\omega)} \\ &= \frac{5+7j\omega+2(j\omega)^2}{(2+j\omega)(3+j\omega)} \end{aligned}$$

$$\begin{aligned} &= 2 - \frac{1}{2+j\omega} - \frac{2}{3+j\omega} \\ h(t) &= 2\delta(t) - (e^{-2t} + 2e^{-3t})u(t) \end{aligned}$$

(c)

$$
\begin{aligned}
X(j\omega) &= \frac{1}{2+j\omega} \\
Y(j\omega) &= \frac{2}{(2+j\omega)^2}
\end{aligned}
$$

$$
\begin{aligned}
H(j\omega) &= \frac{2}{(2+j\omega)} \\
h(t) &= 2e^{-2t}u(t)
\end{aligned}
$$

3.77 (a)

$$
\begin{aligned}
\int_{-\infty}^{\infty} x(t)dt &= X(j0) \\
&= 1
\end{aligned}
$$

(b)

$$
\frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = \int_{-\infty}^{\infty} |x(t)|^2 dt
$$

$$
\begin{aligned}
&= \frac{1}{2\pi}\left[\int_{-5}^{-3}(\omega+5)^2 d\omega + \int_{-3}^{-1}(-\omega-1)^2 d\omega + \int_{-1}^{1}(\omega+1)^2 d\omega + \int_{1}^{3}(-\omega+3)^2 d\omega\right] \\
&= \frac{16}{3\pi}
\end{aligned}
$$

(c)

$$
\begin{aligned}
\int_{-\infty}^{\infty} x(t)e^{j3t}dt &= X(j(-3)) \\
&= 2
\end{aligned}
$$

(d)

$X(j\omega)$ is a real and even function shifted by 1 to the left, i.e. $X(j\omega) = X_e(j(\omega-1))$. Since $X_e(j\omega)$ is real and even, so is $x_e(t)$, thus $x(t) = x_e(t)e^{-j(1)t} = |x_e(t)|e^{-j(1)t}$ which means,

$arg[x(t)] = -t$

(e)

$$
\begin{aligned}
x(0) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)d\omega \\
&= \frac{1}{2\pi}\left[\int_{-5}^{-3}(\omega+5)d\omega + \int_{-3}^{-1}(-\omega-1)d\omega + \int_{-1}^{1}(\omega+1)d\omega + \int_{1}^{3}(-\omega+3)d\omega\right] \\
&= \frac{4}{\pi}
\end{aligned}
$$