2018 Systems and Signals HW7 (No need to submit)

Hw7: 6.21, 6.22, 6.25, 6.27

6.21. A causal LTI filter has the frequency response $H(j\omega)$ shown in Figure P6.21. For each of the input signals given below, determine the filtered output signal $y(t)$.

(a) $x(t) = e^{jt}$ **(b)** $x(t) = (\sin \omega_0 t)u(t)$

(c) $X(j\omega) = \frac{1}{(j\omega)(6+j\omega)}$ **(d)** $X(j\omega) = \frac{1}{2+j\omega}$

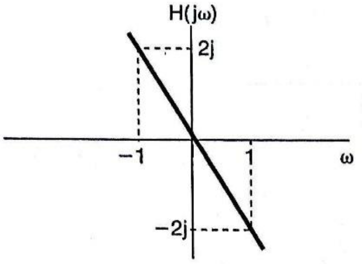


Figure P6.21

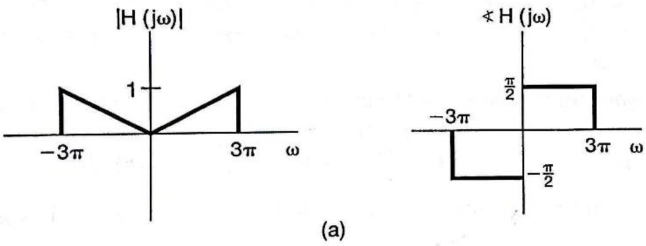
6.22. Shown in Figure P6.22(a) is the frequency response $H(j\omega)$ of a continuous-time filter referred to as a lowpass differentiator. For each of the input signals $x(t)$ below, determine the filtered output signal $y(t)$.

(a) $x(t) = \cos(2\pi t + \theta)$

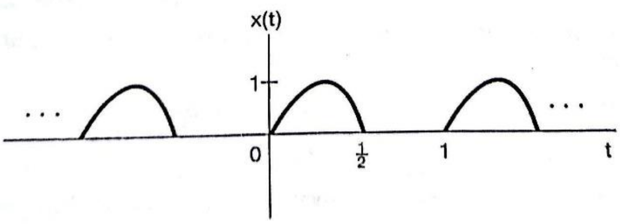
(b) $x(t) = \cos(4\pi t + \theta)$

(c) $x(t)$ is a half-wave rectified sine wave of period, as sketched in Figure P6.22(b).

$$x(t) = \begin{cases} \sin 2\pi t, & m \leq t \leq (m + \frac{1}{2}) \\ 0, & (m + \frac{1}{2}) \leq t \leq m \text{ for any integer } m \end{cases}$$



(a)



(b)

Figure P6.22

6.25. By computing the group delay at two selected frequencies, verify that each of the following frequency responses has nonlinear phase.

(a) $H(j\omega) = \frac{1}{j\omega+1}$ **(b)** $H(j\omega) = \frac{1}{(j\omega+1)^2}$ **(c)** $H(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)}$

6.27. The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t).$$

(a) Determine the frequency response

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

of the system, and sketch its Bode plot.

(b) Specify, as a function of frequency, the group delay associated with this system.

(c) If $x(t) = e^{-t}u(t)$, determine $Y(j\omega)$, the Fourier transform of the output.

(d) Using the technique of partial-fraction expansion, determine the output $y(t)$ for the input $x(t)$ in part (c).