EE 361002 Signal and System HW1

- 1.25(a,c,d), 1.26(b,c,d,e), 1.34(a,b,c), 1.35, 1.36

1.25. Determine whether or not each of the following continuous-time signals is periodic. If the signal is periodic, determine its fundamental period.

(a) $x(t) = 3\cos(4t + \frac{\pi}{3})$ **(b)** $x(t) = e^{j(\pi t - 1)}$

(c) $x(t) = [\cos(2t - \frac{\pi}{3})]^2$ **(d)** $x(t) = \mathcal{E}v\{\cos(4\pi t)u(t)\}$

(e) $x(t) = \mathcal{E}v\{\sin(4\pi t)u(t)\}$ **(f)** $x(t) = \sum_{n=-\infty}^{\infty} e^{-(2t-n)} u(2t - n)$

1.26. Determine whether or not each of the following discrete-time signals is periodic. If the signal is periodic, determine its fundamental period.

(a) $x[n] = \sin(\frac{6\pi}{7}n + 1)$ **(b)** $x[n] = \cos(\frac{n}{8} - \pi)$ **(c)** $x[n] = \cos(\frac{\pi}{8}n^2)$

(d) $x[n] = \cos(\frac{\pi}{2}n)\cos(\frac{\pi}{4}n)$ **(e)** $x[n] = 2\cos(\frac{\pi}{4}n) + \sin(\frac{\pi}{8}n) - 2\cos(\frac{\pi}{2}n + \frac{\pi}{6})$

relationship ... statement. If the statement is not true, produce a counterexample ...

1.34. In this problem, we explore several of the properties of even and odd signals.

(a) Show that if $x[n]$ is an odd signal, then

$$\sum_{n=-\infty}^{+\infty} x[n] = 0.$$

(b) Show that if $x_1[n]$ is an odd signal and $x_2[n]$ is an even signal, then $x_1[n]x_2[n]$ is an odd signal.

(c) Let $x[n]$ be an arbitrary signal with even and odd parts denoted by

$$x_e[n] = \mathcal{E}v\{x[n]\}$$

and

$$x_o[n] = \mathcal{O}d\{x[n]\}.$$

Show that

$$\sum_{n=-\infty}^{+\infty} x^2[n] = \sum_{n=-\infty}^{+\infty} x_e^2[n] + \sum_{n=-\infty}^{+\infty} x_o^2[n].$$

(d) Although parts (a)–(c) have been stated in terms of discrete-time signals, the analogous properties are also valid in continuous time. To demonstrate this, show that

$$\int_{-\infty}^{+\infty} x^2(t)dt = \int_{-\infty}^{+\infty} x_e^2(t)dt + \int_{-\infty}^{+\infty} x_o^2(t)dt,$$

where $x_e(t)$ and $x_o(t)$ are, respectively, the even and odd parts of $x(t)$.

1.35. Consider the periodic discrete-time exponential time signal

$$x[n] = e^{jm(2\pi/N)n}.$$

Show that the fundamental period of this signal is

$$N_0 = N/\gcd(m, N),$$

where $\gcd(m, N)$ is the *greatest common divisor* of m and N—that is, the largest integer that divides both m and N an integral number of times. For example,

$$\gcd(2, 3) = 1, \gcd(2, 4) = 2, \gcd(8, 12) = 4.$$

Note that $N_0 = N$ if m and N have no factors in common.

1.36. Let $x(t)$ be the continuous-time complex exponential signal

$$x(t) = e^{j\omega_0 t}$$

with fundamental frequency ω_0 and fundamental period $T_0 = 2\pi/\omega_0$. Consider the discrete-time signal obtained by taking equally spaced samples of $x(t)$—that is,

$$x[n] = x(nT) = e^{j\omega_0 nT}.$$

(a) Show that $x[n]$ is periodic if and only if T/T_0 is a rational number—that is, if and only if some multiple of the sampling interval *exactly equals* a multiple of the period of $x(t)$.

(b) Suppose that $x[n]$ is periodic—that is, that

$$\frac{T}{T_0} = \frac{p}{q}, \qquad \text{(P1.36–1)}$$

where p and q are integers. What are the fundamental period and fundamental frequency of $x[n]$? Express the fundamental frequency as a fraction of $\omega_0 T$.

(c) Again assuming that T/T_0 satisfies eq. (P1.36–1), determine precisely how many periods of $x(t)$ are needed to obtain the samples that form a single period of $x[n]$.