2018 System and Signal HW10 solution

10.21

10.21

(b) $\delta[n-5]$, By $\delta[n-m] \xleftrightarrow{z} z^{-m}$

$X(z) = z^{5}$, for all z except 0

FT. exist

(d)

$x[n] = (\frac{1}{2})^{n+1} u[n+3]$

$X(z) = \sum_{n=-3}^{\infty} (\frac{1}{2})^{n+1} \cdot z^{-n}$

$= \frac{(\frac{1}{2})^{-2} \cdot z^3}{1 - \frac{1}{2} \cdot \frac{1}{z}}$

$= \frac{4z^3}{1 - \frac{1}{2z}}$, $|\frac{1}{2z}| < 1$

$\Rightarrow |z| > \frac{1}{2}$ FT exist

(e)

$x[n] = (-\frac{1}{3})^n u[-n-2]$

$X(z) = \sum_{n=-\infty}^{-2} x[n] z^{-n}$

$= \frac{(-\frac{1}{3})^{-2} z^2}{1-(-\frac{1}{3})^{-1} z}$

$= \frac{9z^2}{1+3z}$, $|z| < \frac{1}{3}$

FT NOT exist

(g)

$x[n] = 2^n u[-n] + (\frac{1}{4})^n u[n-1]$

分開算

$X(z) = \sum_{n=-\infty}^{0} 2^n z^{-n} + \sum_{n=1}^{\infty} (\frac{1}{4})^n z^{-n}$

$= \frac{1}{1-\frac{1}{2}z} + \frac{\frac{1}{4} z^{-1}}{1-\frac{1}{4} z^{-1}}$

$|z| < 2$ $\quad |z| > \frac{1}{4} \Rightarrow \frac{1}{4} < |z| < 2$, FT exist

(h)

$x[n] = (\frac{1}{3})^{n-2} u[n-2]$

$X(z) = \sum_{n=2}^{\infty} x[n] z^{-n}$

$= \frac{(\frac{1}{3})^0 z^{-2}}{1 - (\frac{1}{3}) z^{-1}}$

$= \frac{\frac{1}{3z^2}}{1 - \frac{1}{3z}}$

$|z| > \frac{1}{3}$, FT exist.

10.22.

(b)

$$x[n] = n\left(\frac{1}{2}\right)^{|n|}$$

令 $x_1[n] = \left(\frac{1}{2}\right)^{|n|}$

$$= \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{2}\right)^{-n} u[-n-1]$$

$$\Rightarrow X_1(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{\frac{1}{2}z}{1-\frac{1}{2}z},$$

$$\Rightarrow \frac{1}{2} < |z| < 2.$$

$$x[n] = n\, x_1[n]$$

$$X(z) = -z\frac{dX_1(z)}{dz}$$

$$= -z \times \left[\frac{2(2z)^{-2}}{(1-\frac{1}{2z})^2} - \frac{\frac{2}{z^2}}{(1-\frac{2}{z})^2}\right]$$

$$= \frac{-\frac{1}{2z}}{(1-\frac{1}{2z})^2} + -\frac{2/z}{(1-\frac{2}{z})^2}$$

$\frac{1}{2} < |z| < 2$, FT exist.

(d)

$$x[n] = 4^n \cos\left[\frac{2\pi}{6}n + \frac{\pi}{4}\right] u[-n-1]$$

$$= 4^n \left\{\frac{e^{j(\frac{2\pi}{6}n+\frac{\pi}{4})} + e^{-j(\frac{2\pi}{6}n+\frac{\pi}{4})}}{2}\right\} u[-n-1]$$

$$= \frac{e^{j\frac{\pi}{4}}}{2}\cdot\left[4^n u[-n-1]\cdot e^{j\frac{2\pi}{6}n}\right] + \frac{e^{-j\frac{\pi}{4}}}{2}\cdot\left[4^n u[-n-1]\cdot e^{-j\frac{2\pi}{6}n}\right]$$

By $e^{j\omega_0 n}x[n] \xleftrightarrow{z} X(e^{-j\omega_0 n}z)$

$$4^n u[-n-1] \xleftrightarrow{z} \frac{\frac{z}{4}}{1-\frac{z}{4}} = \frac{1}{\frac{4}{z}-1}$$

$$X(z) = \frac{e^{j\frac{\pi}{4}}}{2} \times \frac{1}{4(e^{-j\frac{2\pi}{6}n}z)^{-1}-1} + \frac{e^{-j\frac{\pi}{4}}}{2} \times \frac{1}{4(e^{j\frac{2\pi}{6}n}z)^{-1}-1}$$

$|z| < 4$, FT exist.

10.25

(a)

$$X(z)=\frac{1}{(1-\frac{1}{2}z^{-1})(1-z^{-1})}=\frac{a}{1-\frac{1}{2}z^{-1}}+\frac{b}{1-z^{-1}}$$

$$a=\left.\frac{1}{1-z^{-1}}\right|_{z^{-1}=2}=-1,\quad b=\left.\frac{1}{1-\frac{1}{2}z^{-1}}\right|_{z^{-1}=1}=2$$

$$\Rightarrow X(z)=\frac{-1}{1-\frac{1}{2}z^{-1}}+\frac{2}{1-z^{-1}}$$

$$\mathcal{Z}\Big\downarrow$$

$$\underline{x[n]=-\left(\tfrac{1}{2}\right)^n u[n]+2u[n]}\ \#$$

(b)

$$X(z)=\frac{z^2}{(z-\frac{1}{2})(z-1)}=z^2\left(\frac{a}{z-\frac{1}{2}}+\frac{b}{z-1}\right)$$

$$a=\left.\frac{1}{z-1}\right|_{z=\frac{1}{2}}=-2,\quad b=\left.\frac{1}{z-\frac{1}{2}}\right|_{z=1}=2$$

$$\Rightarrow X(z)=2z\left(\frac{-z}{z-\frac{1}{2}}+\frac{z}{z-1}\right)$$

$$\frac{-z}{z-\frac{1}{2}}+\frac{z}{z-1}\xleftrightarrow{\mathcal{Z}}-\left(\tfrac{1}{2}\right)^n u[n]+u[n]$$

$$\Rightarrow X(z)\xleftrightarrow{\mathcal{Z}}-2\left(\tfrac{1}{2}\right)^{n+1}u[n+1]+2u[n+1]$$

$$=-\left(\tfrac{1}{2}\right)^n u[n+1]+2u[n+1]$$

And $x[-1]=0\Rightarrow \underline{x[n]=-\left(\tfrac{1}{2}\right)^n u[n]+2u[n]}$

$x[n]$ is identical to that obtained in part (a) #

10.32

(a) We are given that $h[n] = a^n u[n]$ and $x[n] = u[n] - u[n-N]$. Therefore,

$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= \sum_{k=-\infty}^{\infty} h[n-k]x[k] \\ &= \sum_{k=0}^{N-1} a^{n-k} u[n-k] \end{aligned}$$

Now, $y[n]$ may be evaluated to be

$$y[n] = \begin{cases} 0, & n < 0 \\ \sum_{k=0}^{n} a^n a^{-k}, & 0 \le n \le N-1 \\ \sum_{k=0}^{N-1} a^n a^{-k}, & n > N-1 \end{cases}$$

Simplifying,

$$y[n] = \begin{cases} 0, & n < 0 \\ (a^n - a^{-1})/(1-a^{-1}), & 0 \le n \le N-1 \\ a^n(1-a^{-N})/(1-a^{-1}), & n > N-1 \end{cases}.$$

(b) Using Table 10.2, we get

$$H(z) = \frac{1}{1-az^{-1}}, \quad |z| > |a|$$

and

$$X(z) = \frac{1-z^{-N}}{1-z^{-1}}, \quad \text{All}z.$$

Therefore,

$$Y(z) = X(z)H(z) = \frac{1}{(1-z^{-1})(1-az^{-1})} - \frac{z^{-N}}{(1-z^{-1})(1-az^{-1})}$$

The ROC is $|z| > |a|$. Consider

$$P(z) = \frac{1}{(1-z^{-1})(1-az^{-1})}$$

with ROC $|z| > |a|$. The partial fraction expansion of $P(z)$ is

$$P(z) = \frac{1/(1-a)}{1-z^{-1}} + \frac{1/(1-a^{-1})}{1-az^{-1}}.$$

Therefore,

$$p[n] = \frac{1}{1-a}u[n] + \frac{1}{1-a^{-1}}a^n u[n].$$

Now, note that

$$Y(z) = P(z)[1-z^{-N}].$$

Therefore,

$$y[n] = p[n] - p[n-N] = \frac{1}{1-a}\{u[n] - u[n-N]\} + \frac{1}{1-a^{-1}}\{a^n u[n] - a^{n-N}u[n-N]\}$$

This may be written as

$$y[n] = \begin{cases} 0, & n < 0 \\ (a^n - a^{-1})/(1-a^{-1}), & 0 \le n \le N-1 \\ a^n(1-a^{-N})/(1-a^{-1}), & n > N-1 \end{cases}.$$

This is the same as the result of part (a).