EE 361002 Signal and System HW13 Answer

7.21

(a) The Nyquist rate for the given signal is $2 \times 5000\pi = 10000\pi$. Therefore, in order to be able to recover $x(t)$ from $x_p(t)$, the sampling period must at most be $T_{max} = \frac{2\pi}{10000\pi} = 2 \times 10^{-4}$ sec. Since the sampling period used is $T = 10^{-4} < T_{max}$, $x(t)$ can be recovered from $x_p(t)$.

(b) The Nyquist rate for the given signal is $2 \times 15000\pi = 30000\pi$. Therefore, in order to be able to recover $x(t)$ from $x_p(t)$, the sampling period must at most be $T_{max} = \frac{2\pi}{30000\pi} = 0.66 \times 10^{-4}$ sec. Since the sampling period used is $T = 10^{-4} > T_{max}$, $x(t)$ cannot be recovered from $x_p(t)$.

(c) Here, $\mathcal{Im}\{X(j\omega)\}$ is not specified. Therefore, the Nyquist rate for the signal $x(t)$ is indeterminate. This implies that one cannot guarantee that $x(t)$ would be recoverable from $x_p(t)$.

(d) Since $x(t)$ is real, we may conclude that $X(j\omega) = 0$ for $|\omega| > 5000$. Therefore, the answer to this part is identical to that of part (a).

(e) Since $x(t)$ is real, $X(j\omega) = 0$ for $|\omega| > 15000\pi$. Therefore, the answer to this part is identical to that of part (b).

(f) If $X(j\omega) = 0$ for $|\omega| > \omega_1$, then $X(j\omega) * X(j\omega) = 0$ for $|\omega| > 2\omega_1$. Therefore, in this part, $X(j\omega) = 0$ for $|\omega| > 7500\pi$. The Nyquist rate for this signal is $2 \times 7500\pi = 15000\pi$. Therefore, in order to be able to recover $x(t)$ from $x_p(t)$, the sampling period must at most be $T_{max} = \frac{2\pi}{15000\pi} = 1.33 \times 10^{-4}$ sec. Since the sampling period used is $T = 10^{-4} < T_{max}$, $x(t)$ can be recovered from $x_p(t)$.

(g) If $|X(j\omega)| = 0$ for $\omega > 5000\pi$, then $X(j\omega) = 0$ for $\omega > 5000\pi$. Therefore, the answer to this part is identical to the answer of part (a).

7.23

(a) We may express $p(t)$ as

$$p(t) = p_1(t) - p_1(t-\Delta),$$

where $p_1(t) = \sum_{k=-\infty}^{\infty} \delta(t - k2\Delta)$. Now,

$$P_1(j\omega) = \frac{\pi}{\Delta} \sum_{k=-\infty}^{\infty} \delta(\omega - \pi/\Delta).$$

Therefore,

$$P(j\omega) = P_1(j\omega) - e^{-j\omega\Delta} P_1(j\omega)$$

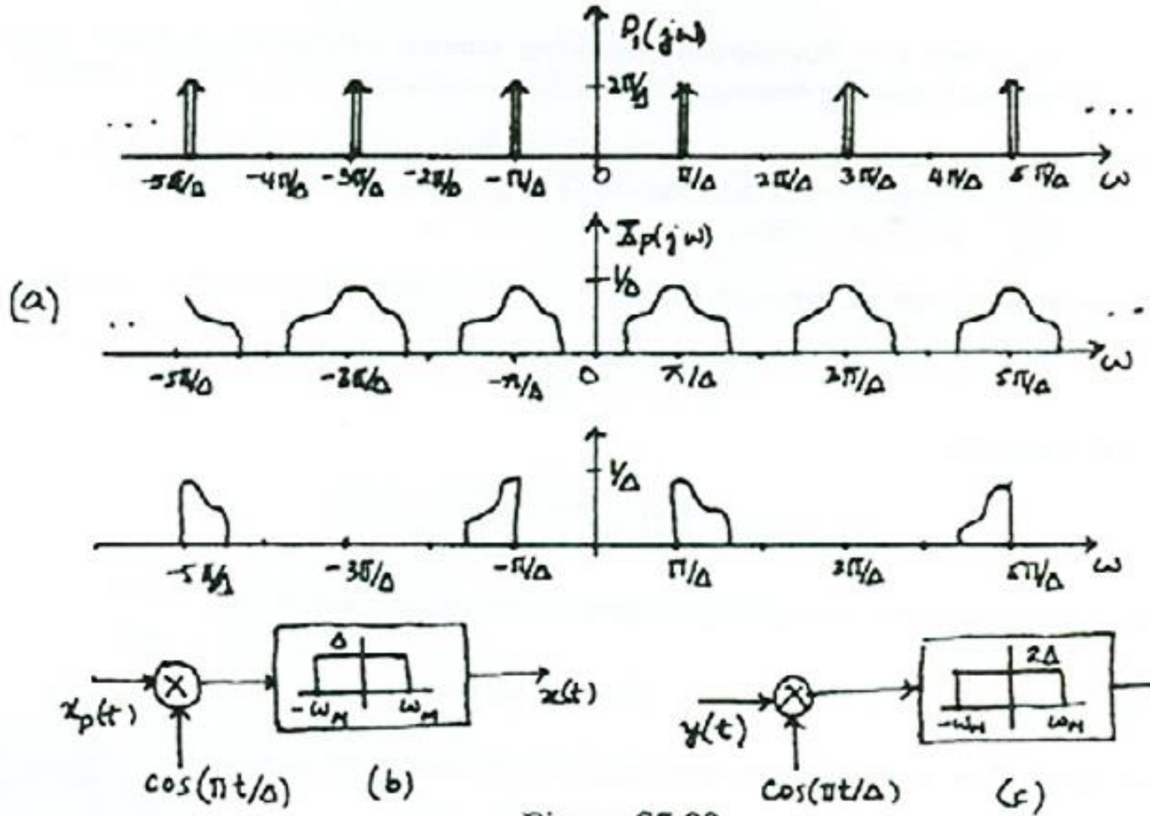

Figure S7.23

is as shown in Figure S7.23.

Now,

$$X_p(j\omega) = \frac{1}{2\pi}\left[X(j\omega) * P(j\omega)\right].$$

Therefore, $X_p(j\omega)$ is as sketched below for $\Delta < \pi/(2\omega_M)$. The corresponding $Y(j\omega)$ is also sketched in Figure S7.23.

(b) The system which can be used to recover $x(t)$ from $x_p(t)$ is as shown in Figure S7.23.

(c) The system which can be used to recover $x(t)$ from $x(t)$ is as shown in Figure S7.23.

(d) We see from the figures sketched in part (a) that aliasing is avoided when $\omega_M \le \pi/\Delta$. Therefore, $\Delta_{max} = \pi/\omega_M$.

7.25

Here, $x_r(kT)$ can be written as

$$x_r(kT) = \sum_{n=-\infty}^{\infty} x(nT)\frac{\sin[\pi(k-n)]}{\pi(k-n)}.$$

Note that when $n \neq k$,

$$\frac{\sin[\pi(k-n)]}{\pi(k-n)} = 0$$

and when $n = k$,

$$\frac{\sin[\pi(k-n)]}{\pi(k-n)} = 1.$$

Therefore,

$$x_r(kT) = x(kT).$$

7.29

From Section 7.1.1 we know that

$$X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega - k2\pi/T)).$$

$X(e^{j\omega})$, $Y(e^{j\omega})$, $Y_p(j\omega)$, and $Y_c(j\omega)$ are as shown in Figure S7.29.

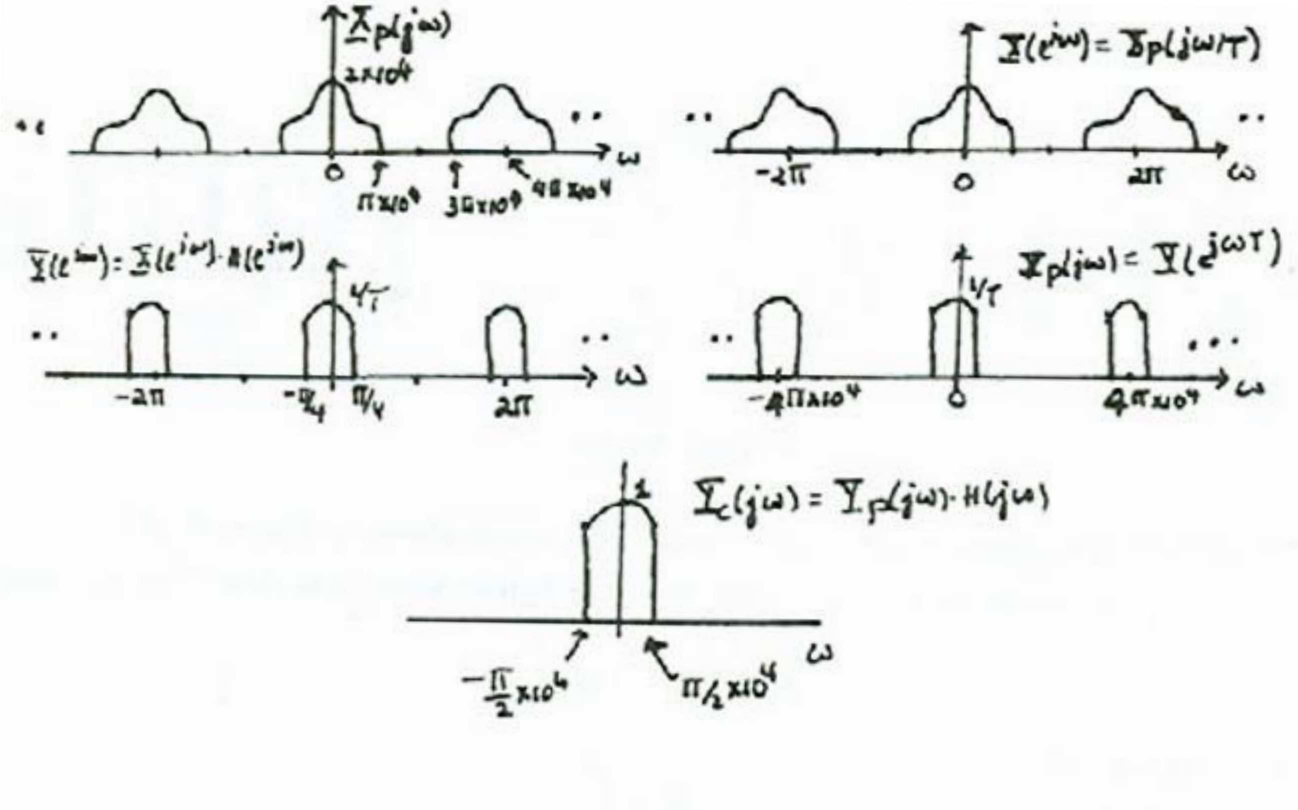

Figure S7.29