2018 Signal and System HW10 (For your practice)

No need to submit ☺

10.21. Determine the z-transform for each of the following sequences. Sketch the pole-zero plot and indicate the region of convergence. Indicate whether or not the Fourier transform of the sequence exists.

(a) $\delta[n+5]$ **(b)** $\delta[n-5]$

(c) $(-1)^n u[n]$ **(d)** $(\frac{1}{2})^{n+1} u[n+3]$

(e) $(-\frac{1}{3})^n u[-n-2]$ **(f)** $(\frac{1}{4})^n u[3-n]$

(g) $2^n u[-n] + (\frac{1}{4})^n u[n-1]$ **(h)** $(\frac{1}{3})^{n-2} u[n-2]$

10.22. Determine the z-transform for the following sequences. Express all sums in closed form. Sketch the pole-zero plot and indicate the region of convergence. Indicate whether the Fourier transform of the sequence exists.

(a) $(\frac{1}{2})^n\{u[n+4] - u[n-5]\}$ **(b)** $n(\frac{1}{2})^{|n|}$

(c) $|n|(\frac{1}{2})^{|n|}$ **(d)** $4^n \cos[\frac{2\pi}{6}n + \frac{\pi}{4}]u[-n-1]$

10.25. Consider a right-sided sequence $x[n]$ with z-transform

$$X(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})(1 - z^{-1})}. \qquad \text{(P10.25–1)}$$

(a) Carry out a partial-fraction expansion of eq. (P10.25–1) expressed as a ratio of polynomials in z^{-1}, and from this expansion, determine $x[n]$.

(b) Rewrite eq. (P10.25–1) as a ratio of polynomials in z, and carry out a partial-fraction expansion of $X(z)$ expressed in terms of polynomials in z. From this expansion, determine $x[n]$, and demonstrate that the sequence obtained is identical to that obtained in part (a).

10.32. Consider an LTI system with impulse response

$$h[n] = \begin{cases} a^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

and input

$$x[n] = \begin{cases} 1, & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases}.$$

(a) Determine the output $y[n]$ by explicitly evaluating the discrete convolution of $x[n]$ and $h[n]$.

(b) Determine the output $y[n]$ by computing the inverse z-transform of the product of the z-transforms of the input and the unit sample response.