2018 Systems and Signals HW8

Deadline: 5/31 after class

7.21. A signal $x(t)$ with Fourier transform $X(j\omega)$ undergoes impulse-train sampling to generate

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t - nT)$$

where $T = 10^{-4}$. For each of the following sets of constraints on $x(t)$ and/or $X(j\omega)$, does the sampling theorem (see Section 7.1.1) guarantee that $x(t)$ can be recovered exactly from $x_p(t)$?

(a) $X(j\omega) = 0$ for $|\omega| > 5000\pi$
(b) $X(j\omega) = 0$ for $|\omega| > 15000\pi$
(c) $\mathcal{Re}\{X(j\omega)\} = 0$ for $|\omega| > 5000\pi$
(d) $x(t)$ real and $X(j\omega) = 0$ for $\omega > 5000\pi$
(e) $x(t)$ real and $X(j\omega) = 0$ for $\omega < -15000\pi$
(f) $X(j\omega) * X(j\omega) = 0$ for $|\omega| > 15000\pi$
(g) $|X(j\omega)| = 0$ for $\omega > 5000\pi$

7.23. Shown in Figure P7.23 is a system in which the sampling signal is an impulse train with alternating sign. The Fourier transform of the input signal is as indicated in the figure.

(a) For $\Delta < \pi/(2\omega_M)$, sketch the Fourier transform of $x_p(t)$ and $y(t)$.
(b) For $\Delta < \pi/(2\omega_M)$, determine a system that will recover $x(t)$ from $x_p(t)$.
(c) For $\Delta < \pi/(2\omega_M)$, determine a system that will recover $x(t)$ from $y(t)$.
(d) What is the *maximum* value of Δ in relation to ω_M for which $x(t)$ can be recovered from either $x_p(t)$ or $y(t)$?

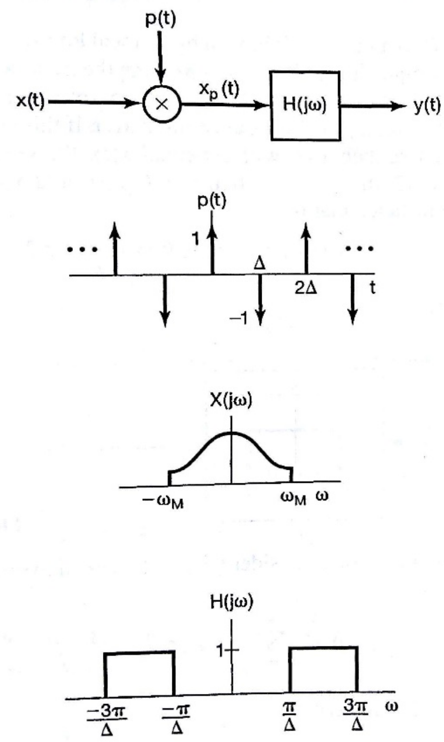

Figure P7.23

7.25. In Figure P7.25 is a sampler, followed by an ideal lowpass filter, for reconstruction of $x(t)$ from its samples $x_p(t)$. From the sampling theorem, we know that if $\omega_s = 2\pi/T$ is greater than twice the highest frequency present in $x(t)$ and $\omega_c = \omega_s/2$, then the reconstructed signal $x_r(t)$ will exactly equal $x(t)$. If this condition on the bandwidth of $x(t)$ is violated, then $x_r(t)$ will *not* equal $x(t)$. We seek to show in this problem that if $\omega_c = \omega_s/2$, then for any choice of T, $x_r(t)$ and $x(t)$ will always be equal at the sampling instants; that is,

$$x_r(kT) = x(kT),\ k = 0, \pm 1, \pm 2, \ldots.$$

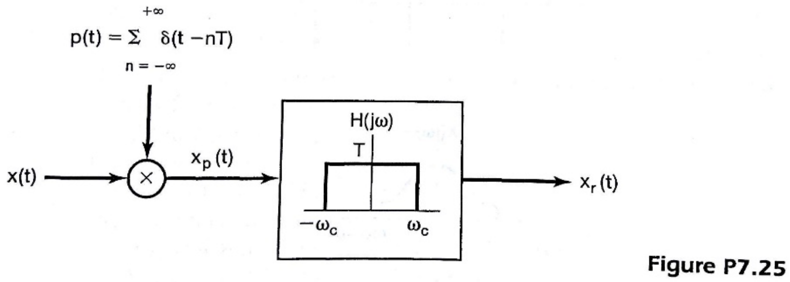

Figure P7.25

To obtain this result, consider eq. (7.11), which expresses $x_r(t)$ in terms of the samples of $x(t)$:

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT)T\frac{\omega_c}{\pi}\frac{\sin[\omega_c(t-nT)]}{\omega_c(t-nT)}.$$

With $\omega_c = \omega_s/2$, this becomes

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT)\frac{\sin\left[\frac{\pi}{T}(t-nT)\right]}{\frac{\pi}{T}(t-nT)}. \quad \text{(P7.25–1)}$$

By considering the values of α for which $[\sin(\alpha)]/\alpha = 0$, show from eq. (P7.25–1) that, without any restrictions on $x(t)$, $x_r(kT) = x(kT)$ for any integer value of k.

7.29. Figure P7.29(a) shows the overall system for filtering a continuous-time signal using a discrete-time filter. If $X_c(j\omega)$ and $H(e^{j\omega})$ are as shown in Figure P7.29(b), with $1/T = 20$ kHz, sketch $X_p(j\omega)$, $X(e^{j\omega})$, $Y(e^{j\omega})$, $Y_p(j\omega)$, and $Y_c(j\omega)$.

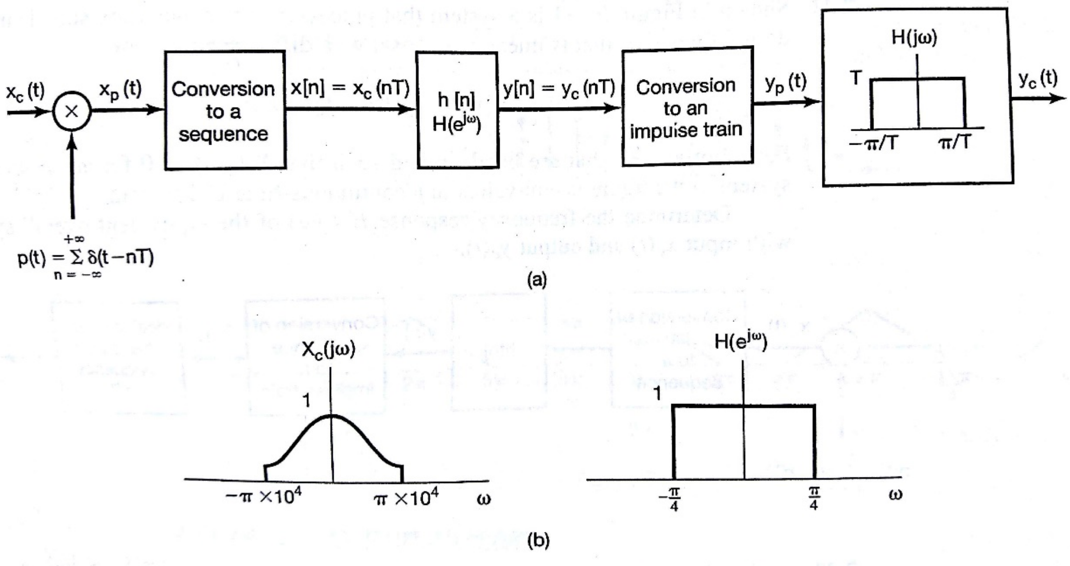

Figure P7.29