Homework No. 5 Solution

1.

(1)

$$X_0(j\omega)=\int_{-\infty}^{\infty}x(t)e^{-jwt}dt=\int_0^1 e^{-t}e^{-jwt}dt=\frac{1}{1+jw}\left(1-e^{-(1+jw)}\right)$$

$$x(t)=x_0(t)+x_0(-t)$$

$$X(j\omega)=X_0(j\omega)+X_0(-j\omega)$$

$$=\frac{1-e^{-(1+jw)}}{1+jw}+\frac{1-e^{-(1-jw)}}{1-jw}=\frac{2-2e^{-1}\cos w+2we^{-1}\sin w}{1+w^2}$$

(2)

$$令\ X_1(jw)=\begin{cases}0 & ,0<w<1\\ w-1 & ,1<w<2\\ 1 & ,2<w<3\\ 0 & ,o.w.\end{cases}$$

$$X(jw)=X_1(jw)-X_1(-jw)\ \Rightarrow\ x(t)=x_1(t)-x_1(-t)$$

$$x_1(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X_1(jw)e^{jwt}dw=\frac{1}{2\pi}\int_1^2(w-1)e^{jwt}dw+\frac{1}{2\pi}\int_2^3 e^{jwt}dw$$

$$=\frac{1}{2\pi}\left[\frac{1}{t^2}\left(e^{j2t}-e^{jt}\right)+\frac{1}{jt}e^{j3t}\right]$$

$$x(t)=x_1(t)-x_1(-t)=\frac{1}{2\pi}\left[\frac{1}{t^2}\left(e^{j2t}-e^{jt}\right)+\frac{1}{jt}e^{j3t}\right]-\frac{1}{2\pi}\left[\frac{1}{t^2}\left(e^{-j2t}-e^{-jt}\right)+\frac{1}{-jt}e^{-j3t}\right]$$

$$=\frac{\cos(3t)}{j\pi t}+\frac{\sin(t)-\sin(2t)}{j\pi t^2}$$

2.

(1)

$$(j\omega)^2 Z(j\omega)-(j\omega)Z(j\omega)-6Z(j\omega)=X(j\omega)$$

$$\Rightarrow H_A(j\omega)=\frac{Z(j\omega)}{X(j\omega)}=\frac{1}{(j\omega)^2-j\omega-6}=\frac{1}{(-3+j\omega)(2+j\omega)}=\frac{0.2}{(-3+j\omega)}+\frac{(-0.2)}{(2+j\omega)}$$

$$\Rightarrow h_A(t)=\frac{1}{5}\left[-e^{3t}u(-t)-e^{-2t}u(t)\right]$$

(2)

$$\frac{dy(t)}{dt}+6y(t)=\frac{dz(t)}{dt}+bz(t)$$

$$\Rightarrow H_B(j\omega)=\frac{Y(j\omega)}{Z(j\omega)}=\frac{(b+j\omega)}{(6+j\omega)}$$

In order to make the system causal, we have to cancel out the non-causal term, i.e. the first term of $H_A(j\omega)$, yields $b=-3$.

3.

(1) The frequency response is

$$H(j\omega)=\frac{Y(j\omega)}{X(j\omega)}=\frac{3(3+j\omega)}{(4+j\omega)(2+j\omega)}.$$

(2) Finding the partial fraction expansion of the answer of part (1) and taking its inverse Fourier transform, we have

$$h(t)=\frac{3}{2}[e^{-4t}+e^{-2t}]u(t).$$

(3) We have

$$\frac{Y(j\omega)}{X(j\omega)}=\frac{9+3j\omega}{8+6j\omega-\omega^2}\Rightarrow Y(j\omega)(8+6j\omega-\omega^2)=X(j\omega)(9+3j\omega).$$

Taking the inverse Fourier transform we obtain

$$\frac{d^2y(t)}{dt^2}+6\frac{dy(t)}{dt}+8y(t)=3\frac{dx(t)}{dt}+9x(t).$$

4.

(1)

$$\begin{aligned} x(t) &= \sin(2\pi t)e^{-t}u(t) \\ &= \frac{1}{2j}e^{j2\pi t}e^{-t}u(t)-\frac{1}{2j}e^{-j2\pi t}e^{-t}u(t) \end{aligned}$$

$$\begin{aligned} e^{-t}u(t) &\xleftrightarrow{FT} \frac{1}{1+j\omega} \\ e^{j2\pi t}s(t) &\xleftrightarrow{FT} S(j(\omega-2\pi)) \\ X(j\omega) &= \frac{1}{2j}\left[\frac{1}{1+j(\omega-2\pi)}-\frac{1}{1+j(\omega+2\pi)}\right] \end{aligned}$$

(2)

$$\frac{\sin(Wt)}{\pi t} \overset{FT}{\longleftrightarrow} \begin{cases} 1 & \omega \le W \\ 0, & \text{otherwise} \end{cases}$$

$$s_1(t)s_2(t) \overset{FT}{\longleftrightarrow} \frac{1}{2\pi} S_1(j\omega) * S_2(j\omega)$$

$$X(j\omega) = \begin{cases} 5 - \frac{|\omega|}{\pi} & \pi < |\omega| \le 5\pi \\ 4 & |\omega| \le \pi \\ 0 & \text{otherwise} \end{cases}$$

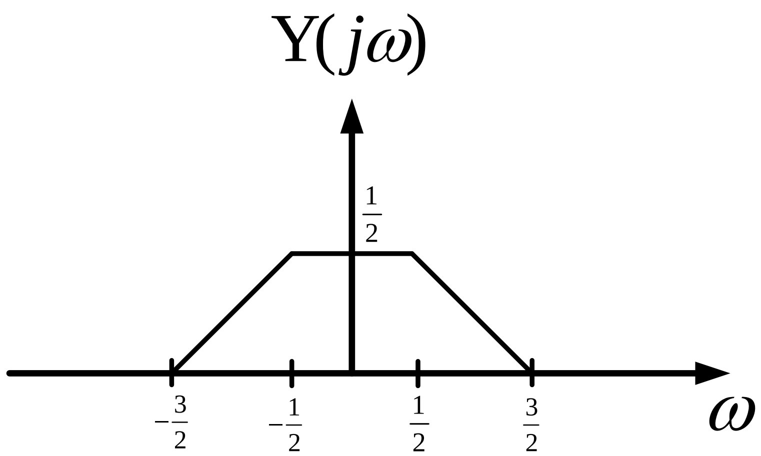
(3)

Since $\frac{1}{(1+j\omega)^2} \overset{F.T.}{\longleftrightarrow} te^{-t}u(t)$ and $j\omega S(\omega) \overset{F.T.}{\longleftrightarrow} \frac{d}{dt}s(t)$

$\therefore x(t) = \frac{d}{dt}\left[te^{-t}u(t)\right] = (1-t)e^{-t}u(t)$

5.

(1)



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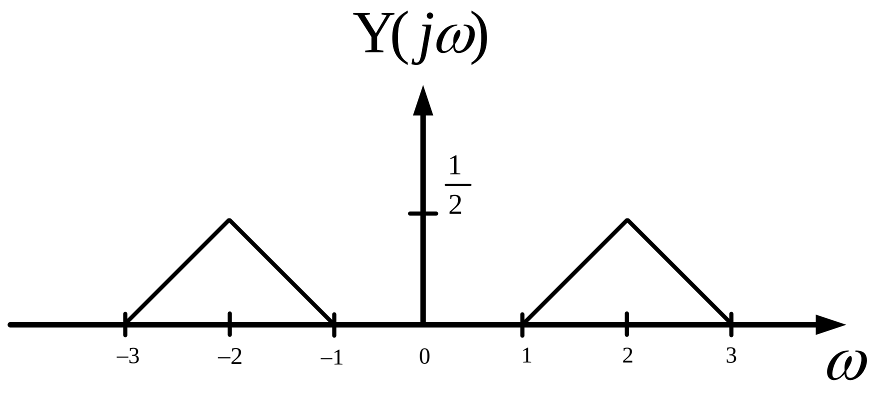

(3)