EE 361002 Signal and System HW1 Answer

1.25

(a) $x(t) = 3\cos(4t + \frac{\pi}{3})$

if $\|$ $x(t+mT) = 3\cos(4t + 4mT + \frac{\pi}{3})$ $\xrightarrow{\text{then}}$ $4mT = 2\pi \Rightarrow T = \frac{2\pi}{4m} = \frac{\pi}{2m}$, $m \in \mathbb{Z}$

$\Rightarrow x(t)$ is periodic, $T_0 = \frac{\pi}{2}$ #

(c) $x(t) = [\cos(2t - \frac{\pi}{3})]^2 = \frac{1}{2}[\cos(4t - \frac{2\pi}{3}) + 1]$

if $\|$ $x(t+mT) = \frac{1}{2}[\cos(4t + 4mT - \frac{2\pi}{3}) + 1]$ $\xrightarrow{\text{then}}$ $4mT = 2\pi \Rightarrow T = \frac{\pi}{2m}$, $m \in \mathbb{Z}$

$\Rightarrow x(t)$ is periodic, $T_0 = \frac{\pi}{2}$ #

(d) $x(t) = \mathcal{E}v\{\cos(4\pi t)u(t)\} = \frac{1}{2}[\cos(4\pi t)u(t) + \cos(-4\pi t)u(-t)]$

if $\|$ $x(t+mT) = \frac{1}{2}[\cos(4\pi t + 4\pi mT)u(t+mT) + \cos(-4\pi t - 4\pi mT)u(-t-mT)]$ $\xrightarrow{\text{then}}$ $4\pi mT = 2\pi \Rightarrow T = \frac{1}{2m}$, $m \in \mathbb{Z}$

$\Rightarrow x(t)$ is periodic, $T_0 = \frac{1}{2}$ #

1.26

(b) $x[n] = \cos(\frac{n}{8} - \pi)$

if $\|$ $x[n+N] = \cos(\frac{n}{8} + \frac{N}{8} - \pi)$ $\xrightarrow{\text{then}}$ $\frac{N}{8} = 2k\pi$, $k \in \mathbb{Z} \Rightarrow N = \frac{k\pi}{4}$, $k \in \mathbb{Z}$

$\Rightarrow$ We can't find any integer k s.t. N is a positive integer.

$\Rightarrow x[n]$ is not periodic. #

(c) $x[n] = \cos(\frac{\pi}{8}n^2)$

if $\|$ $x[n+N] = \cos(\frac{\pi}{8}n^2 + \frac{\pi}{8} \cdot 2nN + \frac{\pi}{8}N^2)$ $\xrightarrow{\text{then}}$ $\frac{2nN+N^2}{8}\pi = 2k\pi$, $k \in \mathbb{Z} \Rightarrow 2nN + N^2 = 16k$, $k \in \mathbb{Z}$

$\Rightarrow x[n]$ is periodic, $N_0 = 8$ #

(d) $x[n] = \cos(\frac{\pi}{2}n)\cos(\frac{\pi}{4}n)$

$= [2\cos^2(\frac{\pi}{4}n) - 1] \cdot \cos(\frac{\pi}{4}n)$

$= 2\cos^3(\frac{\pi}{4}n) - \cos(\frac{\pi}{4}n)$

if $\|$ $x[n+N] = 2\cos^3(\frac{\pi}{4}n + \frac{\pi}{4}N) - \cos(\frac{\pi}{4}n + \frac{\pi}{4}N)$

$\xrightarrow{\text{then}}$ $\frac{\pi}{4}N = 2k\pi$, $k \in \mathbb{Z}$

$\Rightarrow N = 8k$, $k \in \mathbb{Z}$

$\Rightarrow x[n]$ is periodic, $N_0 = 8$ #

(e) $x[n] = 2\cos(\frac{\pi}{4}n) + \sin(\frac{\pi}{8}n) - 2\cos(\frac{\pi}{2}n + \frac{\pi}{6})$

if $\|$ $x[n+N] = 2\cos(\frac{\pi}{4}n + \frac{\pi}{4}N) + \sin(\frac{\pi}{8}n + \frac{\pi}{8}N) - 2\cos(\frac{\pi}{2}n + \frac{\pi}{2}N + \frac{\pi}{6})$

$\xrightarrow{\text{then}}$ $\frac{\pi}{4}N = 2k_1\pi$, $\frac{\pi}{8}N = 2k_2\pi$, $\frac{\pi}{2}N = 2k_3\pi$, $k_1, k_2, k_3 \in \mathbb{Z}$

$\Rightarrow N = 8k_1 = 16k_2 = 4k_3$, $k_1, k_2, k_3 \in \mathbb{Z}$

$\Rightarrow x[n]$ is periodic, $N_0 = 16$ #

1.34

(a) $$\sum_{n=-\infty}^{\infty} x[n] = x[0] + \sum_{n=-\infty}^{-1} x[n] + \sum_{n=1}^{\infty} x[n] = x[0] + \sum_{n=1}^{\infty} x[-n] + \sum_{n=1}^{\infty} x[n]$$

$x[n]$ is odd $\Rightarrow x[n] + x[-n] = 0$, and $x[0] = 0$

$$\Rightarrow \sum_{n=-\infty}^{\infty} x[n] = 0$$

(b) $x_1[n]$: even, $x_2[n]$: odd

$\Rightarrow x_1[n] = x_1[-n] \Rightarrow x_2[-n] = -x_2[n]$

Let $y[n] = x_1[n]x_2[n]$

$$\begin{aligned} y[-n] &= x_1[-n]x_2[-n] \\ &= -x_1[n]x_2[n] \\ &= -y[n] \end{aligned}$$

$\Rightarrow y[n]$ is odd

(c) $$\begin{aligned} \sum_{n=-\infty}^{\infty} x^2[n] &= \sum_{n=-\infty}^{\infty} (x_e[n] + x_o[n])^2 \\ &= \sum_{n=-\infty}^{\infty} (x_e[n])^2 + \sum_{n=-\infty}^{\infty} (x_o[n])^2 + 2\sum_{n=-\infty}^{\infty} x_e[n]x_o[n] \end{aligned}$$

for part (b), $\sum_{n=-\infty}^{\infty} x_e[n]x_o[n]$ is odd, and for part (a)

$$2\sum_{n=-\infty}^{\infty} x_e[n]x_o[n] = 0$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]$$

1.35 Since $x[n]$ is periodic,

$$x[n+N_0] = e^{jm\left(\frac{2\pi}{N}\right)(n+N_0)} = e^{jm\left(\frac{2\pi}{N}\right)n} = x[n]$$

$$\Rightarrow m\left(\frac{2\pi}{N}\right) N_0 = 2\pi k \;, \quad k \in \mathbb{Z}$$

$$\Rightarrow N_0 = \frac{kN}{m}$$

$\frac{m}{k}$ should be an integer and a divisor of both N and m.

Let $N = \gcd(m, N) \cdot N'$

$m = \gcd(m, N) \cdot m'$

$$\Rightarrow N_0 = k \cdot \frac{\gcd(m,N) \cdot N'}{\gcd(m,N) \cdot m'} = \frac{kN'}{m'}$$

k should be m' for the smallest N_0.

$$\Rightarrow N_0 = \frac{m' N}{\gcd(m,N) \cdot m'} = \frac{N}{\gcd(m,N)} \quad *$$

HW1 1.36

$x(t) = e^{j\omega_0 t}$, $T_0 = \frac{2\pi}{\omega_0}$

$x[n] = x(nT) = e^{j\omega_0 nT}$

(a)
($\Rightarrow$)

if $x[n]$ is periodic

$\Rightarrow x[n] = x[n+N]$

$e^{j\omega_0 nT} = e^{j\omega_0 nT + j\omega_0 NT}$

$\Rightarrow \omega_0 NT = 2k\pi$, for $k \in \mathbb{Z}$

$\Rightarrow T = \frac{2k\pi}{\omega_0 N} = T_0 \frac{k}{N}$

$\Rightarrow \frac{T}{T_0} = \frac{k}{N}$, $\therefore \frac{T}{T_0}$ is a rational number

($\Leftarrow$)

if $\frac{T}{T_0}$ is a rational number

$\Rightarrow \frac{T}{T_0} = \frac{k}{N}$

$x[n+N] = e^{j\omega_0 nT} \cdot e^{j\omega_0 NT}$

$= e^{j\omega_0 nT + j\omega_0 \frac{k}{T} \times T_0 \cdot T}$

$= e^{j\omega_0 nT + j\omega_0 k T_0}$ $\quad (T_0 = \frac{2\pi}{\omega_0})$

$= e^{j\omega_0 nT + j2\pi k}$

$= e^{j\omega_0 nT} = x[n]$

$\therefore x[n]$ is periodic

(b). $x[n]$ is periodic

$e^{j\omega_0 nT + j\omega_0 NT} = e^{j\omega_0 nT}$

$\Rightarrow 2k\pi = \omega_0 N \times \frac{p}{q} T_0$

$N = \frac{2k\pi}{\omega_0 T_0} \times \frac{q}{p} = k \times \frac{q}{p}$, $k \in \mathbb{Z}$

fundamental period : $N_0 = \frac{q}{\gcd(p,q)}$

fundamental freq. $\omega_0 = 2\pi \times \frac{\gcd(p,q)}{q}$

$= 2\pi \times \frac{p}{q} \times \frac{1}{p} \times \gcd(p,q)$

$= \omega_0 T \times \frac{\gcd(p,q)}{p}$

(c)

How many period?

$m \times T_0 = N_0 \times T$

$m = \frac{N_0}{T_0} \times T$

$= \frac{q}{\gcd(p,q)} \times \frac{p}{q}$

$= \frac{p}{\gcd(p,q)}$ #