# Homework No. 4 Solution

**1.**

**(a)** fundamental frequency $w_0 = \frac{2\pi}{6} = \frac{\pi}{3}$

**(b)**

$$x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right)$$
$$= 2 + \frac{1}{2}e^{j\left(\frac{2\pi}{3}t\right)} + \frac{1}{2}e^{j\left(\frac{-2\pi}{3}t\right)} + 2j\,e^{j\left(\frac{-5\pi}{3}t\right)} - 2j\,e^{j\left(\frac{5\pi}{3}t\right)}$$
$$= 2 + \frac{1}{2}e^{j2\left(\frac{2\pi}{6}t\right)} + \frac{1}{2}e^{-j2\left(\frac{2\pi}{6}t\right)} + 2j\,e^{-j5\left(\frac{2\pi}{6}t\right)} - 2j\,e^{j5\left(\frac{2\pi}{6}t\right)}$$

$$\rightarrow a_0 = 2,\quad a_2 = a_{-2} = \frac{1}{2},\quad a_5 = -2j,\quad a_{-5} = 2j$$

**2.**

**(a)**

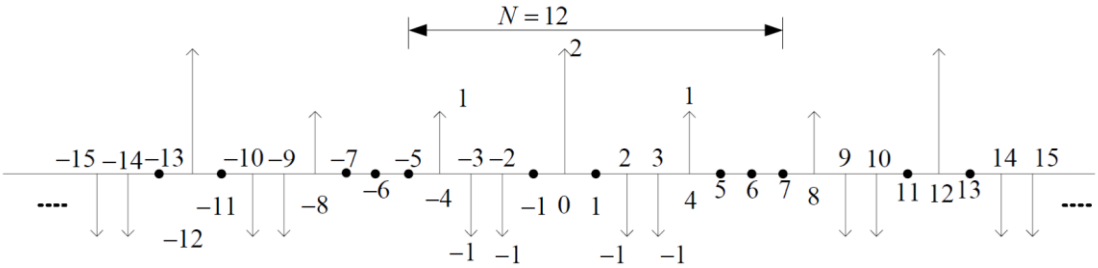
$$x[n] = \cos^2\left(\frac{6\pi}{17}n + \frac{\pi}{3}\right) = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{12\pi}{17}n + \frac{2\pi}{3}\right) = x_1[n] + x_2[n]$$

$$\left.\begin{aligned} &x_1[n] = \frac{1}{2} \Rightarrow N_1 = 1 \\ &x_2[n] = \frac{1}{2}\cos\left(\frac{12\pi}{17}n + \frac{2\pi}{3}\right) \Rightarrow N_2 = 2\pi \Big/ \frac{12\pi}{17} = \frac{17}{6} \end{aligned}\right\} \Rightarrow \therefore N = 17$$

$$x[n] = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{12\pi}{17}n + \frac{2\pi}{3}\right) = \frac{1}{2} + \frac{1}{4}\left[e^{j\left(\frac{12\pi}{17}n + \frac{2\pi}{3}\right)} + e^{-j\left(\frac{12\pi}{17}n + \frac{2\pi}{3}\right)}\right]$$
$$= \frac{1}{2} + \frac{1}{4}\left[e^{j\frac{2\pi}{3}}e^{j6\frac{2\pi}{17}n} + e^{-j\frac{2\pi}{3}}e^{-j6\frac{2\pi}{17}n}\right]$$

$$a_k = \begin{cases} \frac{1}{2},\ k = 0 \\ \frac{1}{4}e^{j\frac{2\pi}{3}},\ k = 6 \\ \frac{1}{4}e^{-j\frac{2\pi}{3}},\ k = -6 \\ 0,\ \text{otherwise on } k = \{-8, -7, \ldots, 8\} \end{cases}$$

**(b)** $x[n]=\sum_{m=-\infty}^{\infty}(-1)^m\left(\delta[n-2m]+\delta[n+3m]\right)$




$N=12$

$$a_k=\frac{1}{12}\sum_{n=-5}^{6}x[n]e^{-jk\frac{\pi}{6}n}$$

$$=\frac{1}{12}\left[e^{-j(-4)\frac{\pi}{6}k}-e^{-j(-3)\frac{\pi}{6}k}-e^{-j(-2)\frac{\pi}{6}k}+2-e^{-j(2)\frac{\pi}{6}k}-e^{-j(3)\frac{\pi}{6}k}+e^{-j(4)\frac{\pi}{6}k}\right]$$

$$=\frac{1}{6}\left[\cos\left(\frac{2\pi}{3}k\right)-\cos\left(\frac{\pi}{2}k\right)-\cos\left(\frac{\pi}{3}k\right)+1\right]$$

**3.**

$$a_k=\frac{1}{2}\int_{-1}^{1}\mathrm{e}^{-t}\mathrm{e}^{-j2\pi k\frac{1}{2}t}dt$$

$$=\frac{1}{2}\int_{-1}^{1}\mathrm{e}^{-(1+j\pi k)t}dt$$

$$=\frac{1}{2}\frac{\left(\mathrm{e}^{-(1+j\pi k)}-\mathrm{e}^{(1+j\pi k)}\right)}{-(1+j\pi k)}$$

$$=\frac{1}{2}\frac{\mathrm{e}^{(1+j\pi k)}-\mathrm{e}^{-(1+j\pi k)}}{1+j\pi k}$$

$$=\frac{1}{2}\frac{(-1)^k\left(\mathrm{e}-\mathrm{e}^{-1}\right)}{1+j\pi k}$$

$$\rightarrow x(t)=\sum_{k=-\infty}^{\infty}a_k e^{jk\pi t}$$

**4.**

**(a)**

$$x[n]=\sum_{k=-\infty}^{+\infty}\delta[n-8k]=\sum_{k=<N>}a_k e^{j2\pi\frac{kn}{N}}$$

$$a_k=\frac{1}{N}\sum_{n=<N>}x[n]e^{-j2\pi\frac{kn}{N}}$$
$$=\frac{1}{8}\sum_{n=0}^{7}x[n]e^{-j2\pi\frac{kn}{N}}$$
$$=\frac{1}{8}$$

$$x[n]=\sum_{k=<N>}\frac{1}{8}e^{j\pi\frac{kn}{4}}$$

**(b)**

$$x[n]=\sum_{k=<N>}\frac{1}{8}e^{j\pi\frac{kn}{4}}$$
$$\rightarrow y[n]=\sum_{k=<N>}\frac{1}{8}H(e^{j\pi\frac{k}{4}})e^{j\pi\frac{kn}{4}}$$
$$=1+\sin(\frac{9\pi}{4}n+\frac{\pi}{4})+\cos(\frac{5\pi}{2}n+\frac{\pi}{4})$$
$$=1+\left(\frac{-j}{2}e^{j\frac{\pi}{4}}\right)e^{j\frac{\pi}{4}n}+\left(\frac{j}{2}e^{-j\frac{\pi}{4}}\right)e^{-j\frac{\pi}{4}n}+\left(\frac{1}{2}e^{j\frac{\pi}{4}}\right)e^{j\frac{\pi}{2}n}+\left(\frac{1}{2}e^{-j\frac{\pi}{4}}\right)e^{-j\frac{\pi}{2}n}$$

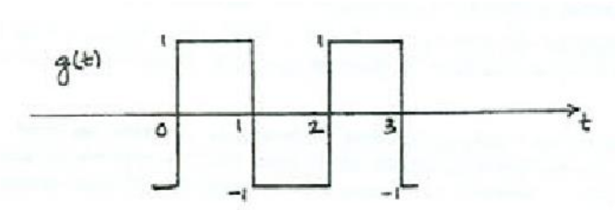
$$\rightarrow H(e^{j\pi\frac{k}{4}})|_{k=0}=H(e^{j\pi\frac{0}{4}})=8$$
$$H(e^{j\pi\frac{k}{4}})|_{k=1}=H(e^{j\pi\frac{1}{4}})=-4je^{j\frac{\pi}{4}}$$
$$H(e^{j\pi\frac{k}{4}})|_{k=-1}=H(e^{j\pi\frac{-1}{4}})=4je^{-j\frac{\pi}{4}}$$
$$H(e^{j\pi\frac{k}{4}})|_{k=2}=H(e^{j\pi\frac{1}{2}})=4e^{j\frac{\pi}{4}}$$
$$H(e^{j\pi\frac{k}{4}})|_{k=-2}=H(e^{j\pi\frac{-1}{2}})=4e^{-j\frac{\pi}{4}}$$
$$H(e^{j\pi\frac{k}{4}})|_{k=\pm 3}=H(e^{j\pi\frac{\pm 3}{4}})=0$$

**5.**

**(a)**

$$a_0 = \frac{1}{2}\int_0^1 t\,dt + \frac{1}{2}\int_1^2 (2-t)\,dt = \frac{1}{2}$$

**(b)**




$$b_0 = \frac{1}{2}\int_0^1 dt - \frac{1}{2}\int_1^2 dt = 0$$

and

$$b_k = \frac{1}{2}\int_0^1 e^{-j\pi kt}dt - \frac{1}{2}\int_1^2 e^{-j\pi kt}dt$$

$$= \frac{1}{j\pi k}\left(1-e^{-j\pi k}\right)$$

**(c)**

$$g(t) = \frac{dx(t)}{dt} \Leftrightarrow b_k = j\pi k a_k$$

$$\rightarrow a_k = \frac{b_k}{j\pi k} = \frac{-1}{\pi^2 k^2}\left(1-e^{-j\pi k}\right) \quad ;k \neq 0$$

$$\rightarrow a_k = \begin{cases} 1/2 & ;k=0 \\ \dfrac{-1}{\pi^2 k^2}\left(1-e^{-j\pi k}\right) & ;k \neq 0 \end{cases}$$