Homework No. 6 Solution

1.

(1)

$$e^{-t}e^{jt}u(t)\xrightarrow{L}\frac{1}{s+1-j},\mathrm{Re}\{s\}>-1$$

$$e^{-t}e^{-jt}u(t)\xrightarrow{L}\frac{1}{s+1+j},\mathrm{Re}\{s\}>-1$$

$$e^{-t}\sin(t)u(t)=\frac{1}{2j}[e^{-t}e^{jt}-e^{-t}e^{-jt}]u(t)\xrightarrow{L}\frac{1}{(s+1)^2+1}$$

ROC: Re{s}> -1

(2)

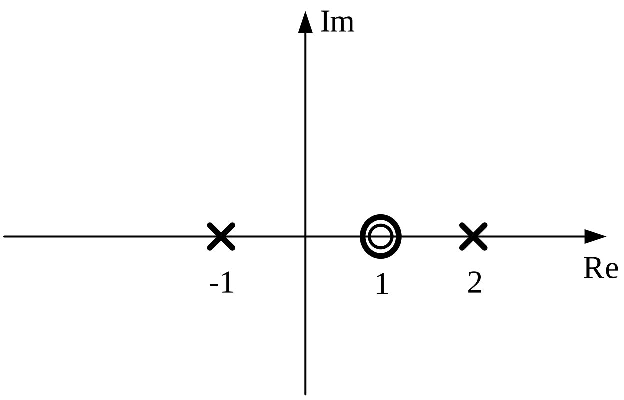
$$\underset{\text{right-sided}}{A(s)=\frac{1}{s}}\xleftrightarrow{\mathcal{L}}a(t)=u(t)$$

$$B(s)=e^{-3s}A(s)\xleftrightarrow{\mathcal{L}}b(t)=a(t-3)=u(t-3)$$

$$X(s)=\frac{d}{ds}B(s)\xleftrightarrow{\mathcal{L}}x(t)=-tb(t)=-tu(t-3)$$

2.

(1)



The possible ROCs are:

- ROC_1: $\mathrm{Re}\{s\}>2$
- ROC_2: $-1<\mathrm{Re}\{s\}<2$
- ROC_3: $\mathrm{Re}\{s\}<-1$

(2)

- ROC_1: $\mathrm{Re}\{s\}>2\Rightarrow$ causal, unstable.

- ROC2: $-1 < \text{Re}\{s\} < 2 \Rightarrow$ noncausal, stable.
- ROC3: $\text{Re}\{s\} < -1 \Rightarrow$ noncausal, unstable.

(3)

The inverse is $H_{inv}(s) = \dfrac{s^2 - s - 2}{s^2 - 2s + 1}$.

By long division, we obtain $H_{inv}(s) = 1 + \dfrac{s-3}{(s-1)^2}$.

By the partial-fraction expansion method, $H_{inv}(s) = 1 + \dfrac{1}{s-1} + \dfrac{(-2)}{(s-1)^2}$.

The inverse system is known to be stable, thus

$h_{inv}(t) = \delta(t) - e^t u(-t) + 2te^t u(-t)$.

3. Taking the unilateral Laplace transform of both sides of the given differential equation, we get

$$s^3Y(s) - s^2y(0^-) - sy'(0^-) - y''(0^-) + 6s^2Y(s) - 6sy(0^-)$$
$$-6y(0^-) + 11sY(s) - 11y(0^-) + 6Y(s) = X(s).$$

(1) For the zero state response, assume that all the initial conditions are zero. Furthermore, from the given $z(t)$ we may determine

$$X(\text{s}) = \frac{1}{s+4}, \qquad \text{Re}\{s\} > -4.$$

Then we have

$$Y(s)\{s^3 + 6s^2 + 11s + 6\} = \frac{1}{s+4}$$

Therefore,

$$Y(s) = \{s^3 + 6s^2 + 11s + 6\} = \frac{1}{(s+4)(s^3 + 6s^2 + 11s + 6)}$$

Taking the inverse unilateral Laplace transform of the partial fraction expansion of the above equation, we get

$$y(t) = \frac{1}{6}e^{-t}u(t) - \frac{1}{6}e^{-4t}u(t) - \frac{1}{2}e^{-2t}u(t) + \frac{1}{2}e^{-3t}u(t).$$

(2) For the zero-input response, we assume that $X(s) = 0$. Assuming that the initial conditions are as given, we obtain

$$Y(s) = \frac{s^2 + 5s + 6}{s^3 + 6s^2 + 11s + 6} = \frac{1}{s+1}.$$

Taking the inverse unilateral Laplace transform of the above equation, we get

$$y(t) = e^{-t}u(t).$$

(3) The total response is the sum of the zero-state and zero-input responses.

$$y(t) = \frac{7}{6}e^{-t}u(t) - \frac{1}{6}e^{-4t}u(t) - \frac{1}{2}e^{-2t}u(t) + \frac{1}{2}e^{-3t}u(t).$$

4. If $x(t) = e^{2t}$ produces $y(t) = (1/6)e^{2t}$, then $H(2)$=1/6. Also, by taking the Laplace transform of both sides of the given differential equation we get

$$H(s) = \frac{s + b(s+4)}{s(s+4)(s+2)}.$$

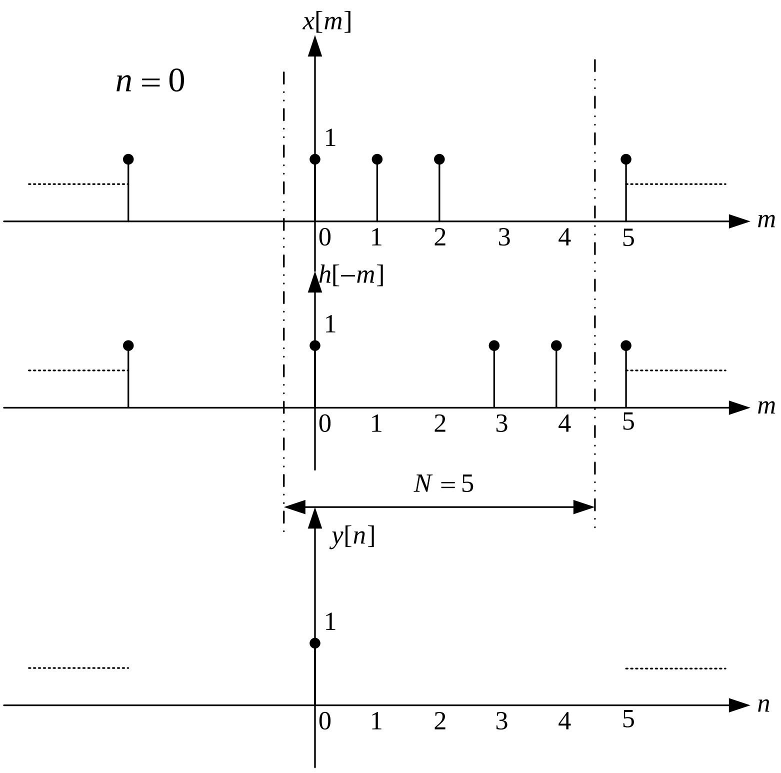
Since $H(2)$=1/6, we may deduce that b=1. Therefore

$$H(s) = \frac{2(s+2)}{s(s+4)(s+2)} = \frac{2}{s(s+4)}.$$

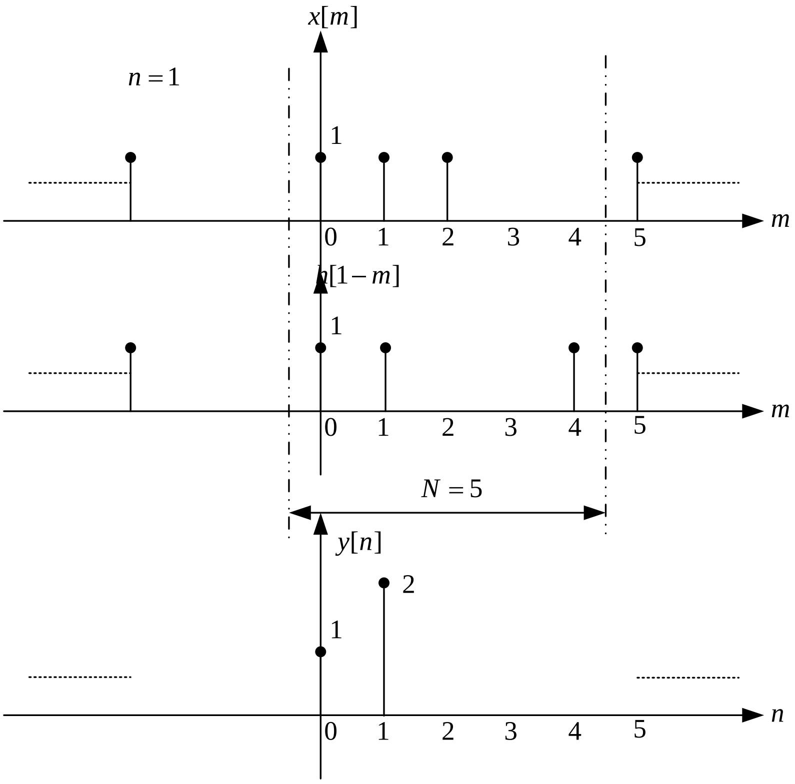
5.

(1) $N \geq 3+3-1=5$

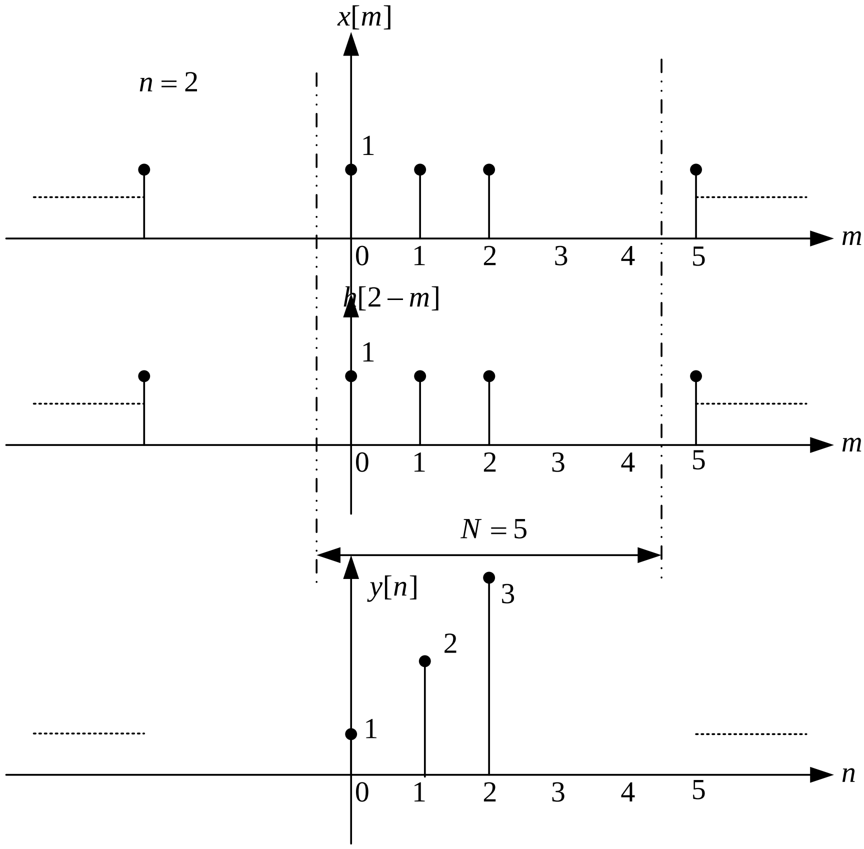
(2)



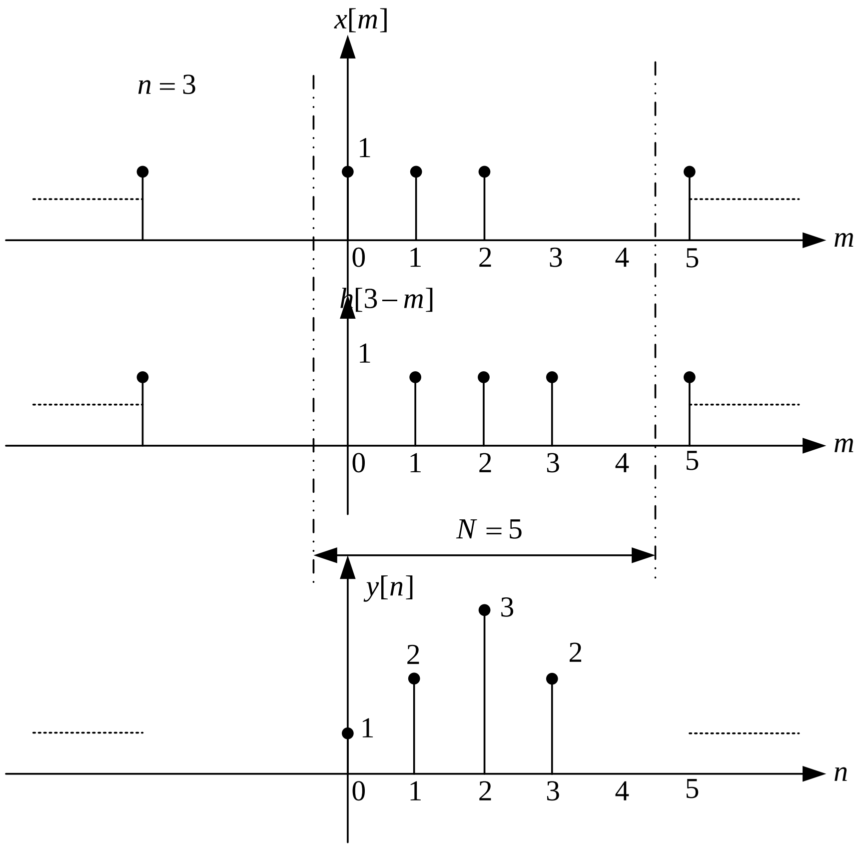
$n = 0 \Rightarrow y[0] = 1$



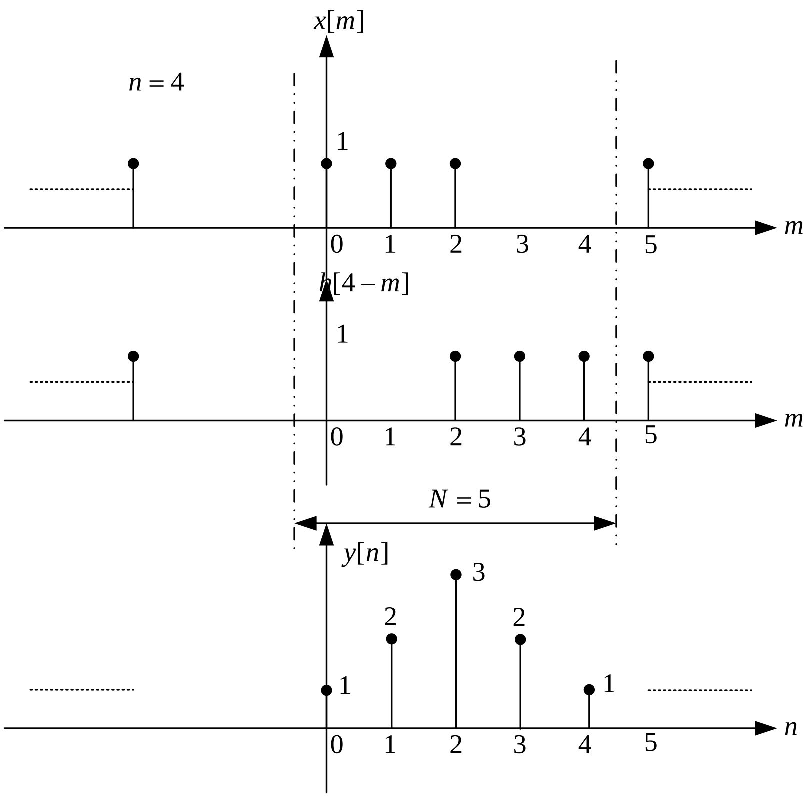
$n = 1 \Rightarrow y[1] = 2$



$n = 2 \Rightarrow y[2] = 3$



$n = 3 \Rightarrow y[3] = 2$



$n = 4 \Rightarrow y[4] = 1$

(3)

i) First, calculate DFT of $x[n]$ and $h[n]$, respectively.

ii) Multiply $X[k]$ and $H[k]$

iii) Finally, inverse $Y[k]$ and get the result