Homework No. 4

Due 17:20, Nov. 24, 2016

1. Consider the following continuous-time periodic signal:

$$x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right).$$

(a) Determine the fundamental frequency of the signal.

(b) Determine the Fourier series coefficients of the signal.

2. Determine the discrete-time Fourier series representation for each of the following signals:

(a) $x[n] = \cos^2\left(\frac{6\pi}{17}n + \frac{\pi}{3}\right).$

(b) $x[n] = \sum_{m=-\infty}^{\infty} (-1)^m \left(\delta[n-2m] + \delta[n+3m]\right).$

3. Consider a continuous-time periodic signal $x(t)$ with fundamental period equal to 2, where

$$x(t) = e^{-t} \quad \text{for} \quad -1 < t < 1.$$

Determine the corresponding Fourier series representation.

4. Consider a discrete-time linear time-invariant system with impulse response $h[n]$, frequency response $H(e^{j\Omega})$, input $x[n]$, and output $y[n]$ given as follows:

$$H(e^{j\Omega}) = \sum_{-\infty}^{+\infty} h[k]e^{-jk\Omega}, \quad x[n] = \sum_{-\infty}^{+\infty} \delta[n-8k], \text{ and } y[n] = 1 + \sin\left(\frac{9\pi}{4}n + \frac{\pi}{4}\right) + \cos\left(\frac{5\pi}{2}n + \frac{\pi}{4}\right).$$

(a) Determine the Fourier series representation of $x[n]$.

(b) Determine the values of $H(e^{jk\pi/4})$ for $k = 0, \pm1, \pm2$, and ±3.

5. Let

$$x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2-t, & 1 \le t \le 2 \end{cases}.$$

be a periodic signal with fundamental period $T = 2$ and Fourier coefficients a_k.

(a) Determine the value of a_0.

(b) Determine the Fourier series representation of $dx(t)/dt$.

(c) Use the result of part (b) and the differentiation property of the continuous-time Fourier series to help determine the Fourier series coefficients of $x(t)$.