Homework #6

(Due by 17:20, December 29, 2016)

1. Determine the bilateral Laplace transform and the corresponding region of convergence (ROC) or the inverse Laplace transform for each of the following signals:

   (1) $x(t) = e^{-t}(\sin t)u(t)$.

   (2) $X(s) = \dfrac{d}{ds}\left(\dfrac{e^{-3s}}{s}\right)$ with ROC $\mathrm{Re}\{s\} > 0$.

2. Consider a continuous-time linear time-invariant (LTI) system with system function

$$H(s) = \frac{s^2 - 2s + 1}{s^2 - s - 2}.$$

   (1) Plot the poles and zeros of $H(s)$, and indicate all possible ROCs.

   (2) For each ROC identified in part (1), specify whether the associated system is stable and/or causal.

   (3) Determine the impulse response $h_{inv}(t)$ of the corresponding stable inverse system.

3. Consider a system S characterized by the differential equation

$$\frac{d^3y(t)}{dt^3} + 6\frac{d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 6y(t) = x(t).$$

   (1) Determine the zero-state response of this system for the input $x(t) = e^{-4t}u(t)$.

   (2) Determine the zero-input response of this system for $t > 0^-$, given that

$$y(0^-) = 1, \quad \left.\frac{dy(t)}{dt}\right|_{t=0^-} = -1, \quad \left.\frac{d^2y(t)}{dt^2}\right|_{t=0^-} = 1.$$

   (3) Determine the output of S when the input is $x(t) = e^{-4t}u(t)$ and the initial conditions are the same as those specified in (2).

4. A causal LTI system with impulse response $h(t)$ has the following properties:

   (1) When the input to the system is $x(t) = e^{2t}$ for all t, the output is $y(t) = (1/6)e^{2t}$ for all t.

   (2) The impulse response $h(t)$ satisfies the differential equation

$$\frac{dh(t)}{dt} + 2h(t) = (e^{-4t})u(t) + bu(t),$$

   where b is an unknown constant.

   Determine the constant b and the corresponding system function $H(s)$.

5. Suppose that we have two three-point sequences $x[n]$ and $h[n]$ as follows:

$$x[n] = h[n] = \begin{cases} 1, & 0 \le n \le 2 \\ 0, & \text{otherwise} \end{cases}.$$

Two periodic sequences $\tilde{x}[n]$ and $h[n]$ are constructed from $x[n]$ and $h[n]$ in the following way:

$$\tilde{x}[n] = \sum_{r=-\infty}^{\infty} x[n+Nr],\ h[n] = \sum_{r=-\infty}^{\infty} h[n+Nr].$$

(1) Let $\tilde{y}[n] = \tilde{x}[n] \circledast \tilde{h}[n]$ (periodic convolution). How should we choose N such that $y[n] = x[n] * h[n]$ (linear convolution) is equal to $\tilde{y}[n]$ for $0 \le n \le N-1$?

(2) Determine $y[n]$ by computing the periodic convolution of $\tilde{y}[n] = \tilde{x}[n] \circledast \tilde{h}[n]$ directly.

(3) Another method for determining $y[n]$ is through the use of the N-point discrete Fourier transforms of $x[n]$ and $h[n]$. Describe the corresponding procedure.