Homework No. 5

Due 17:20, Dec. 8, 2016

1. (1) Determine the Fourier transform of the signal $x(t)$ shown in Fig. 1, where

$$x_0(t) = \begin{cases} e^{-t}, & 0 \le t \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

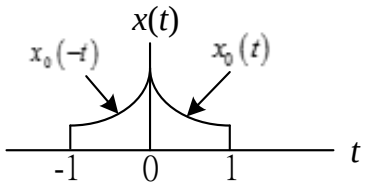


Fig. 1

(2) Determine the inverse Fourier transform of the signal shown in Fig. 2.

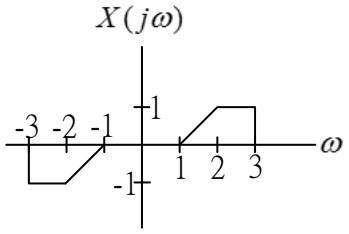
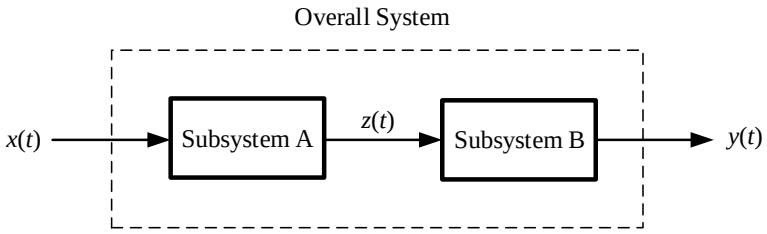


Fig. 2

2. Consider the following system:



The input-output relation of Subsystem A is given by

$$\frac{d^2 z(t)}{dt^2} - \frac{dz(t)}{dt} - 6z(t) = x(t)$$

and the input-output relation of Subsystem B is given by

$$\frac{dy(t)}{dt} + 6y(t) = \frac{dz(t)}{dt} + bz(t).$$

(1) Determine the frequency response and the impulse response of Subsystem A.

(2) Determine b such that the overall system is causal. Justify your answer.

3. Consider a linear time-invariant system whose response to the input

$$x(t) = [e^{-t} + e^{-3t}]u(t)$$

is

$$y(t) = [2e^{-t} - 2e^{-4t}]u(t).$$

(1) Find the frequency response of this system.

(2) Determine the system's impulse response.

(3) Find the differential equation relating the input and the output of this system.

4. Find the Fourier transform or the inverse Fourier transform of each of the following signals:

(1) $x(t) = \sin(2\pi t)e^{-t}u(t)$; (2) $x(t) = \left[\frac{\sin(2\pi t)}{\pi t}\right]\left[\frac{2\sin(3\pi t)}{\pi t}\right]$; (3) $X(\omega) = \frac{j\omega}{(1+j\omega)^2}$.

5. Consider a continuous-time signal $x(t)$ with Fourier transform $X(j\omega)$ shown in Fig. 3. Let $y(t)=x(t)p(t)$. Sketch the spectrum of $y(t)$ for each of the following cases:

(a) $p(t) = \cos(t/2)$; (b) $p(t) = \cos(2t)$; (c) $p(t) = \sum_{k=-\infty}^{+\infty} \delta\left(t - \frac{3}{2}k\right)$.

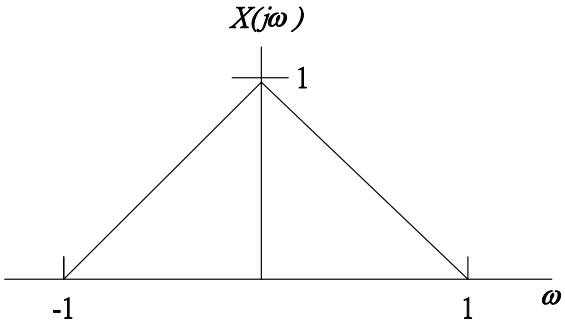


Fig. 3