Signals and Systems

Min Sun

Spring, 2018

Logistics

- Instructor
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- TAs
  - 胡展維 Chanwei Hu
  - Office hour: Mon. 1:30pm-3:00pm
- Course website
  - http://aliensunmin.github.io/teaching/ss2018/index.html
- Online discussion, **lecture slides**, and grading
  - http://lms.nthu.edu.tw/course/33168

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- Weekly homework
  - -Out Wed.
  - -Due next Wed. in class
- Bi-weekly 10-15 minutes quiz in class
- 2 midterms and 1 final

Topic	Weeks	Slides	Homeworks
Intro.	1.5	TBA	Hw1: Hw2
LTI System	1.5	TBA	Hw3 Hw4
Fourier Series	2	TBA	Hw5 Hw6
Midterm-1	Tentative Date: 3/28/2018		Cover: Chap1-3
Continuous-time Fourier Transform	1.5	TBA	Hw7 Hw8
Discrete-time Fourier Transform	1.5	TBA	Hw9 Hw10
Time and Frequency Characterization	2.5	TBA	Hw11
Midterm-2	Tentative Date: 5/16/2018		Cover: Chap4-6
Sampling	1.5	TBA	Hw12
Laplace Transform	2	TBA	Hw13
Z Transform	2	TBA	Hw14
Final	Tentative Date: 6/20/2018		Cover: All Taught Chaps

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- Textbook
  - Alan V. Oppenheim and Alan S. Willsky, with S. Hamid Nawab, Signals and Systems, 2nd Ed., Pearson New International Ed., Eurasia Book Co. (歐亞), 2014 or 東華代理版本 are both fine.
- Grading

  Your final grade will be made up from
  - 40% homework assignments
  - 30% two midterms
  - 30% final
  - 5% quizzes
  - 5% extra credit

Any other question?

Course Overview

- What is a signal?

“A **function** that conveys information about the behavior or attributes of some phenomenon”.

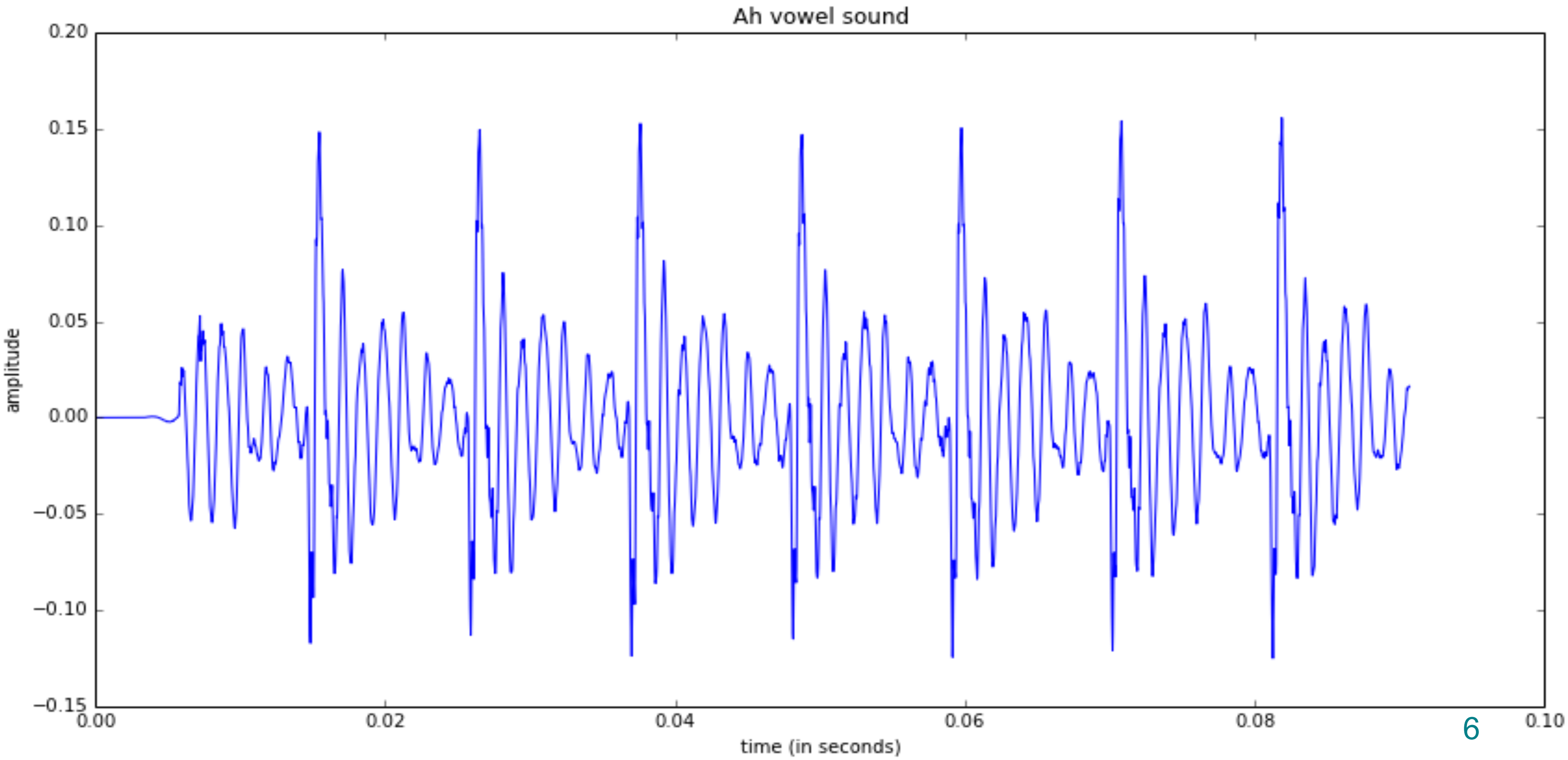
-Roland Priemer (1991). Introductory Signal Processing

“"signal" includes, among others, audio, video, speech, image, communication, geophysical, sonar, radar, medical and musical signals.”

-The IEEE Transactions on Signal Processing

Example of signals

- A sound signal (1D signal)

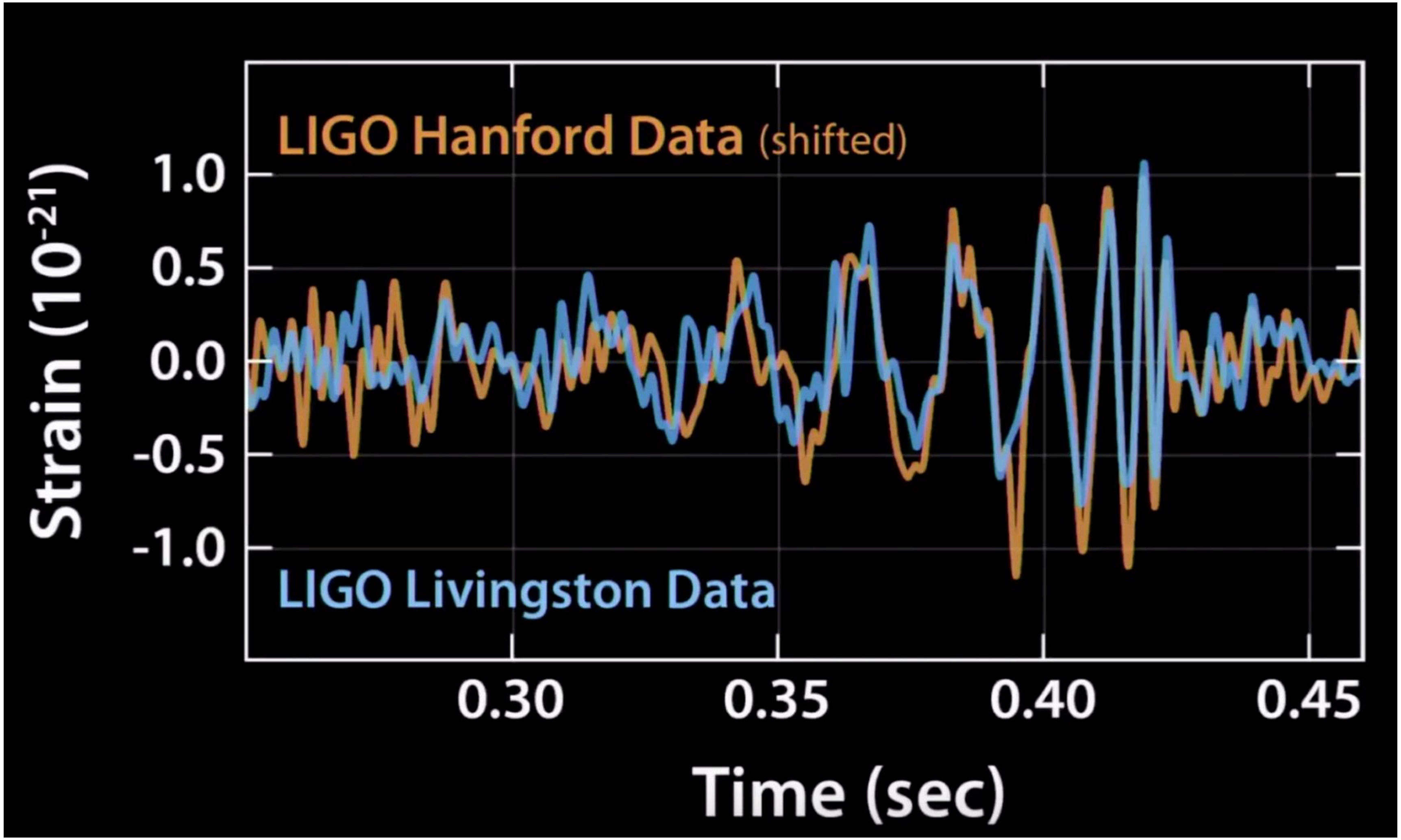


Example of signals：Electrocardiography (ECG)

速度

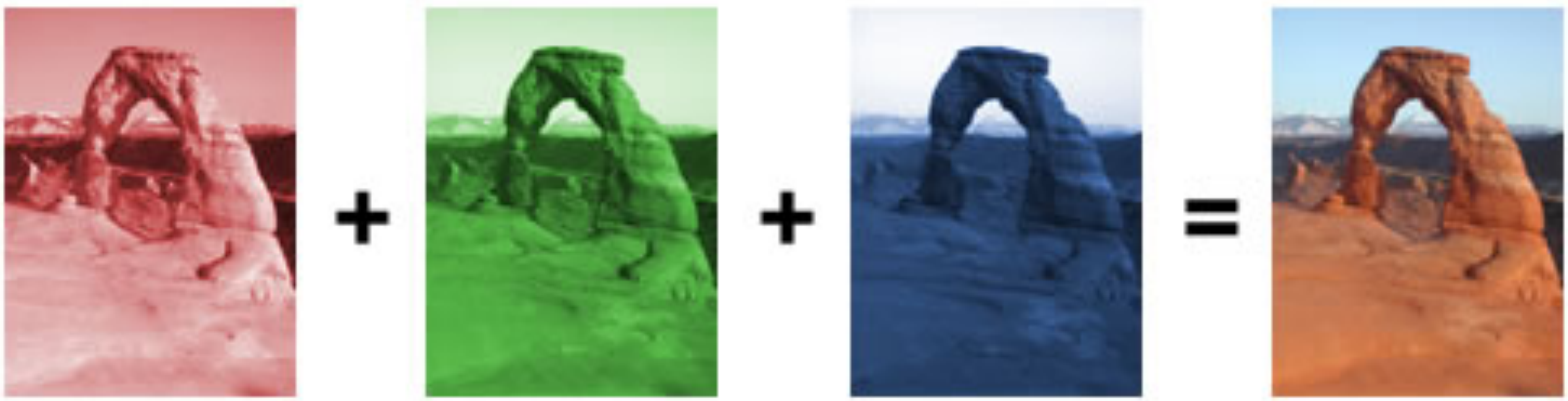
心跳速率、功率

Example of signals：gravitational wave



Example of signals : Image/Visual

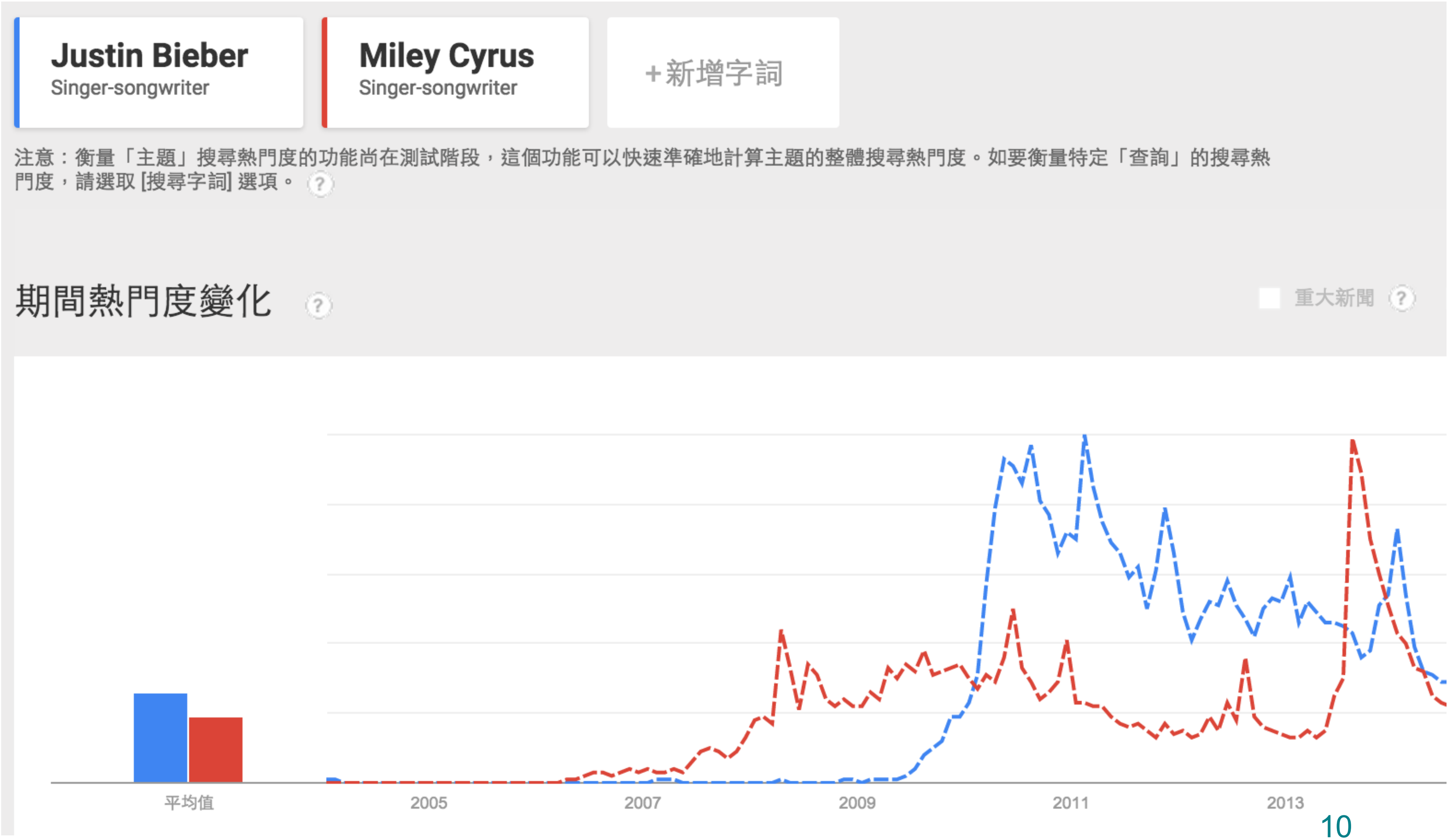
- An image signal (2D signal with RGB channels)



Red
Green
Blue
RGB color

Example of signals

- Signals from the web



Example of signals

- Signals from the web

Course Overview

- What is a system?

  The *things* that interact with signals or process the signals.

Examples of systems: filter

Passive, first order low-pass RC filter

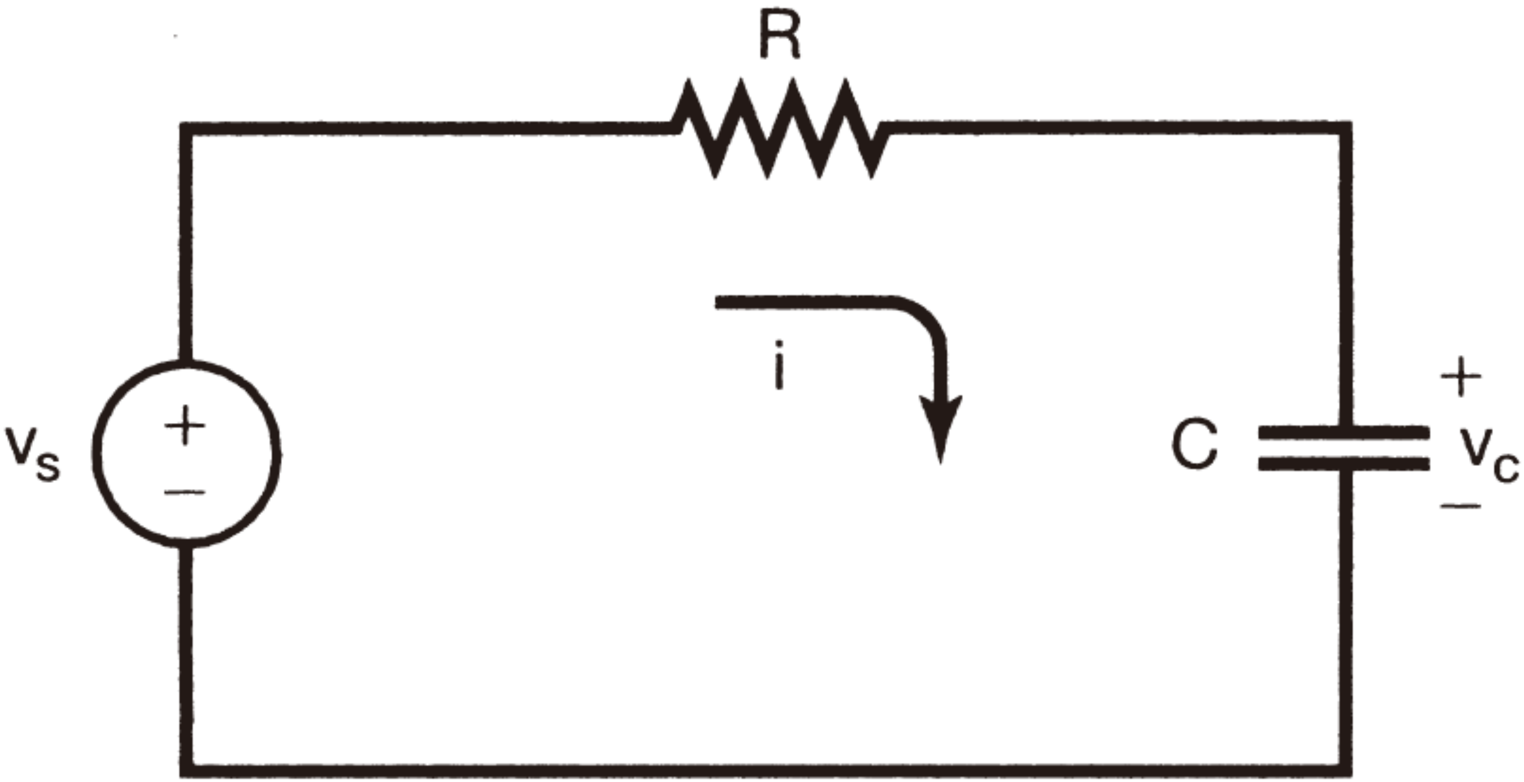
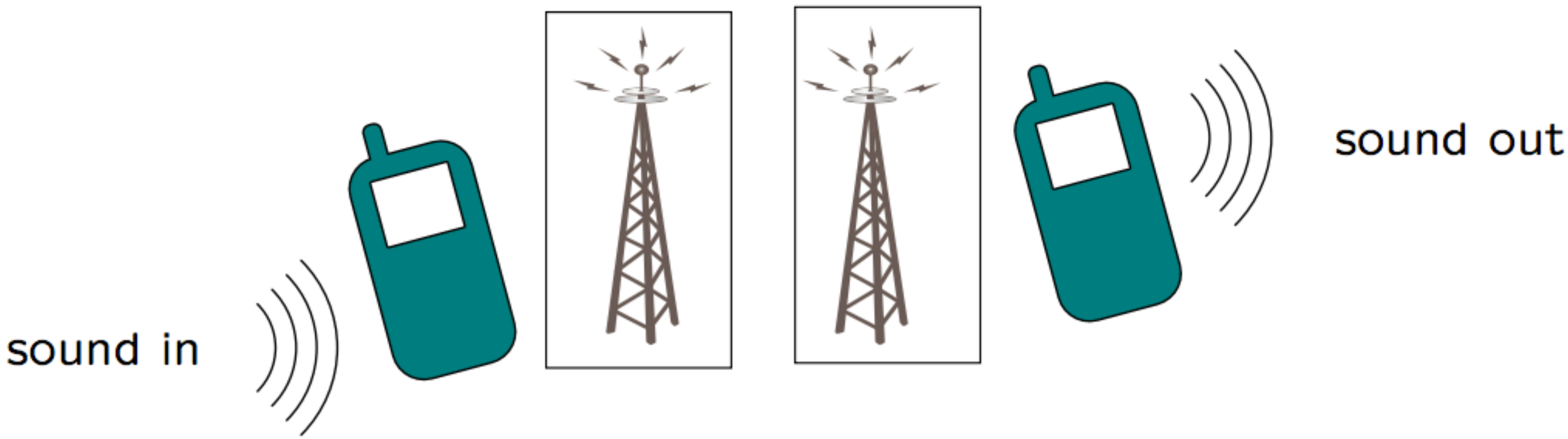
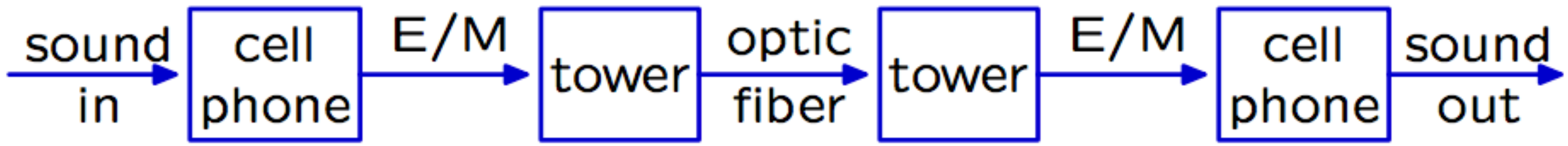


Figure 1.1 A simple *RC* circuit with source voltage v_s and capacitor voltage v_c.

Examples of systems: communication





Examples of systems: control

Input f, output v

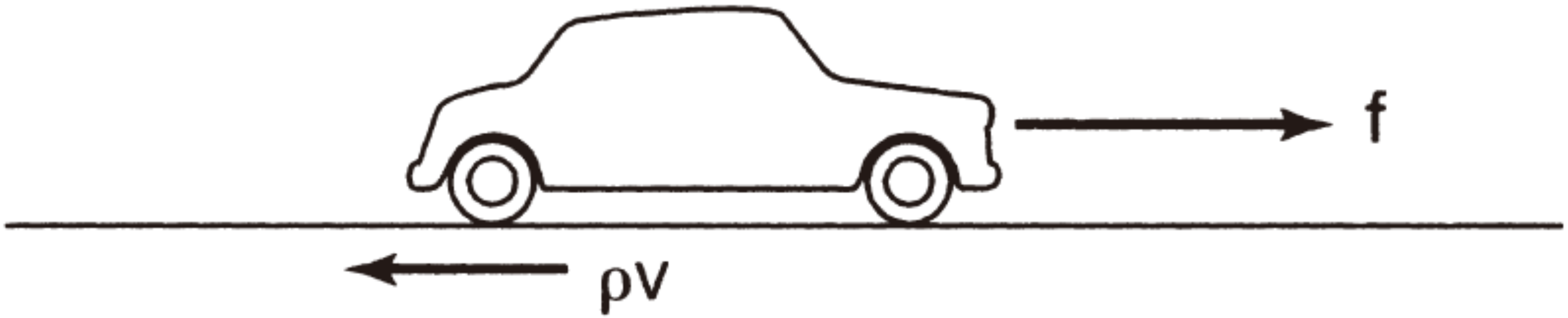
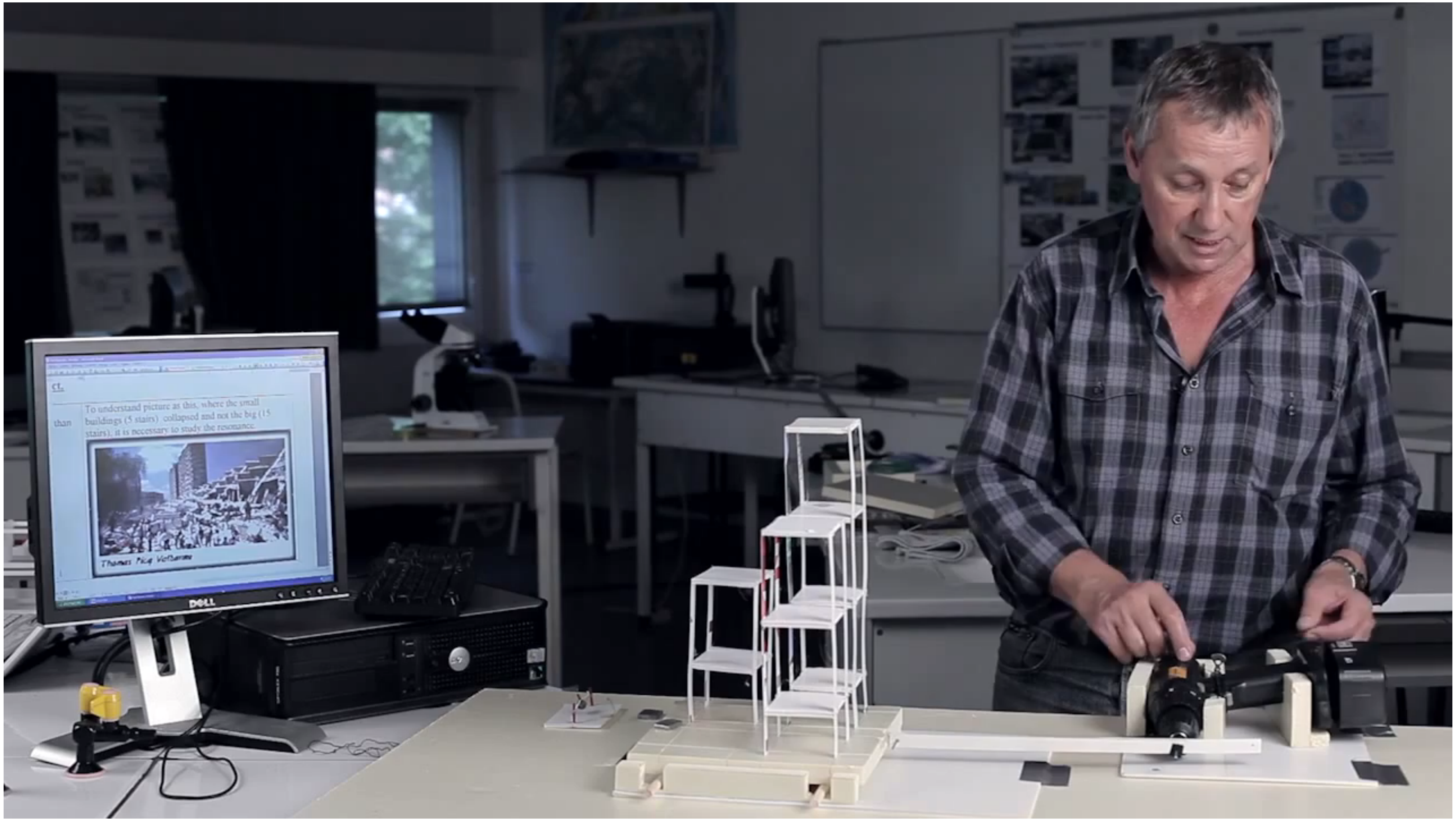


Figure 1.2 An automobile responding to an applied force f from the engine and to a retarding frictional force ρv proportional to the automobile's velocity v.

Examples of systems: system identification

Resonance and pole of a system

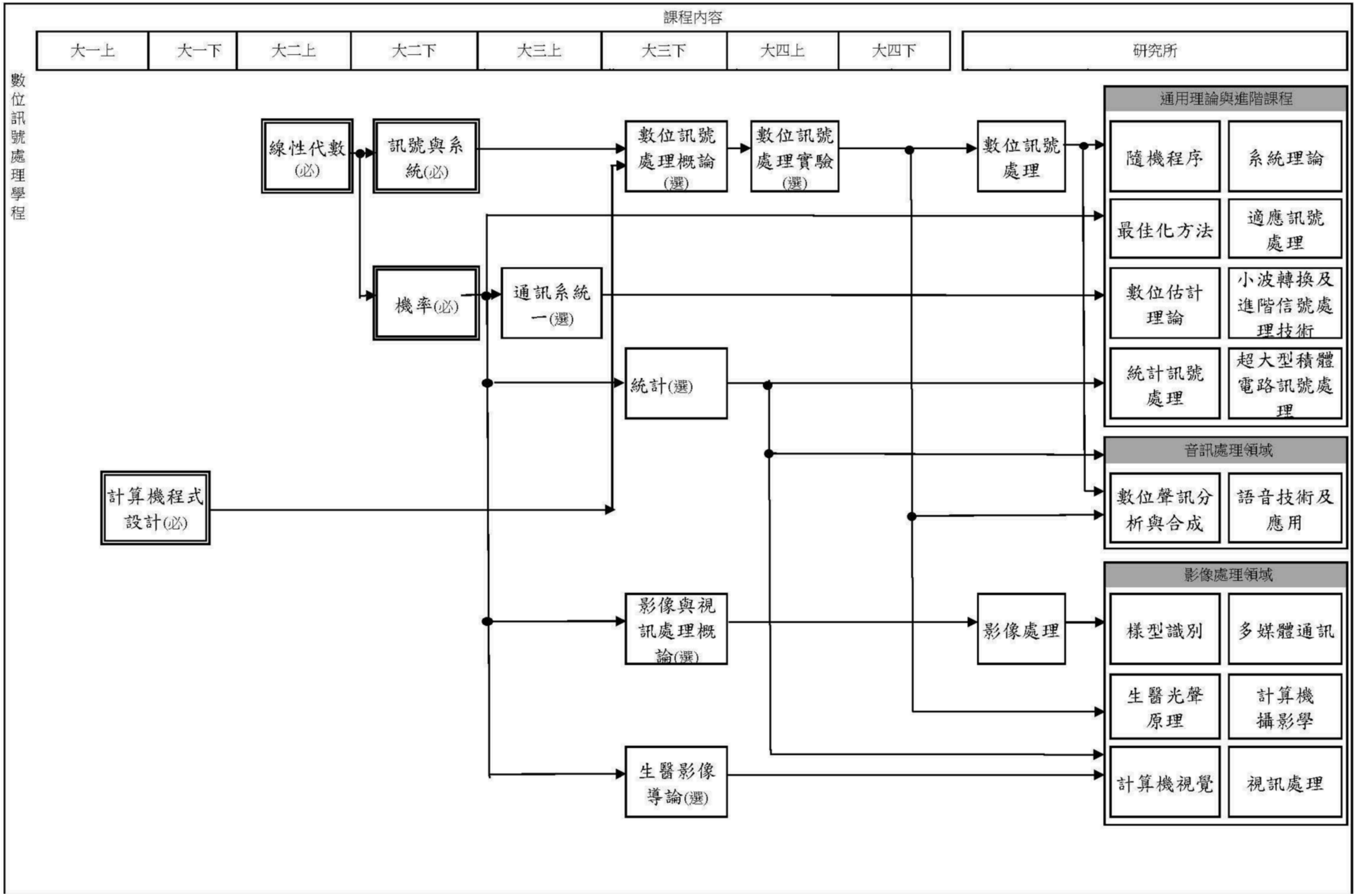


Goals

- Concepts and methods of signals and systems
- Analyze and solve problems involving signals and systems
- Applications
- How to write codes to do the analysis or visualization?
  - Python tutorial **RSVP** (TBA)

Related Courses

EE3510 控制系統
EE4070 數值分析



Back to Signals

1.1.1Examples and Mathematical Representation

- Two basic types of signals
  - Continuous-time signals (連續時間訊號)
    - The independent variable is continuous, and thus these signals are defined for a continuum of values of the independent variable.
      連續時間的獨立變數為連續的，所以訊號的定義是在獨立變數軸(時間軸)上連續的數值
  - Discrete-time signals (離散時間訊號)
    - Fore these signals, the independent variable takes on only a discrete set of values.
      離散時間訊號只定義在離散的時間點上所得的一組離散的數值

1.1.1Examples and Mathematical Representation

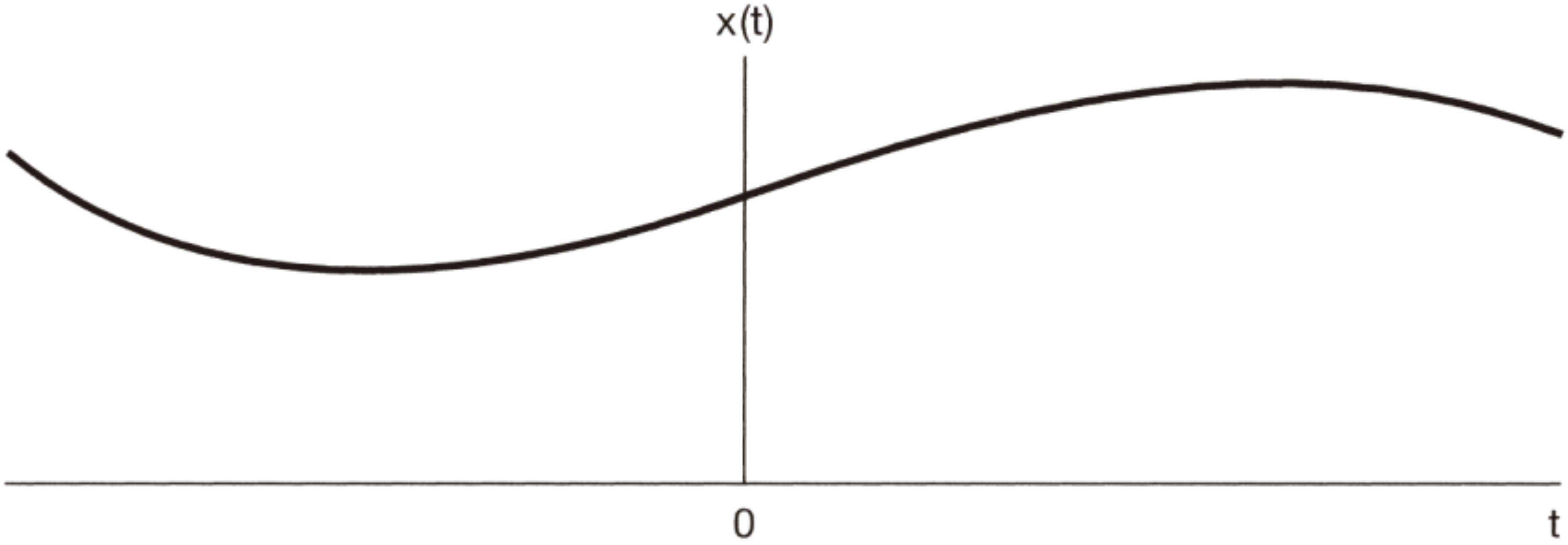
- To distinguish between continuous-time and discrete-time signals, we will use the symbol *t* to denote the continuous-time independent variable and *n* to denote the discrete-time independent variable.
  為了有所區別，我們以*t*代表連續時間的獨立變數，離散時間的獨立變數為*n*

1.1.1Examples and Mathematical Representation

- For continuous-time signals we will enclose the independent variable in parentheses (·), whereas for discrete-time signals we will use brackets[·]

  連續時間的訊號以小括號(·)表示；離散時間的訊號則以中括號表示[·]

1.1.1Examples and Mathematical Representation



(a)

1.1.1Examples and Mathematical Representation

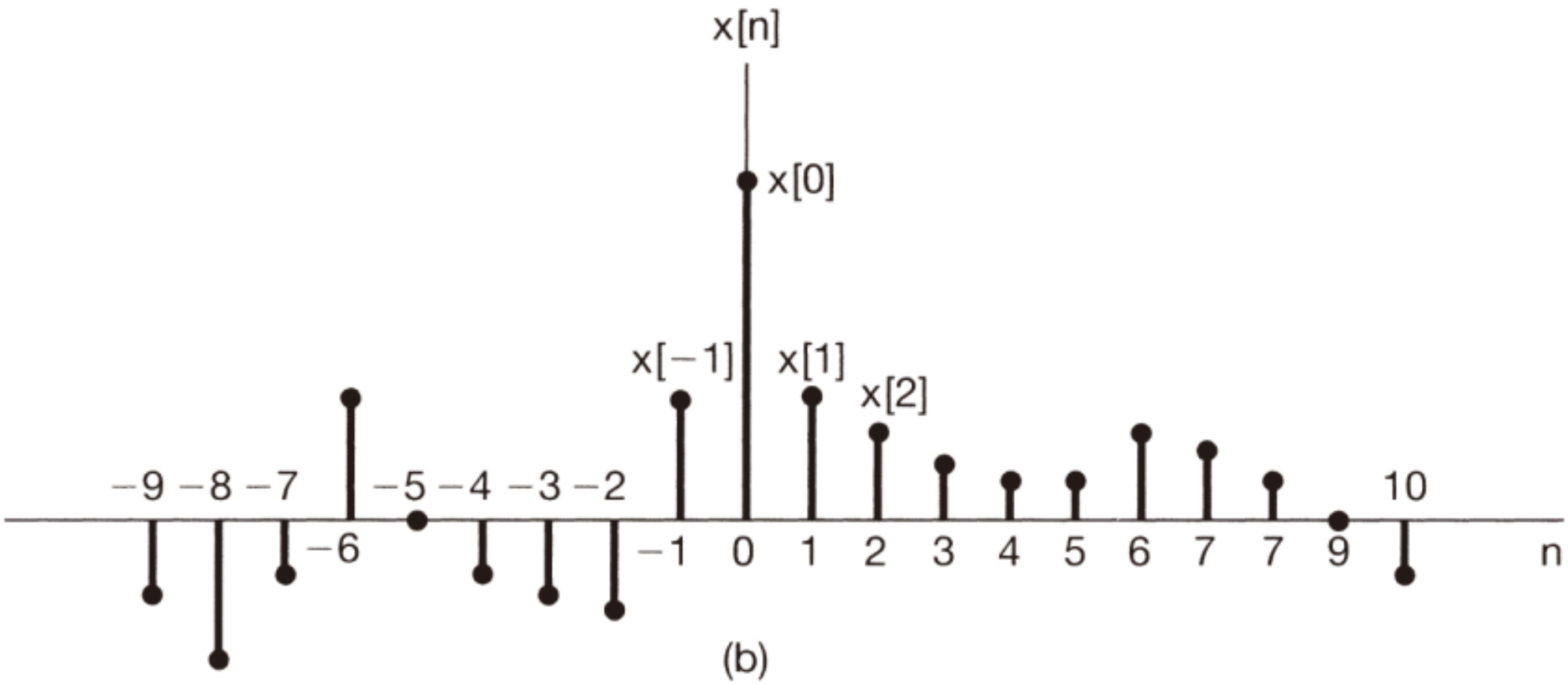


Figure 1.7 Graphical representations of (a) continuous-time and (b) discrete-time signals.

1.2 Transformations of the Independent Variable

- **Elementary signal transformations**
  - Time shift
  - Time reversal
  - Time scaling
- Why?
  - introducing several **basic properties** of signals and system.
  - defining and **characterizing far richer and important classes of systems**.

1.2.1 Time shift

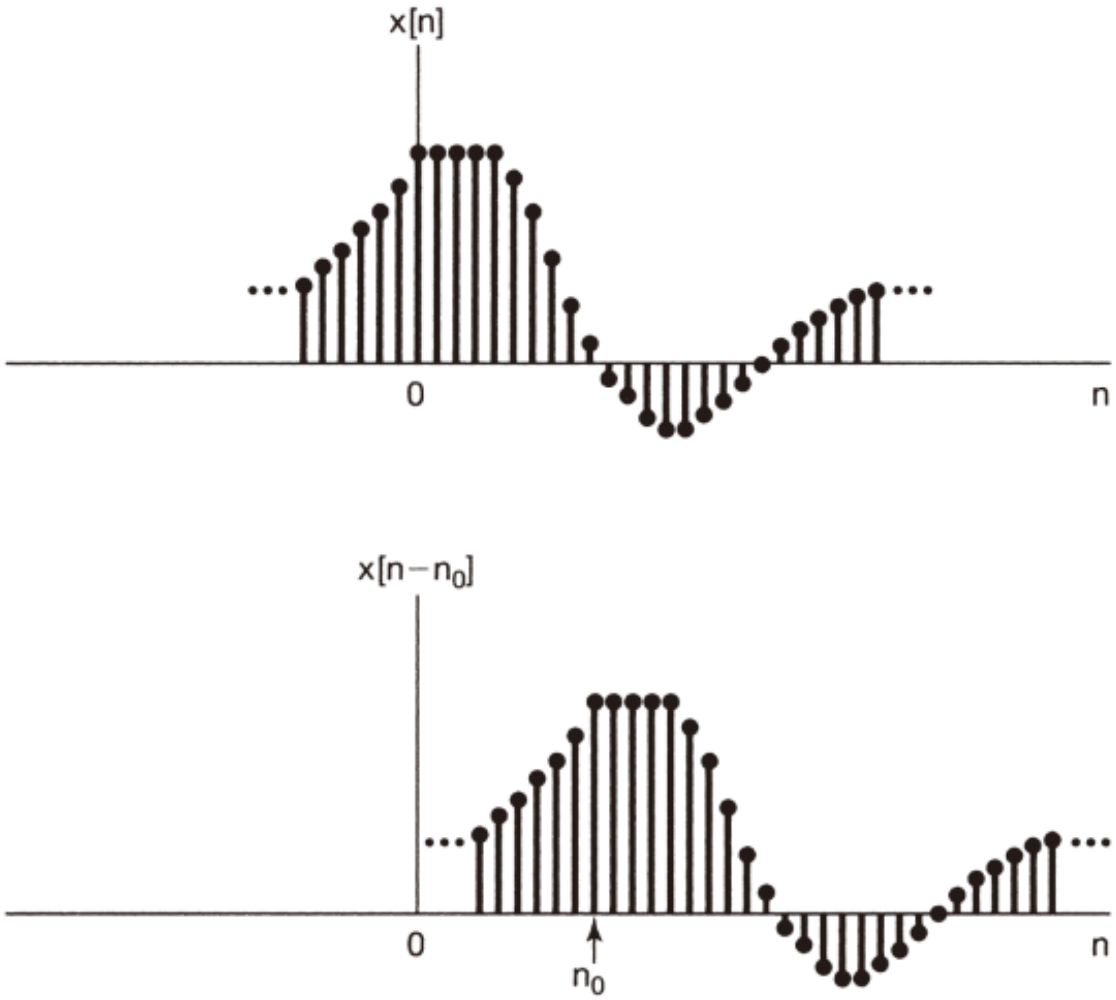


圖 1.8 爲離散時間訊號的時間移位。

Figure 1.8 Discrete-time signals related by a time shift. In this figure $n_0 > 0$, so that $x[n - n_0]$ is a delayed verson of $x[n]$ (i.e., each point in $x[n]$ occurs later in $x[n - n_0]$).

1.2.1 Time shift

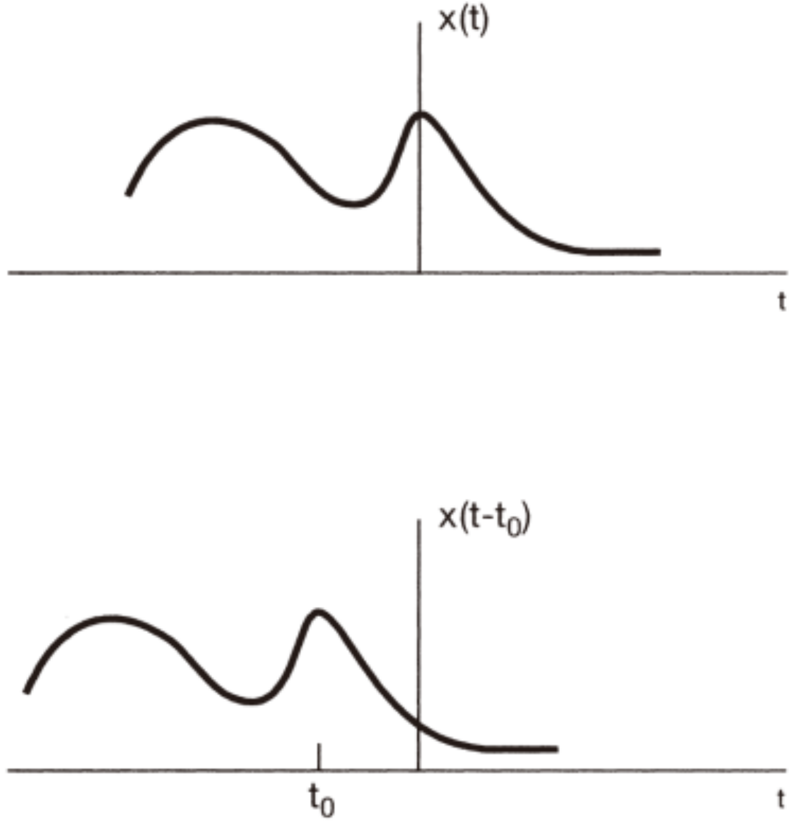


Figure 1.9 Continuous-time signals related by a time shift. In this figure $t_0 < 0$, so that $x(t - t_0)$ is an advanced version of $x(t)$ (i.e., each point in $x(t)$ occurs at an earlier time in $x(t - t_0)$).

1.2.1 Time reversal

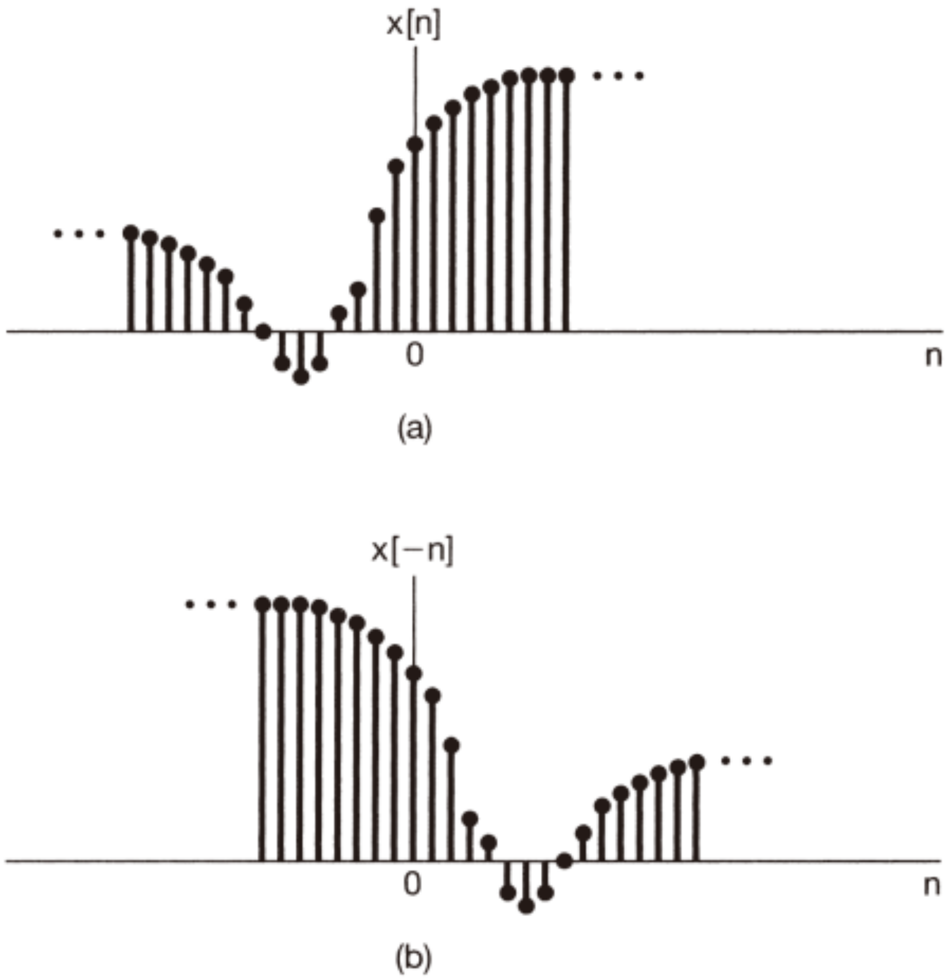


Figure 1.10 (a) A discrete-time signal $x[n]$; (b) its reflection $x[-n]$ about $n = 0$.

1.2.1 Time reversal

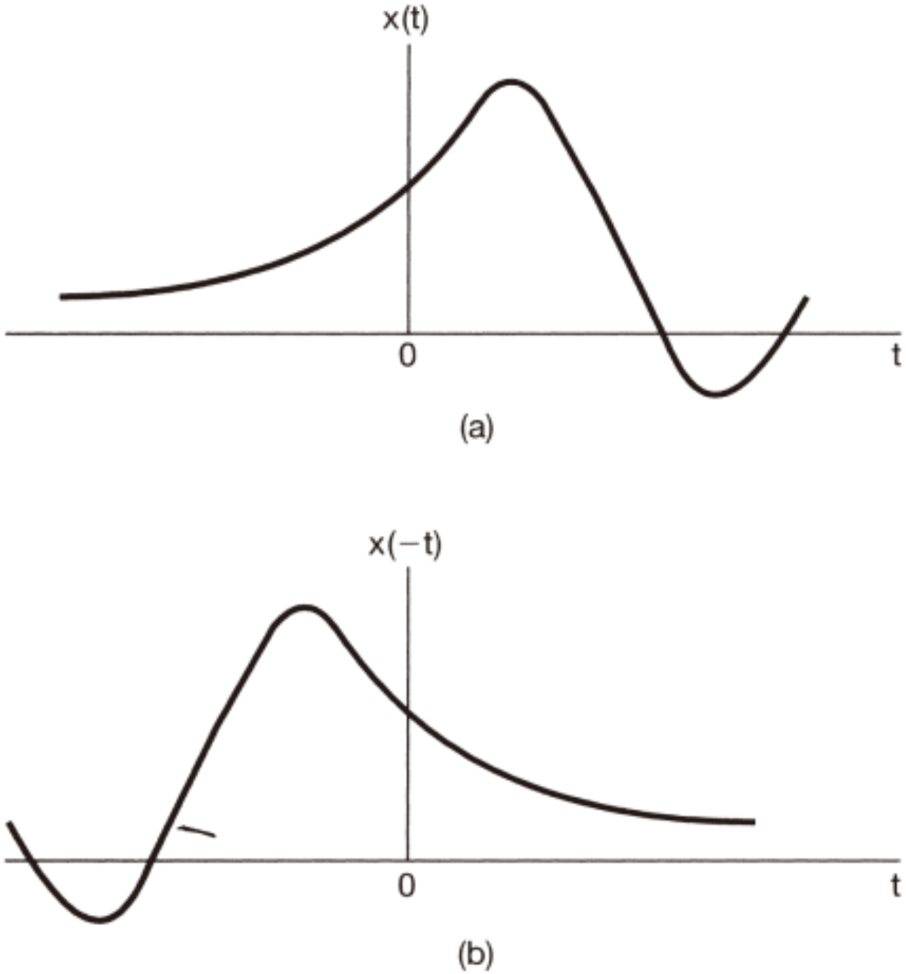


Figure 1.11 (a) A continuous-time signal $x(t)$; (b) its reflection $x(-t)$ about $t = 0$.

1.2.1 Time scaling

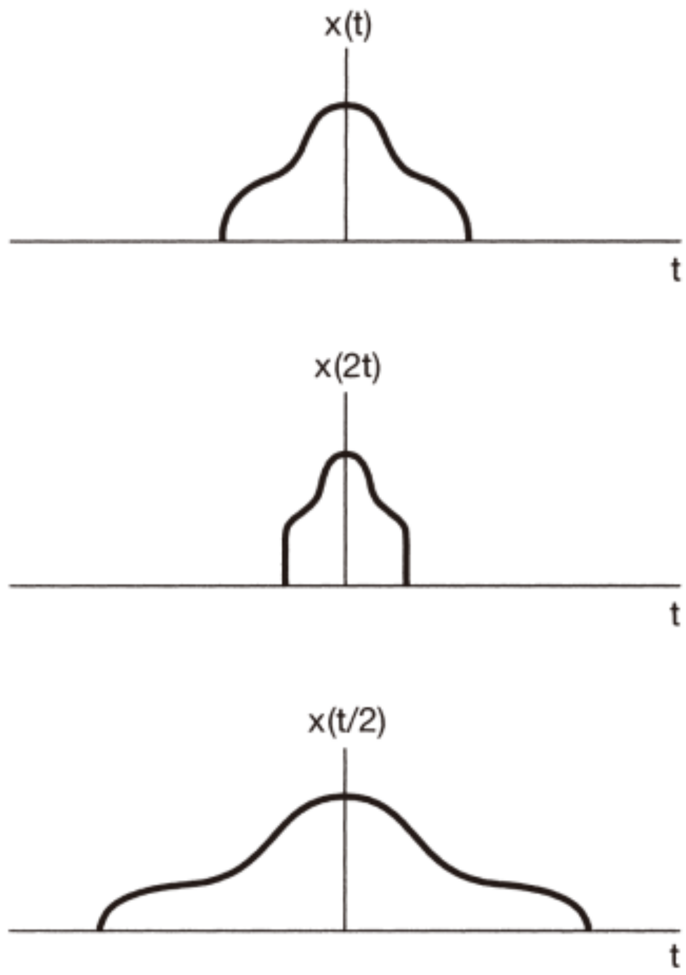
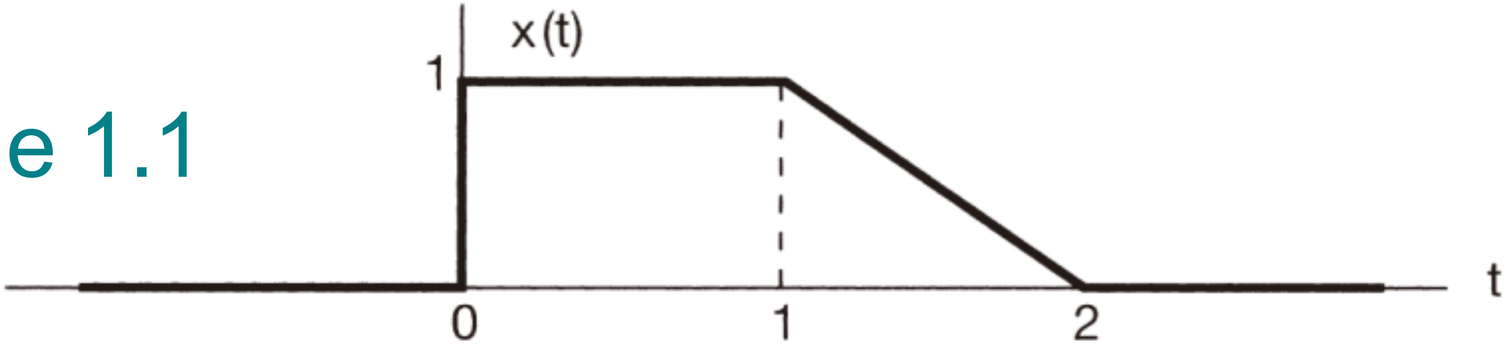
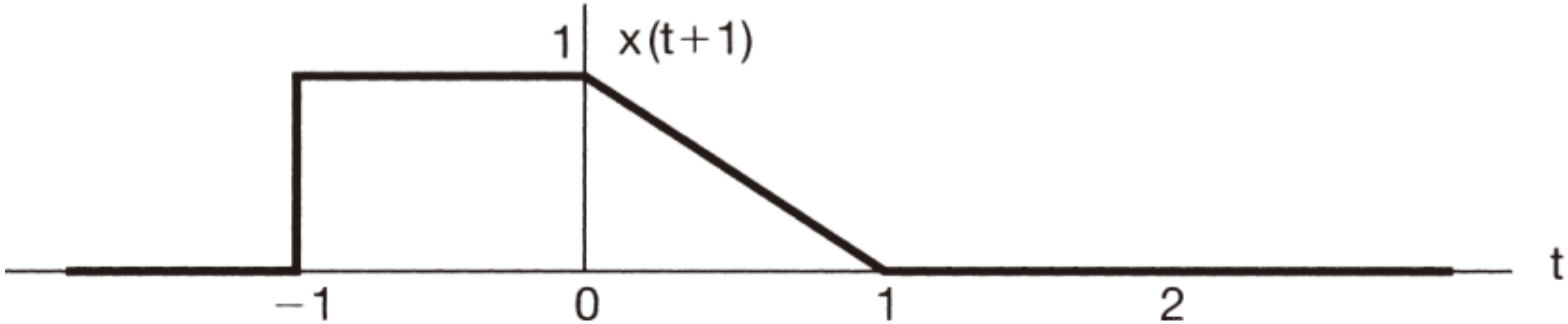


Figure 1.12 Continuous-time signals related by time scaling.

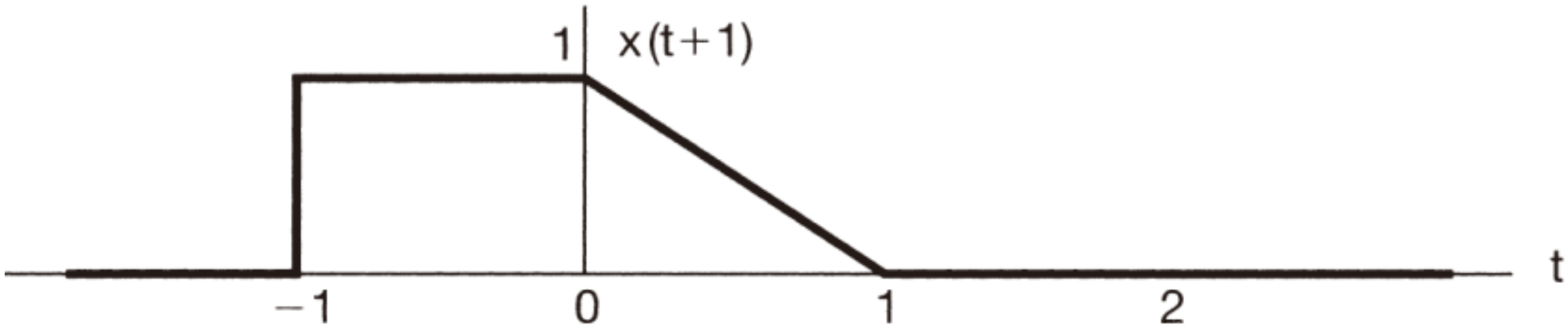
Example 1.1



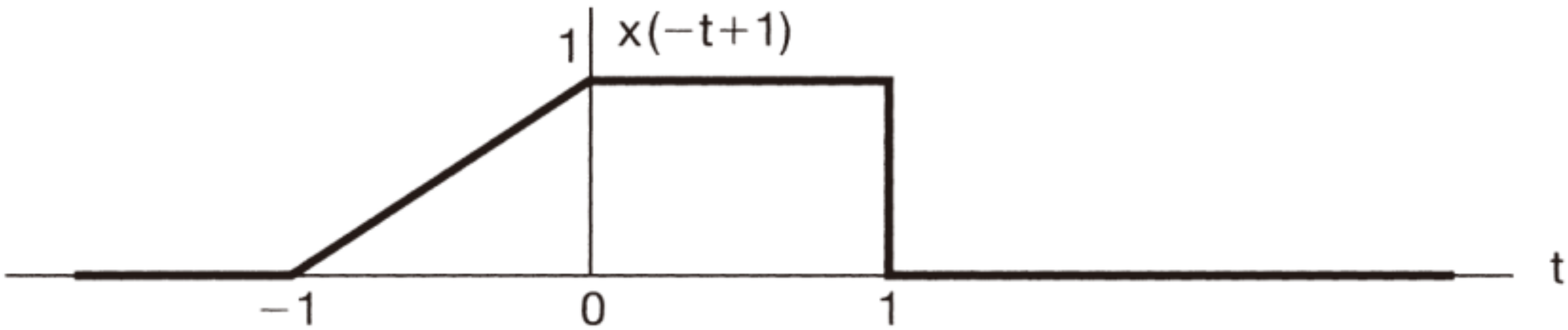
Given the signal $x(t)$ shown in Figure 1.13(a), the signal $x(t+1)$ corresponds to an advance by one unit along the t axis as illustrated in Figure 1.13(b).



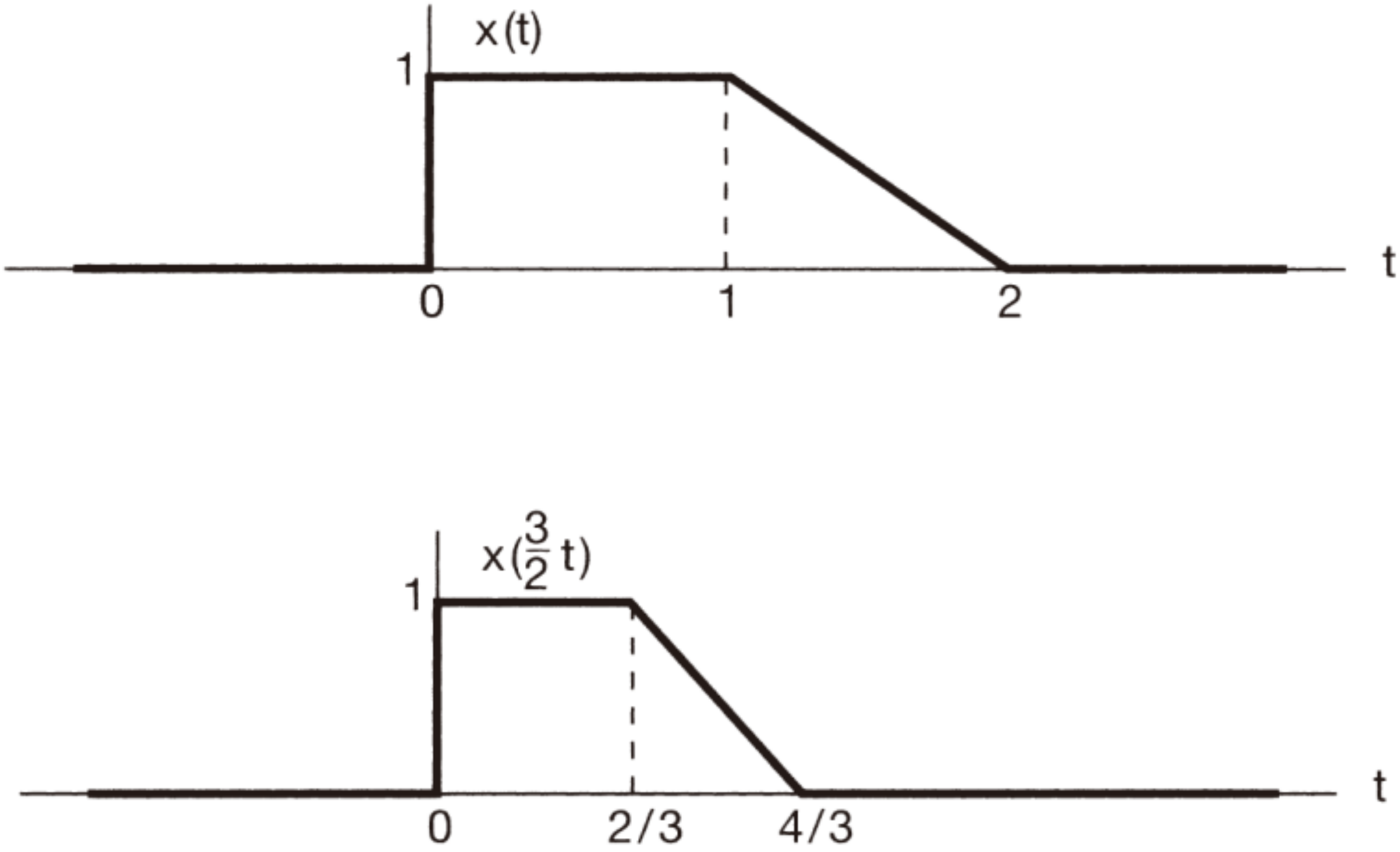
For example, the value at $t = 1$ is found in $x(t+1)$ at $t = 1\text{-}1 = 0$. Also, since $x(t)$ is zero for $t < 0$, we have $x(t+1)$ zero for $t < \text{-}1$. Similarly, since $x(t)$ is zero for $t > 2$, $x(t+1)$ is zero for $t > 1$



Let us also consider the signal $x(-t+1)$, which may be obtained by replacing t with $-t$ in $x(t+1)$. That is, $x(-t+1)$ is the time reversed version of $x(t+1)$. Thus, $x(-t+1)$ may be obtained graphically be reflecting $x(t+1)$ about the t axis as shown in Figure 1.13(c)

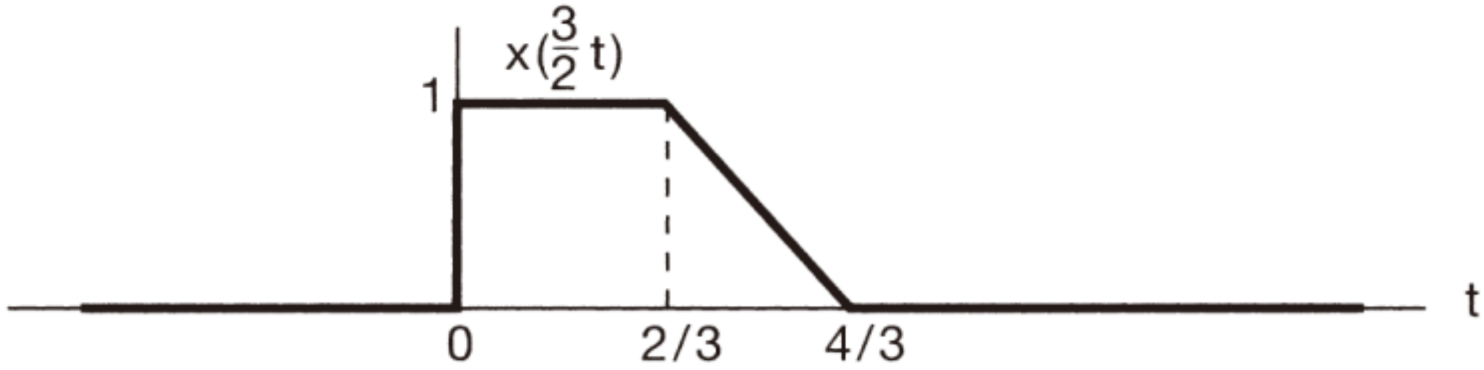


Example 1.1 Scaling

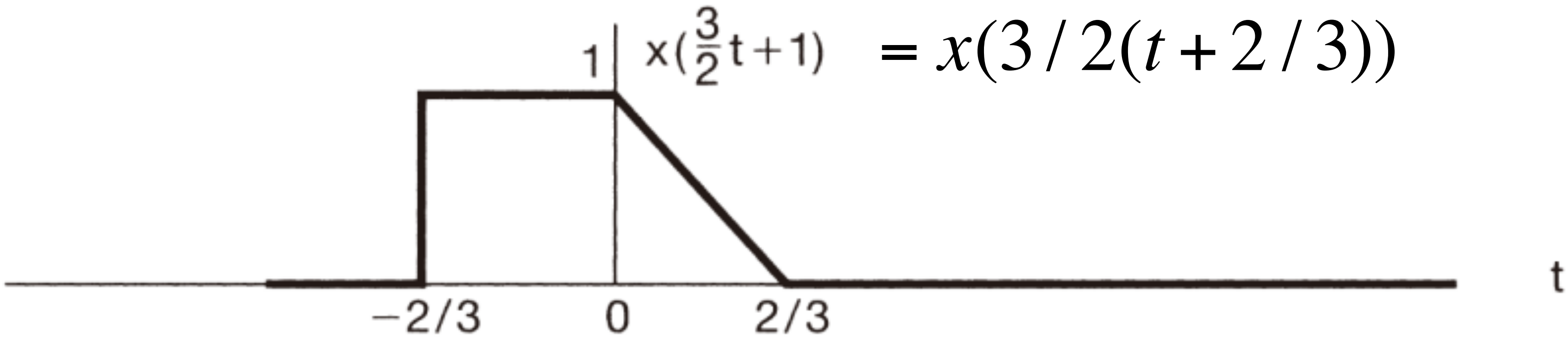


(d)

Example 1.1

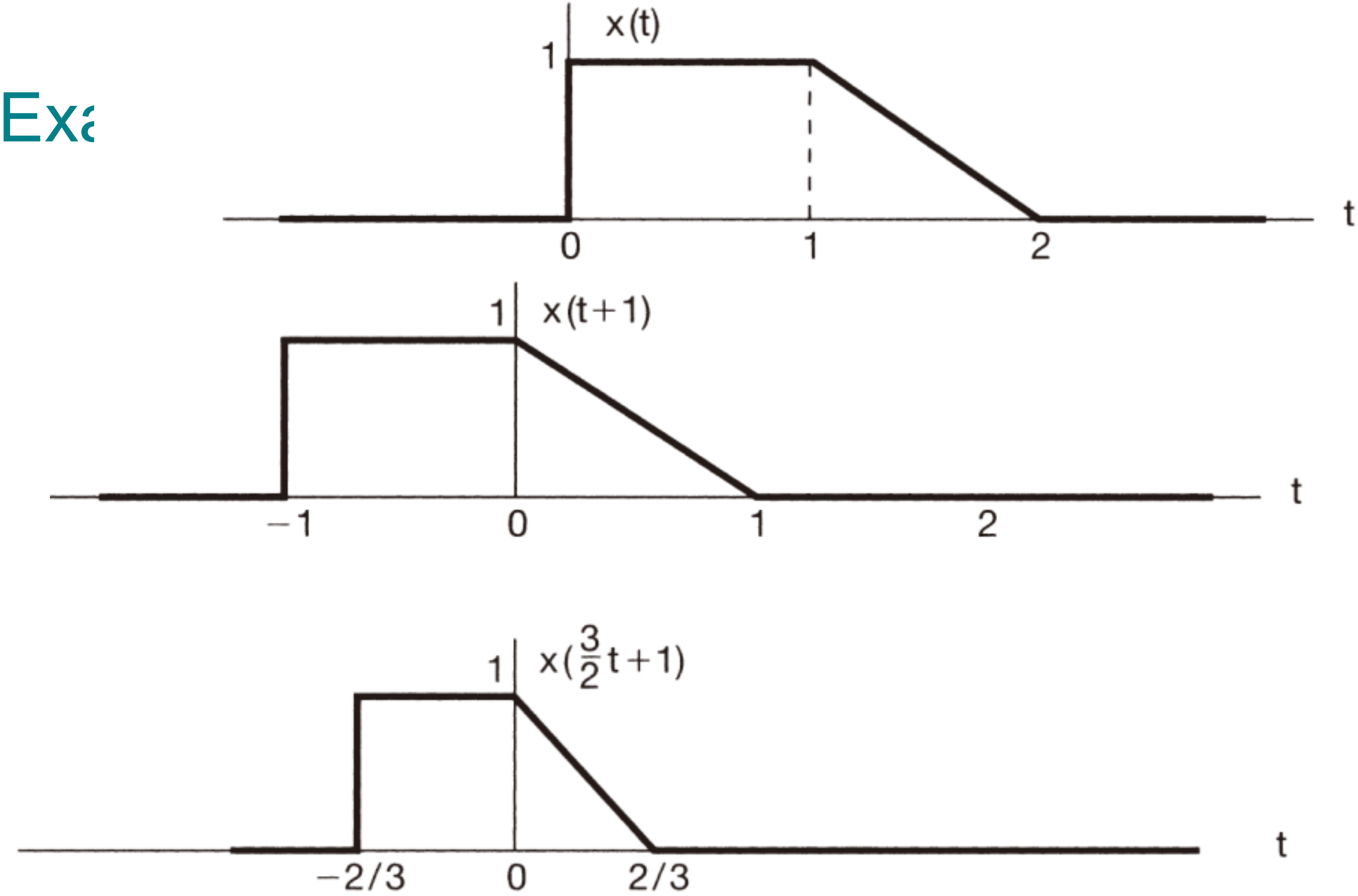


(d)



$$= x(3/2(t+2/3))$$

Exa



1.2.2 Periodic Signals

- A periodic continuous-time signal $x(t)$ has the property that there is a **positive** value of T for which

$$x(t) = x(t+T) \quad (1.11)$$

for all values of t. In other words, a periodic signal has the property that it is unchanged by a time shift of T. We say that $x(t)$ is periodic with *period* T.
連續時間週期訊號可表示為在某一個正數T之下，對任何時間t可得$x(t)=x(t+T)$。亦即週期訊號在時間軸上移位T時間其波形均不變。T稱為週期

1.2.2 Periodic Signals

- We can readily deduce that if x(t) is periodic with period T, then $x(t) = x(t+mT)$ for all t and for any integer m. Thus, x(t) is also periodic with period $2T$, $3T$, $4T$,….The **fundamental period** $\boldsymbol{T_0}$ of $x(t)$ is the **smallest positive** value of T for which eq. (1.11) holds.

  若$x(t)$為週期訊號，其週期為T，則對任何時間t及任意整數m，$x(t) = x(t+mT)$。基本週期T_0為可使(1.11)式成立的最小正數T。

1.2.2 Periodic Signals

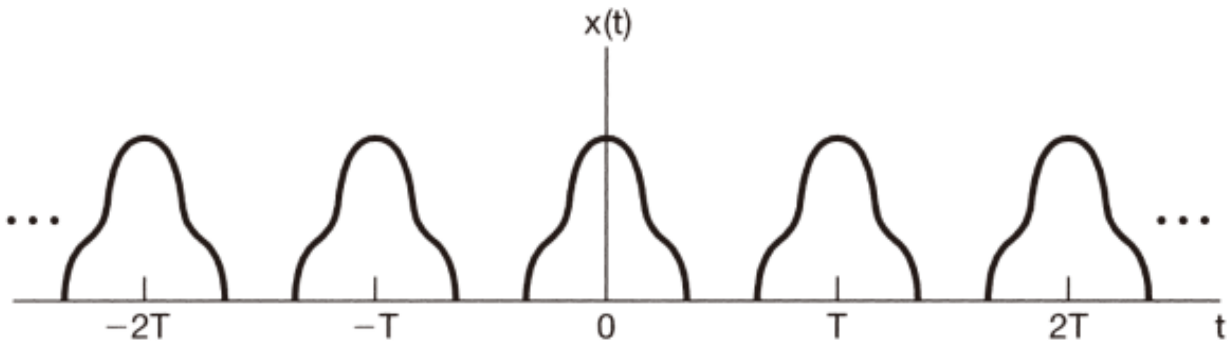


Figure 1.14 A continuous-time periodic signal.

Fundamental period T

1.2.2 Periodic Signals

- Periodic signals are defined analogously in discrete time. Specifically, a discrete-time signal $x[n]$ is periodic with period N, where N is a positive **integer**, if it is unchanged by a time shift of N, i.e., if

$$x[n] = x[n+N] \qquad (1.12)$$

for all values of n.

離散時間週期訊$x[n]$，其週期為一正整數N，且對任何時間n之下，可滿足$x[n]=x[n+N]$。基本週期N_0為可使(1.12)式成立最小正數N。

1.2.2 Periodic Signals

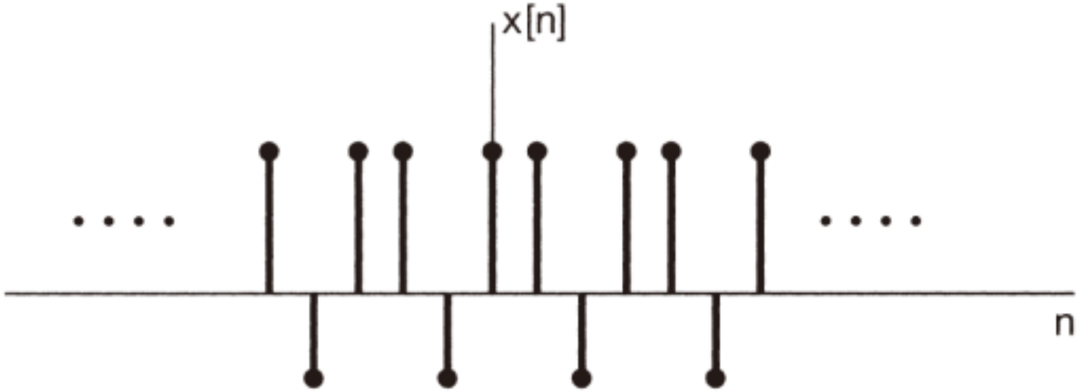


Figure 1.15 A discrete-time periodic signal with fundamental period

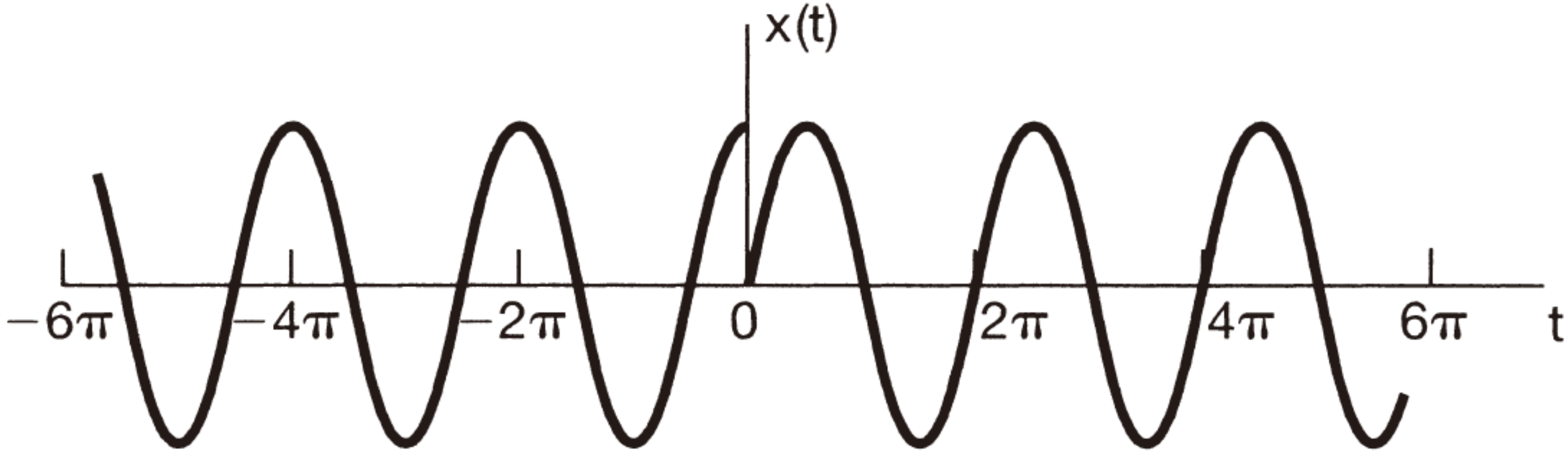
Example 1.4

The signal whose periodicity we wish to check is given by

$$x(t) = \begin{cases} \cos(t) & \text{if } t < 0 \\ \sin(t) & \text{if } t \geq 0 \end{cases} \tag{1.13}$$

We know that $\cos(t+2\pi) = \cos(t)$ and $\sin(t+2\pi) = \sin(t)$. Thus, considering $t > 0$ and $t < 0$ **separately**, we see that $x(t)$ does repeat itself over every interval of length 2π.

$$x(t) = \begin{cases} \cos(t) & \text{if } t < 0 \\ \sin(t) & \text{if } t \geq 0 \end{cases} \tag{1.13}$$



However, as illustrated in Figure 1.16, $x(t)$ also has a **discontinuity** at the time origin that does not recur at any other time. Since every feature in the shape of a periodic signal must recur periodically, we conclude that the signal $x(t)$ is not periodic.

1.2.3 Even and Odd Signals

In continuous time a signal is even if

$$x(-t) = x(t) \qquad (1.14)$$

while a discrete-time signal is even if

$$x[-n] = x[n] \qquad (1.15)$$

A signal is referred to as odd if

$$x(-t) = -x(t) \qquad (1.16)$$

$$x[-n] = -x[n] \qquad (1.17)$$

1.2.3 Even and Odd Signals

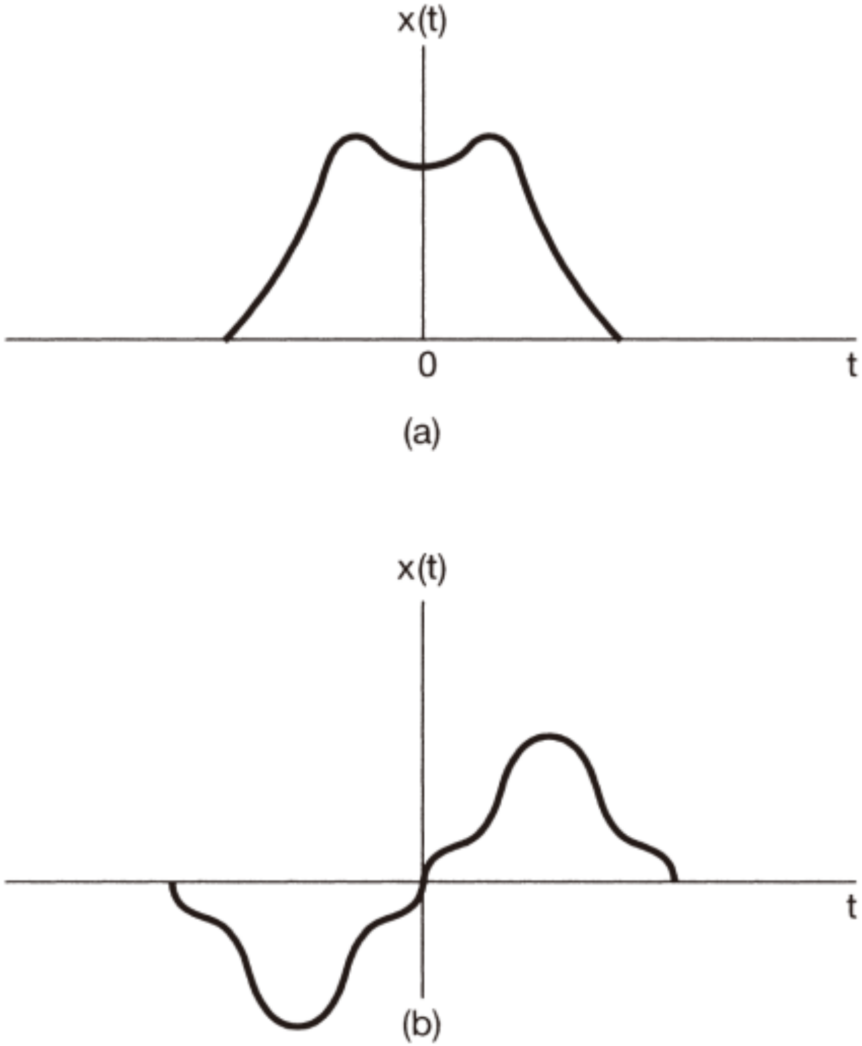


Figure 1.17 (a) An even continuous-time signal; (b) an odd continuous-time signal.

1.2.3 Even and Odd Signals

An **important fact** is that any signal can be broken into a sum of two signals:

$$x(t) = Ev\{x(t)\} + Od\{x(t)\}$$

One is even $Ev\{x(t)\}$

,and one is odd. $Od\{x(t)\}$

1.2.3 Even and Odd Signals

The even signal can be obtained as

$$Ev\{x(t)\} = \frac{1}{2}\left[x(t) + x(-t)\right]$$

The odd signal can be obtained as

$$Od\left\{x(t)\right\} = \frac{1}{2}\left[x(t) - x(-t)\right]$$

$$Ev\{x(t)\} = \frac{1}{2}\left[x\left(t\right) + x\left(-t\right)\right]$$

Check if even $\quad Ev\{x(-t)\} = Ev\{x(t)\}$

$$Ev\{x(-t)\} = \frac{1}{2}\left[x\left(-t\right) + x\left(t\right)\right]$$

$$= \frac{1}{2}\left[x\left(t\right) + x\left(-t\right)\right] = Ev\{x(t)\}$$

$$Od\left\{x\left(t\right)\right\} = \frac{1}{2}\left[x\left(t\right) - x\left(-t\right)\right]$$

Check if odd $Od\{x(-t)\} = -Od\{x(t)\}$

$$Od\{x(-t)\} = \frac{1}{2}\left[x\left(-t\right) - x\left(t\right)\right]$$

$$= -\frac{1}{2}\left[-x\left(-t\right) + x\left(t\right)\right] = -\frac{1}{2}\left[x\left(t\right) - x\left(-t\right)\right]$$

$$= -Od\{x(t)\}$$

1.2.3 Even and Odd Signals

Check if $Ev\{x(t)\}+Od\{x(t)\}=x(t)$

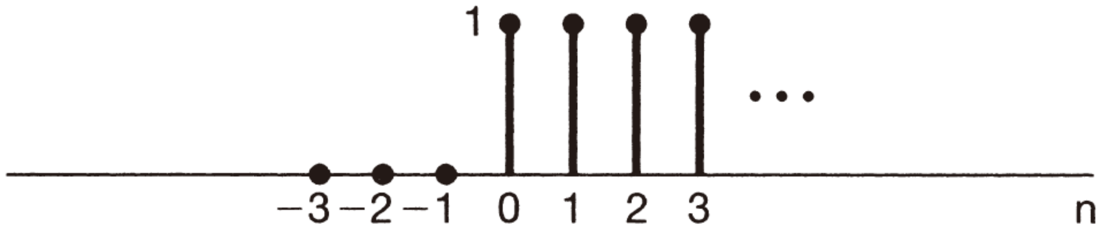
$$Ev\{x(t)\}+Od\{x(t)\}$$

$$=\frac{1}{2}\left[x(t)+x(-t)\right]+\frac{1}{2}\left[x(t)-x(-t)\right]$$

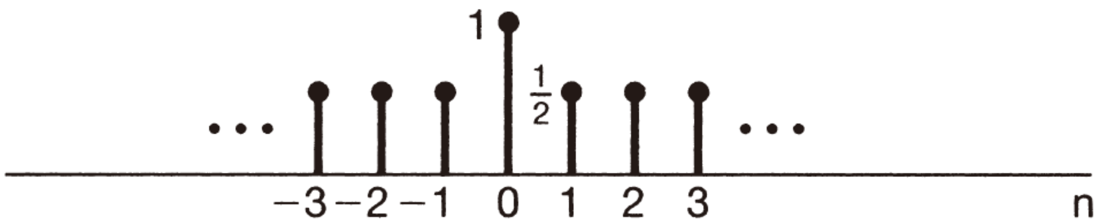
$$=\frac{1}{2}\left[x(t)+x(t)\right]=x(t)$$

1.2.3 Even and Odd Signals

$$x[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



$$\mathcal{E}v\{x[n]\} = \begin{cases} \frac{1}{2}, & n < 0 \\ 1, & n = 0 \\ \frac{1}{2}, & n > 0 \end{cases}$$



1.2.3 Even and Odd Signals

$$\mathcal{O}d\{x[n]\} = \begin{cases} -\frac{1}{2}, & n < 0 \\ 0, & n = 0 \\ \frac{1}{2}, & n > 0 \end{cases}$$

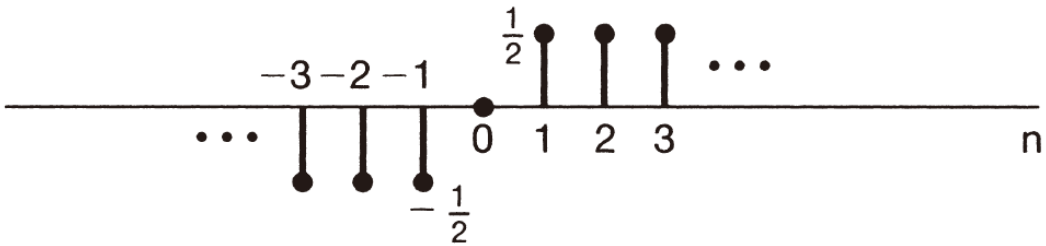


Figure 1.18 Example of the even-odd decomposition of a discrete-time signal.

1.1.2 Signal Energy and Power

- Sometime the signals we consider are directly related to physical quantities capturing power and energy in a physical system.
- If $v(t)$ and $i(t)$ are, respectively, the voltage and current across a resistor with resistance R, then the instantaneous power is

$$p(t) = v(t)i(t) = \frac{1}{R}v^2(t) \qquad (1.1)$$

1.1.2 Signal Energy and Power

The total energy expended over the time interval is $t_1 \le t \le t_2$

$$\int_{t_1}^{t_2} p(t)dt = \int_{t_1}^{t_2} \frac{1}{R} v^2(t)dt \qquad (1.2)$$

and the average power over this time interval is

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p(t)dt = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \frac{1}{R} v^2(t)dt \qquad (1.3)$$

1.1.2 Signal Energy and Power

The total energy over the time interval $t_1 \le t \le t_2$ in a continuous-time signal $x(t)$ is defined as

$$\int_{t_1}^{t_2} \left| x(t) \right|^2 dt \qquad (1.4)$$

where $|x|$ denotes the magnitude of number x. Dividing by the duration t_2-t_1 yields the average **power** over the duration.

1.1.2 Signal Energy and Power

The total energy in a discrete-time signal x[n] over the time interval $n_1 \le n \le n_2$ is defined as

$$\sum_{n=n_1}^{n_2} \left| x[n] \right|^2 \qquad (1.5)$$

and dividing by the number of points in the interval, $n_2 - n_1 + 1$, yields the average **power** over the interval.

1.1.2 Signal Energy and Power

We define the **total energy** as limits of eqs.(1.4) and (1.5) as the time interval increases without bound. In continuous time,

$$E_\infty \triangleq \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2\,dt = \int_{-\infty}^{+\infty} |x(t)|^2\,dt \qquad (1.6)$$

and in discrete time,

$$E_\infty \triangleq \lim_{N\to\infty} \sum_{n=-N}^{+N} |x[n]|^2 = \sum_{n=-\infty}^{+\infty} |x[n]|^2 \qquad (1.7)$$

1.1.2 Signal Energy and Power

In an analogous fashion, we can define the **time-averaged power** over an infinite interval as

$$p_\infty \triangleq \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 \, dt \qquad (1.8)$$

and

$$p_\infty \triangleq \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{+N} |x[n]|^2 \qquad (1.9)$$

1.1.2 Signal Energy and Power

$$p_\infty \triangleq \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2\,dt \quad (1.8) \qquad E_\infty \triangleq \lim_{T\to\infty}\int_{-T}^{T}|x(t)|^2\,dt(1.6)$$

We see from eq.(1.8) that if E_∞ is finite

$$p_\infty = \lim_{T\to\infty}\frac{E_\infty}{2T} = 0 \qquad (1.10)$$

An example of **finite-energy** signal is a signal that takes on the value 1 for $0 \le t \le 1$ and 0 otherwise. In this case,

$$E_\infty = 1 \text{ and } P_\infty = 0.$$

Abstraction: from samples to a signal

- Lumping all of the (possibly infinite) samples into a single object —the signal — simplifies its manipulation.

- This lumping is an abstraction that is analogous to
  - representing coordinates in three-space as points
  - representing lists of numbers as vectors in linear algebra
  - creating an object in Python

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals

- The continuous-time complex exponential signal is of the form

$$x(t) = Ce^{at} \qquad (1.20)$$

where C and a are, in general, complex numbers.

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals

$$x(t) = Ce^{at} \qquad (1.20)$$

- Real Exponential Signals

  If C and a are real [in which case $x(t)$ is called a real exponential]

  - If a is positive, then as t increase $|x(t)|$ is a growing exponential
  - If a is negative, then $|x(t)|$ is a decaying exponential
  - $a=0$, $x(t)$ is constant

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals

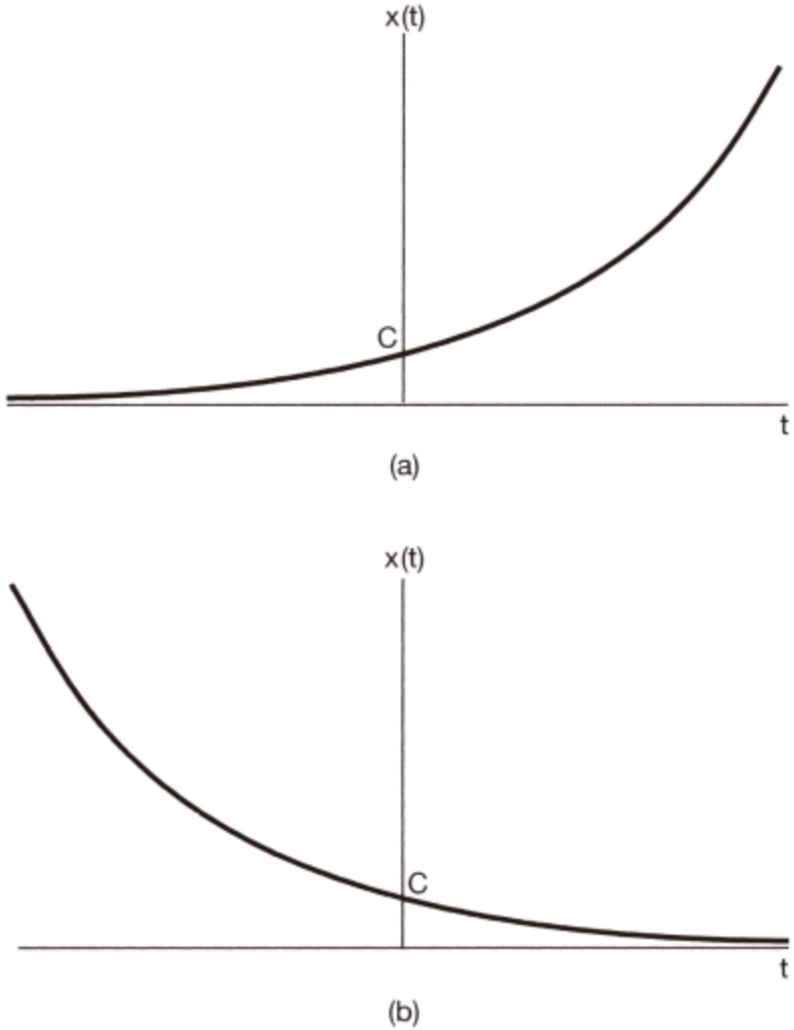


Figure 1.19 Continuous-time real exponential $x(t) = Ce^{at}$: (a) $a > 0$; (b) $a < 0$.

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

- Periodic Complex Exponential:
  A second important class of complex exponential is obtained by constraining **a=jw$_0$** to be purely imaginary.

$$x(t) = e^{j\omega_0 t} \qquad (1.21)$$

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals

Property: it is **periodic**

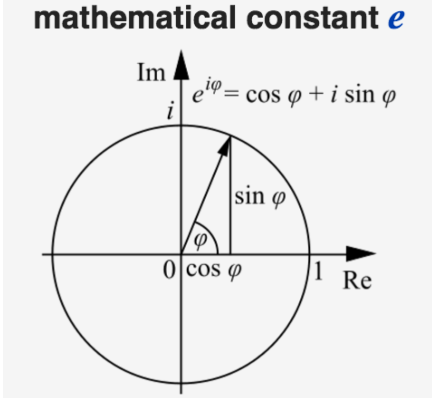
$$e^{j\omega_0 t} = e^{j\omega_0 (t+T)} \qquad (1.22)$$

$$e^{j\omega_0 (t+T)} = e^{j\omega_0 t} e^{j\omega_0 T}$$

it follows that for periodicity, we must have

$$e^{j\omega_0 T} = 1 \qquad (1.23)$$

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals



$$e^{j\omega_0 T} = 1 \qquad (1.23)$$

- If $\omega_0 = 0$, then $x(t) = 1$, which is periodic for any value of T. If $\omega_0 \neq 0$, then the fundamental period T_0 of $x(t)$ is

$$T_0 = \frac{2\pi}{|\omega_0|} \qquad (1.24)$$

Thus, the signals $e^{j\omega_0 t}$ and $e^{-j\omega_0 t}$ have the same fundamental period.

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals

A signal closely related to the periodic complex exponential is the ***sinusoidal signal***

$$x(t) = A\cos(\omega_0 t + \varphi) \qquad (1.25)$$

with seconds as the units of t, the units of φ and ω_0 are radians and radians per second. It is also common to write $\omega_0 = 2\pi f_0$, where f_0 has the units of cycles per second, or hertz (Hz).

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals

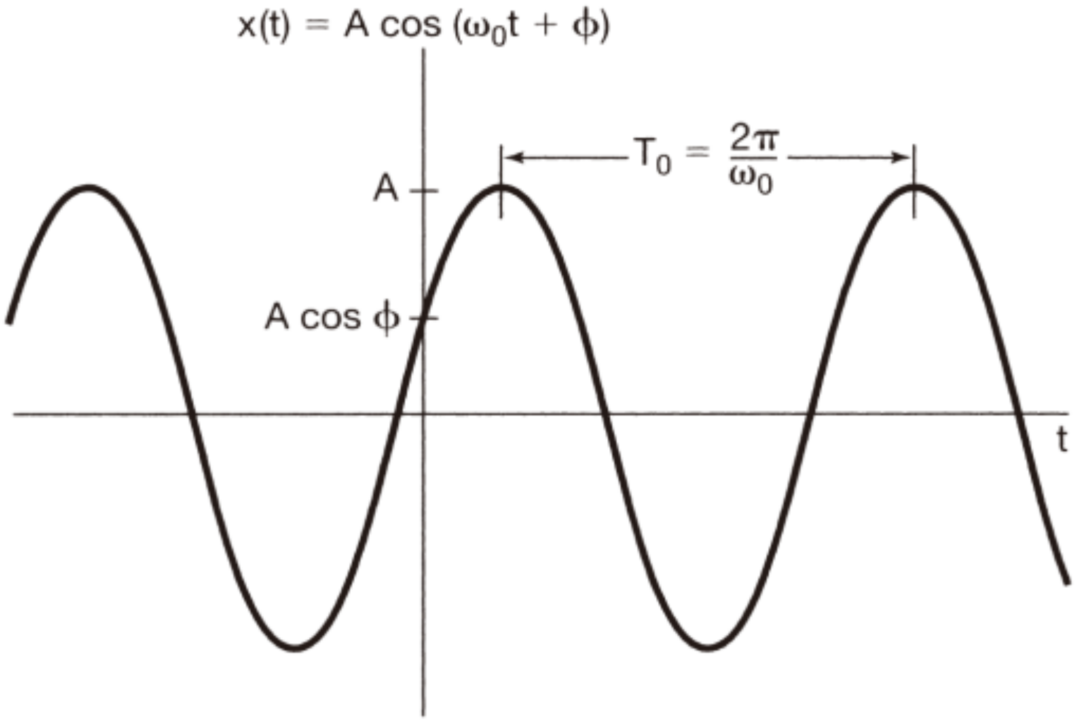
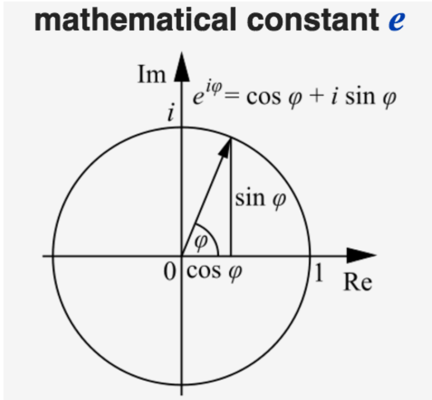


Figure 1.20 Continuous-time sinusoidal signal.

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals



$$x(t) = A\cos(\omega_0 t + \varphi) \qquad (1.25)$$

By using Euler's relation, the complex exponential in eq.(1.21) can be written in terms of ***sinusoidal signals***, again with the same fundamental period:

$$e^{j\omega_0 t} = \cos\omega_0 t + j\sin\omega_0 t \qquad (1.26)$$

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals

$$x(t) = A\cos(\omega_0 t + \varphi) \qquad (1.25)$$

the sinusoidal signal of eq. (1.25) can be written in terms of periodic complex exponentials

$$A\cos(\omega_0 t + \varphi) = \frac{A}{2} e^{j\varphi} e^{j\omega_0 t} + \frac{A}{2} e^{-j\varphi} e^{-j\omega_0 t} \qquad (1.27)$$

$$\cos x = \mathrm{Re}\{e^{ix}\} = \frac{e^{ix} + e^{-ix}}{2}$$

$$\sin x = \mathrm{Im}\{e^{ix}\} = \frac{e^{ix} - e^{-ix}}{2i}$$

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals

we can express a sinusoid in terms of a complex exponential signal as

$$A\cos(\omega_0 t + \varphi) = A\,\mathrm{Re}\left\{e^{j(\omega_0 t + \varphi)}\right\} \qquad (1.28)$$

where, if c is a complex number, $\mathrm{Re}\{c\}$ denotes its real part. We will also use the notation $\mathrm{Jm}\{c\}$ for the imaginary part of c, so that, for example,

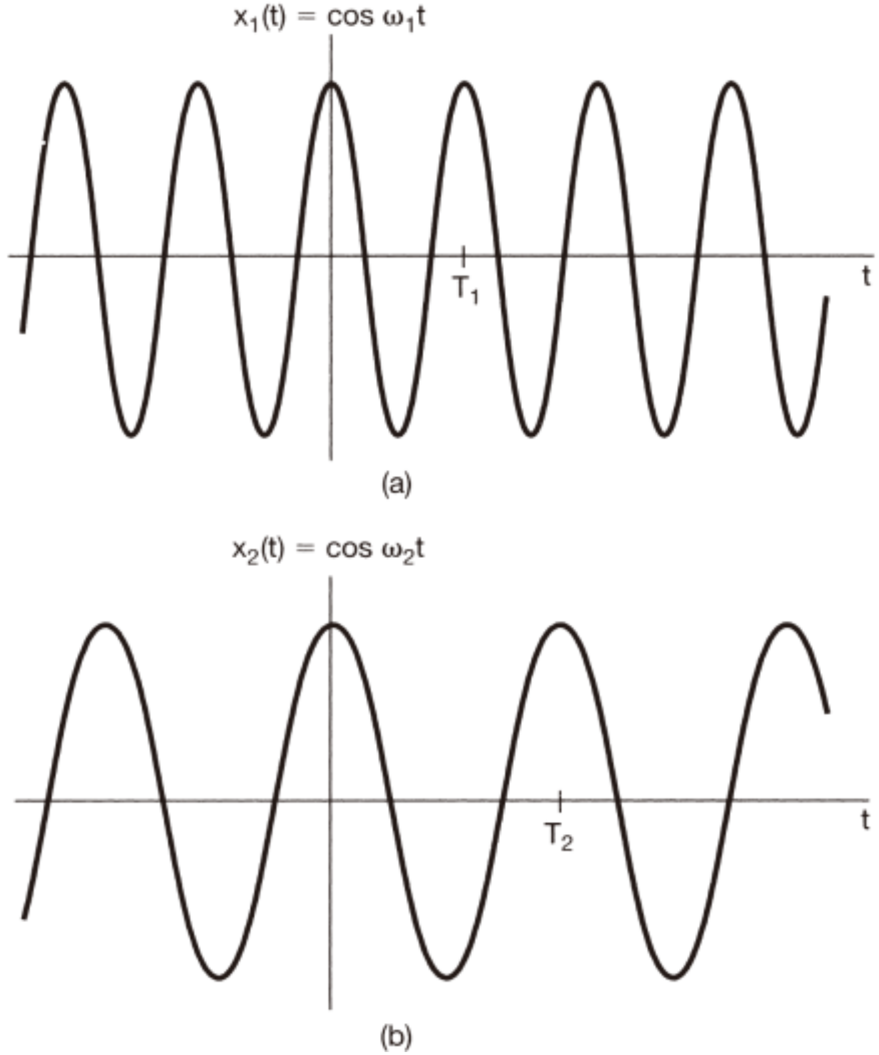
$$A\sin(\omega_0 t + \varphi) = A\,\mathrm{Jm}\left\{e^{j(\omega_0 t + \varphi)}\right\} \qquad (1.29)$$

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals

$$T_0 = \frac{2\pi}{|\omega_0|} \qquad (1.24)$$

- We see that the **fundamental period** T_0 of a continuous-time sinusoidal signal or a periodic complex exponential is inversely proportional to $|\omega_0|$, which we will refer to as the ***fundamental frequency***.
- $\omega_0 = 0$. We mentioned earlier, *x(t)* is constant and therefore is periodic with period T for any positive value of *T*.

1.3.1 Check Yourself



(a)

(b)

$$T_1 < T_2$$

$$\omega_1 ? \omega_2$$

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals

$$E_{\text{period}} = \int_0^{T_0} \left| e^{j\omega_0 t} \right|^2 dt$$

$$= \int_0^{T_0} 1 \cdot dt = T_0 \qquad (1.30)$$

$$P_{\text{period}} = \frac{1}{T_0} E_{\text{period}} = 1 \qquad (1.31)$$

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals

It has infinite energy:

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} \left| e^{j\omega_0 t} \right|^2 dt = \lim_{T\to\infty} 2T = \infty$$

It has infinite average power equal to

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left| e^{j\omega_0 t} \right|^2 dt = 1 \qquad (1.32)$$

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

a necessary condition for a complex exponential $e^{j\omega t}$ to be **periodic with period** T_0 is that

$$e^{j\omega T_0} = 1 \qquad (1.33)$$

which implies that ωT_0 is a multiple of 2π,i.e.,

$$\omega T_0 = 2\pi k, \qquad k = 0,\pm 1,\pm 2,\ldots. \qquad (1.34)$$

if we define

$$\omega_0 = \frac{2\pi}{T_0} \qquad (1.35)$$

$$\omega = \omega_0 k$$

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals

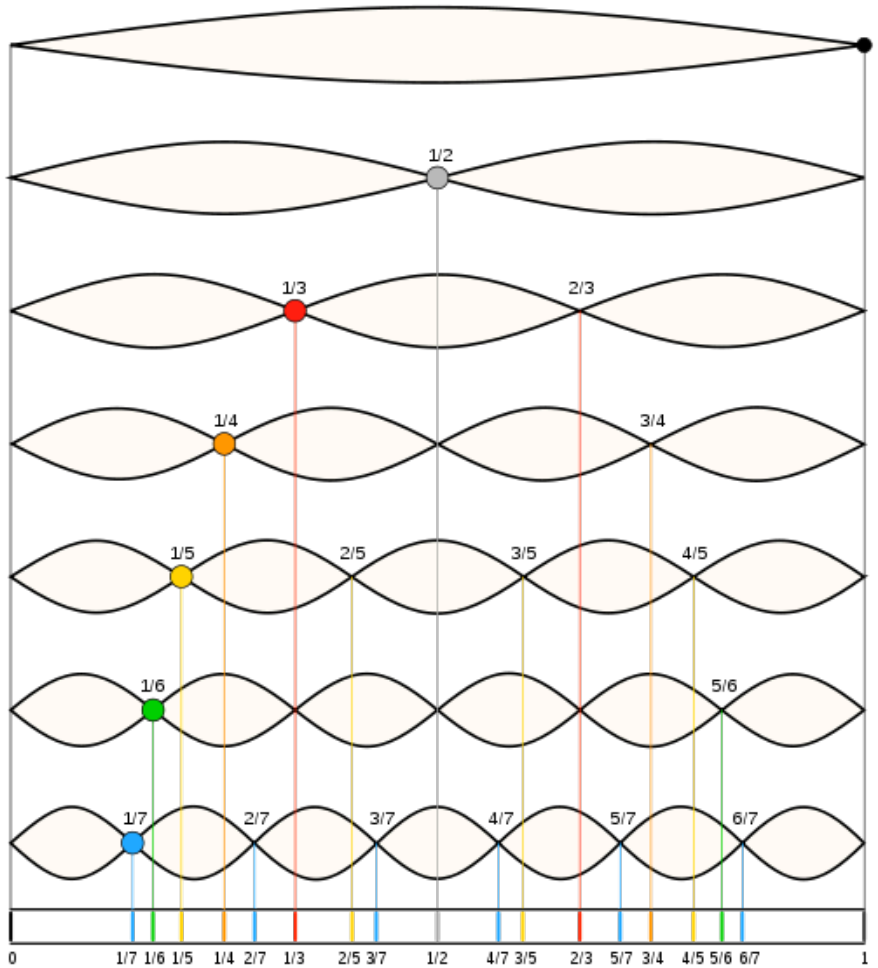
Harmonically related set of complex exponentials is a set of periodic exponentials with fundamental frequencies that are all multiples of a single positive frequency ω_0 :

$$\varphi_k(t) = e^{jk\omega_0 t}, \qquad k = 0, \pm 1, \pm 2, \ldots \qquad (1.36)$$

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals

$$\varphi_k(t) = e^{jk\omega_0 t}, \qquad k = 0, \pm 1, \pm 2, \ldots \tag{1.36}$$

They can be superimposed into a rich set of period signals (see Chap. 3)



$$\frac{2\pi}{|k|\omega_0} = \frac{T_0}{|k|} \tag{1.37}$$

Announcement

- 1st quiz on March 2, 2016
  - 15 minutes
  - Related to Chapter 1

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals

- General Complex Exponential Signals

$$C = |C|e^{j\theta}$$

$$a = r + j\omega_0$$

$$Ce^{at} = |C|e^{j\theta}e^{(r+j\omega_0)t} = |C|e^{rt}e^{j(\omega_0 t+\theta)} \qquad (1.42)$$

Using Euler's relation, we can expand this further as

$$Ce^{at} = |C|e^{rt}\cos(\omega_0 t+\theta) + j|C|e^{rt}\sin(\omega_0 t+\theta) \qquad (1.43)$$

1.3.1Continuous-Time Complex Exponential and Sinusoidal Signals

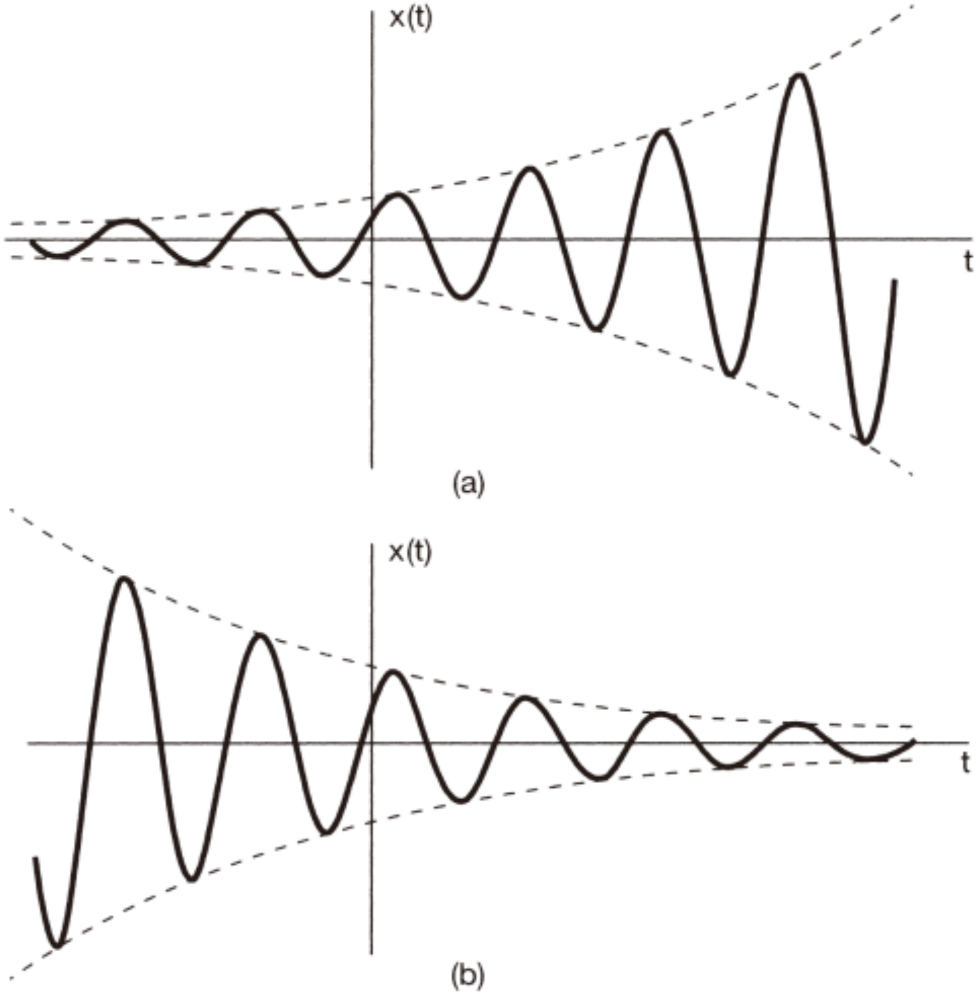


圖 1.23(a) 爲漸增弦波訊號 ($r > 0$)。
圖 1.23(b) 爲衰減弦波訊號 ($r < 0$)。

Figure 1.23 (a) Growing sinusoidal signal $x(t) = Ce^{rt} \cos(\omega_0 t + \theta)$, $r > 0$; (b) decaying sinusoid $x(t) = Ce^{rt} \cos(\omega_0 t + \theta)$, $r < 0$.

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals

$x(t) = Ce^{at}$ (1.20)

As in continuous time, an important signal in discrete time is the complex exponential signal or sequence, defined by

$$x[n] = C\alpha^n \qquad (1.44)$$

Where C and α are, in general, complex numbers. This could alternatively be expressed in the form

$$x[n] = Ce^{\beta n} \qquad (1.45)$$

Where

$$\alpha = e^{\beta}$$

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals

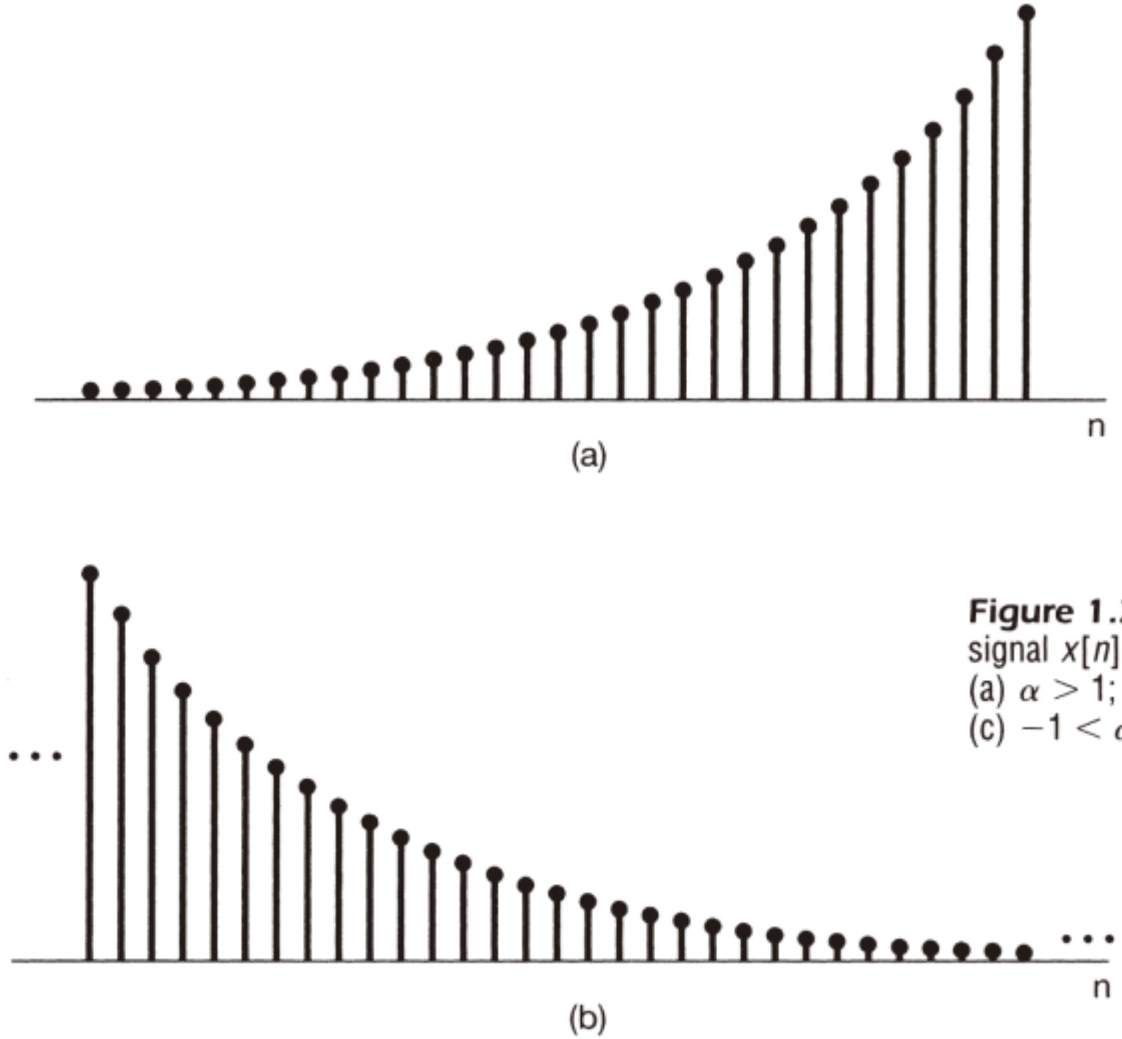


Figure 1.24 The real exponential signal $x[n] = C\alpha^n$: (a) $\alpha > 1$; (b) $0 < \alpha < 1$; (c) $-1 < \alpha < 0$; (d) $\alpha < -1$.

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals

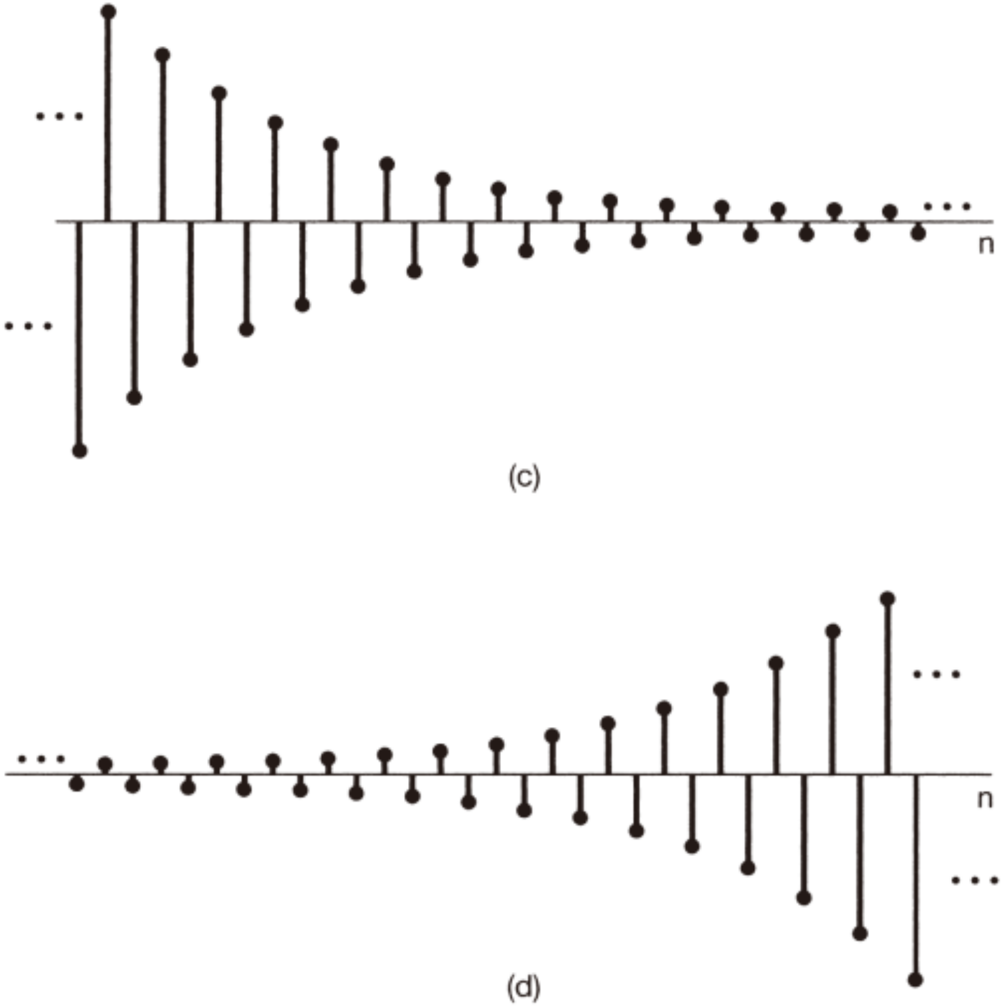


Figure 1.24 The real exponential signal $x[n] = C\alpha^n$:
(a) $\alpha > 1$; (b) $0 < \alpha < 1$;
(c) $-1 < \alpha < 0$; (d) $\alpha < -1$.

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals

$$x[n] = Ce^{\beta n} \qquad (1.45)$$

- **Sinusoidal-related** complex exponential

$$x[n] = e^{j\omega_0 n} \qquad (1.46)$$

As in the continuous-time case, this signal is closely related to the sinusoidal signal

$$x[n] = A\cos(\omega_0 n + \varphi) \qquad (1.47)$$

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals

As before, Euler's relation allows us to relate complex exponentials and sinusoids:

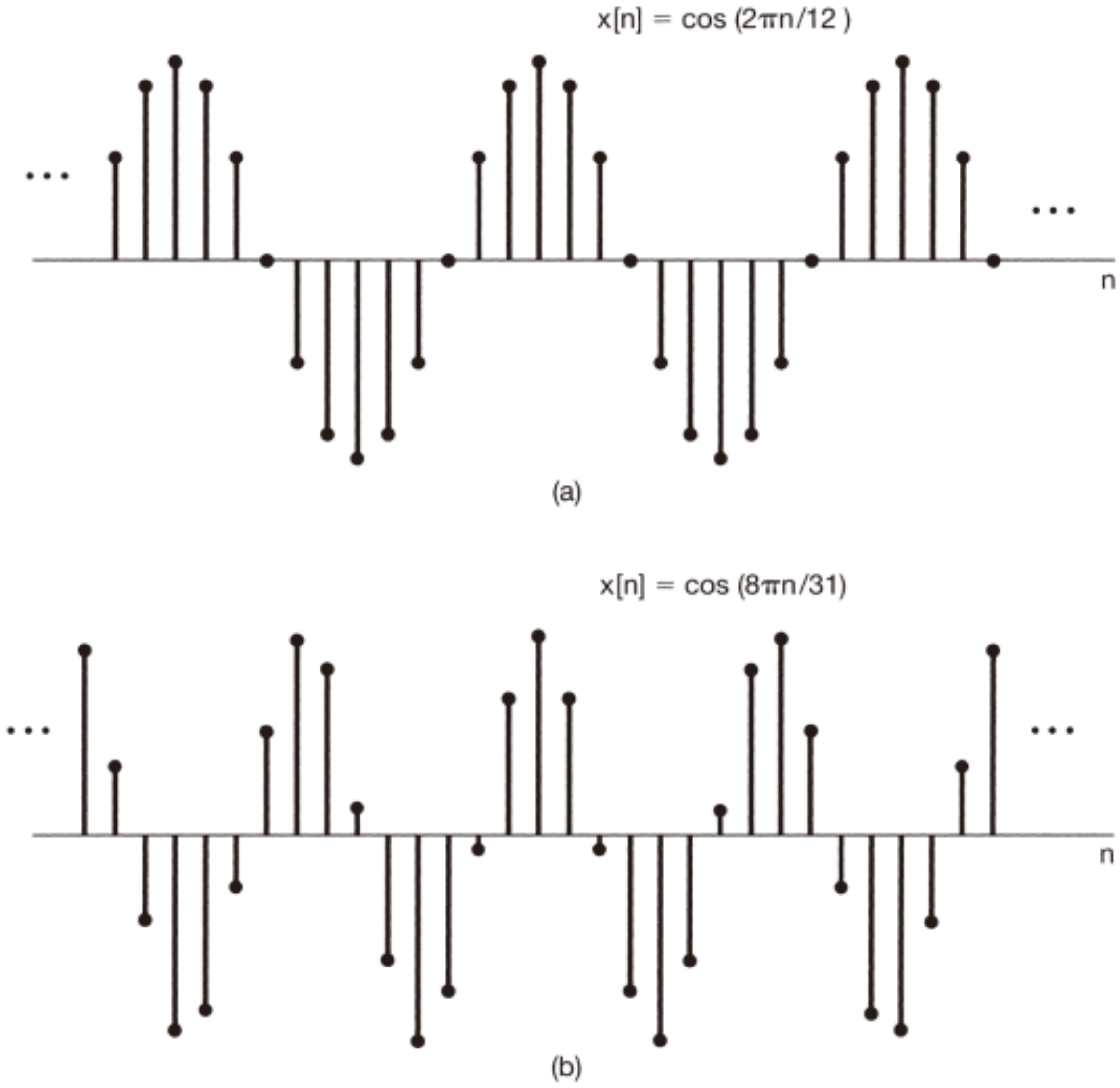
$$e^{j\omega_0 n} = \cos\omega_0 n + j\sin\omega_0 n \qquad (1.48)$$

and

$$A\cos(\omega_0 n + \varphi) = \frac{A}{2}e^{j\varphi}e^{j\omega_0 n} + \frac{A}{2}e^{-j\varphi}e^{-j\omega_0 n} \qquad (1.49)$$

Are they periodic?

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals



1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals

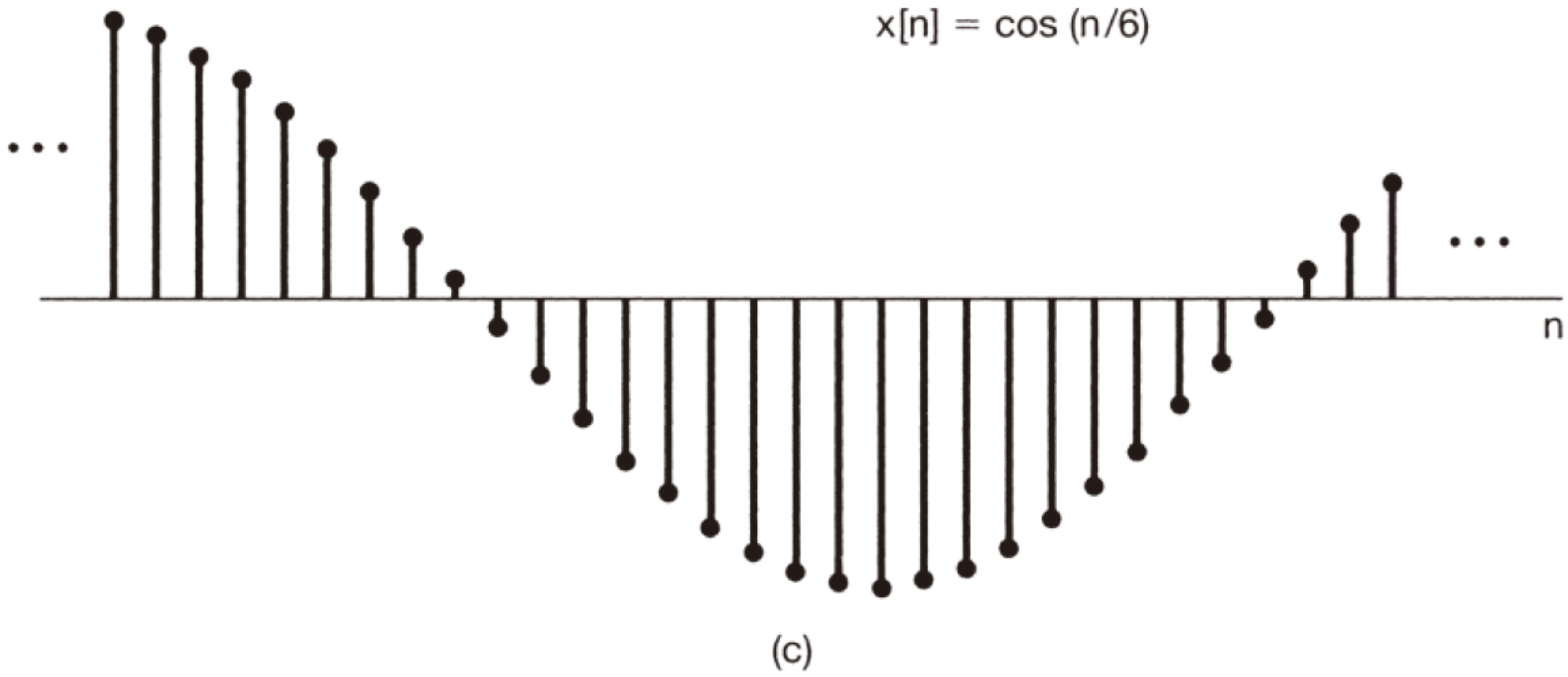


Figure 1.25 Discrete-time sinusoidal signals.

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals

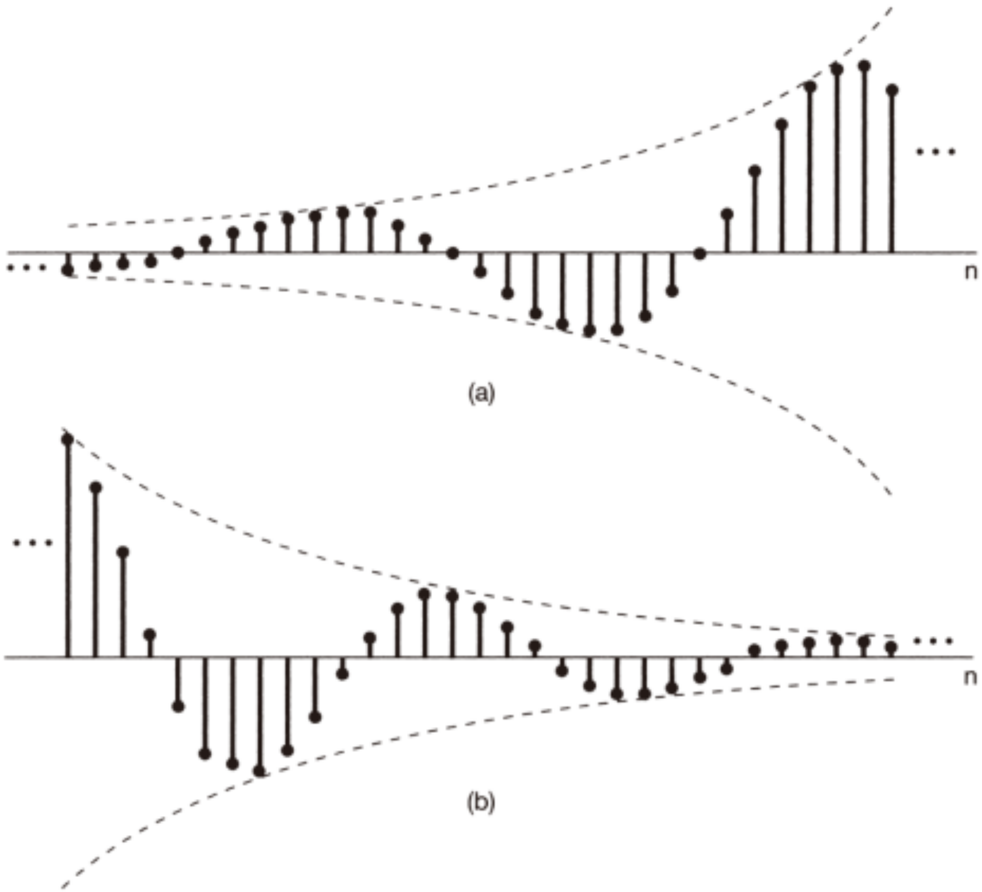
- General Complex Exponential Signals

$$C = |C|e^{j\theta}$$

$$\alpha = |\alpha|e^{j\omega_0}$$

$$C\alpha^n = |C||\alpha|^n \cos(\omega_0 n + \theta) + j|C||\alpha|^n \sin(\omega_0 n + \theta) \qquad (1.50)$$

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals



$|\alpha| ? 1$

$|\alpha| ? 1$

Figure 1.26 (a) Growing discrete-time sinusoidal signals; (b) decaying discrete-time sinusoid.

$$C\alpha^n = |C||\alpha|^n \cos(\omega_0 n + \theta) + j|C||\alpha|^n \sin(\omega_0 n + \theta) \qquad (1.50)$$

1.3.3 Periodicity Properties of Discrete-Time Complex Exponentials

- Recall in Sec. 1.3.1, we identified the two properties of its continuous-time counterpart $e^{j\omega_0 t}$:
  1. The larger the magnitude of ω_0 , the higher is the rate of oscillation in the signal
  2. $e^{j\omega_0 t}$ is periodic for any value of ω_0 .

1.3.3 Periodicity Properties of Discrete-Time Complex Exponentials

- **The first difference**:

$$e^{j(\omega_0+2\pi)n} = e^{j2\pi n}e^{j\omega_0 n} = e^{j\omega_0 n} \qquad (1.51)$$

We see that the exponential at frequency $\omega_0 + 2\pi$ is the same as that at frequency ω_0 .

- **Distinct Discrete-time Complex Exponentials**

Any interval of length 2 π will do, on most occasions we will use the interval $0 \le \omega_0 \le 2\pi$ or the interval $-\pi \le \omega_0 \le \pi$

1.3.3 Periodicity Properties of Discrete-Time Complex Exponentials

$$0 \le \omega_0 \le 2\pi$$

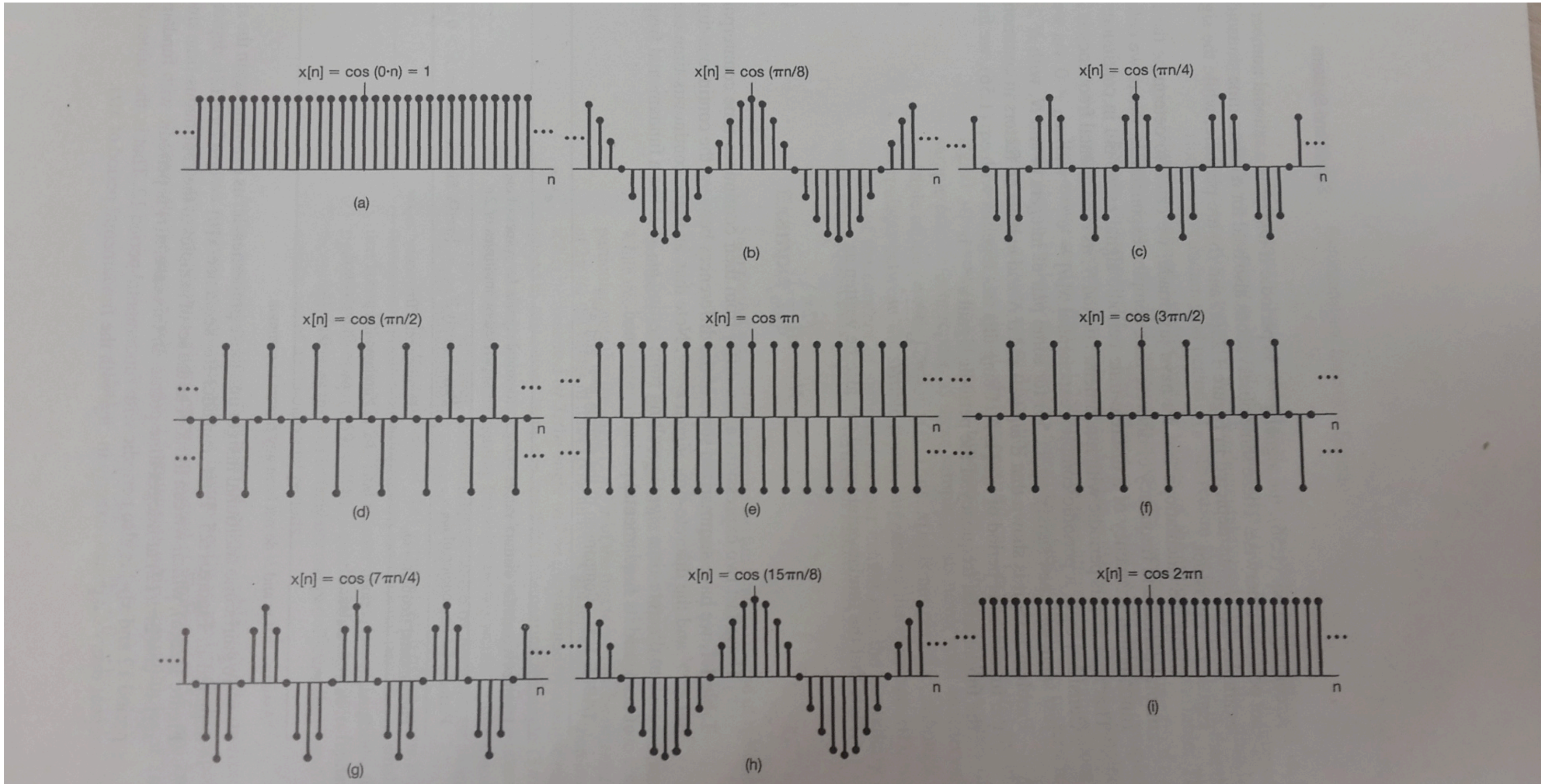


Figure 1.27 Discrete-time sinusoidal sequences for several different frequencies.

$$e^{j\pi n} = \left(e^{j\pi}\right)^n = (-1)^n \qquad (1.52)$$

1.3.3 Periodicity Properties of Discrete-Time Complex Exponentials

- **The second difference**:

  In order for the signal $e^{j\omega_0 n}$ to be **periodic with period** $N>0$, we must have

$$e^{j\omega_0(n+N)} = e^{j\omega_0 n} \qquad (1.53)$$

Or equivalently,

$$e^{j\omega_0 N} = 1 \qquad (1.54)$$

1.3.3 Periodicity Properties of Discrete-Time Complex Exponentials

$\omega_0 N$ must be a multiple of 2π. That is, there must be integer m such that

$$\omega_0 N = 2\pi m \qquad (1.55)$$

or equivalently,

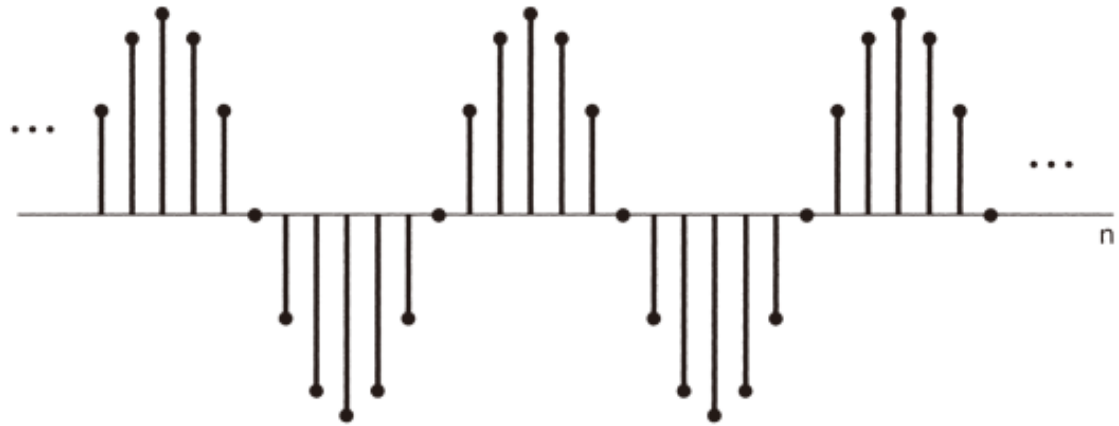
$$\frac{\omega_0}{2\pi} = \frac{m}{N} \qquad (1.56)$$

must be a rational number

Periodic

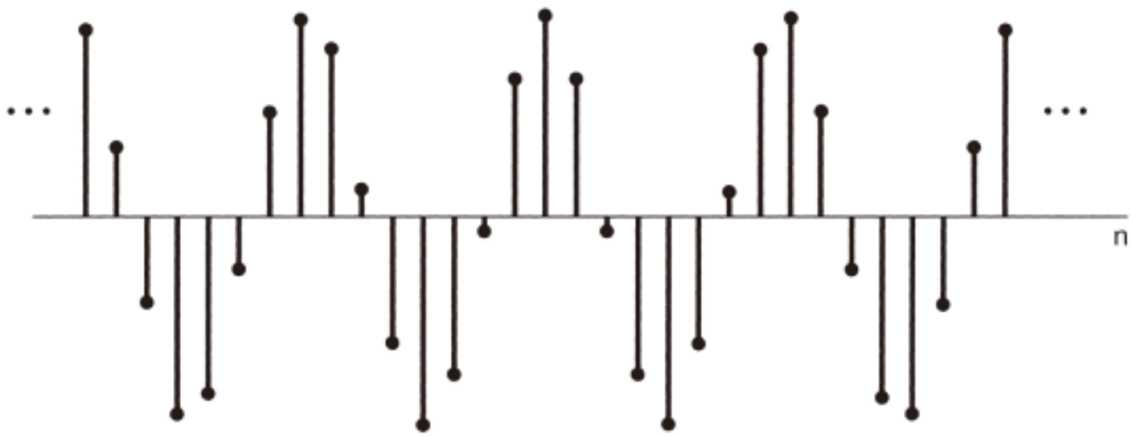
$$\frac{\omega_0}{2\pi} = \frac{m}{N} \qquad (1.56)$$

x[n] = cos (2πn/12)



(a)

x[n] = cos (8πn/31)



(b)

$$\frac{\omega_0}{2\pi} = \frac{m}{N} \qquad (1.56)$$

Aperiodic (not periodic)

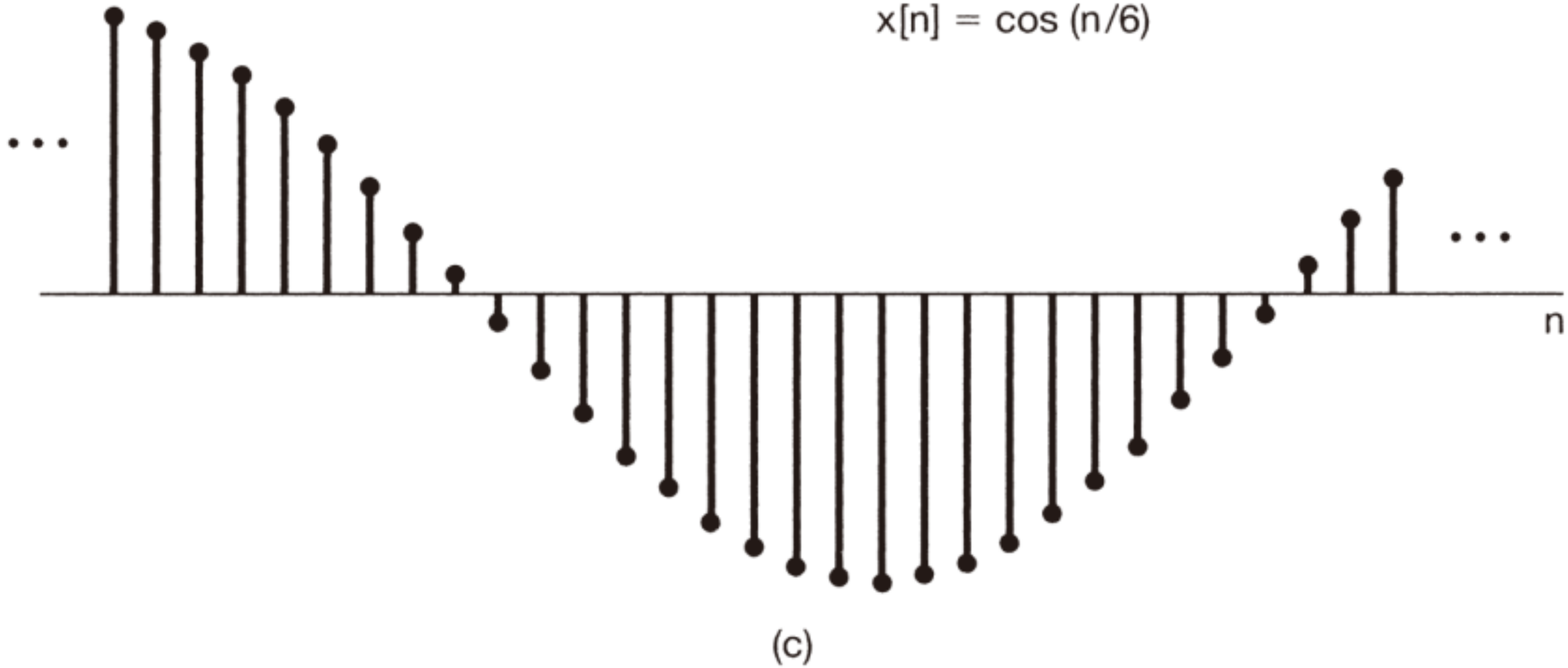


Figure 1.25 Discrete-time sinusoidal signals.

1.3.3 Periodicity Properties of Discrete-Time Complex Exponentials

We find that the fundamental frequency of the periodic signal $e^{j\omega_0 n}$ is

$$\frac{2\pi}{N} = \frac{\omega_0}{m} \qquad (1.57)$$

Note that the fundamental period can also be written as

$$N = m\left(\frac{2\pi}{\omega_0}\right) \qquad (1.58)$$

*assumes N and m has no factor in common

1.3.3 Periodicity Properties of Discrete-Time Complex Exponentials

TABLE 1.1 Comparison of the signals $e^{j\omega_0 t}$ and $e^{j\omega_0 n}$. 表 1.1 $e^{j\omega_0 t}$ 與 $e^{j\omega_0 n}$ 的各種比較

$e^{j\omega_0 t}$	$e^{j\omega_0 n}$
Distinct signals for distinct values of ω_0	Identical signals for values of ω_0 separated by multiples of 2π
Periodic for any choice of ω_0	Periodic only if $\omega_0 = 2\pi m/N$ for some integers $N > 0$ and m.
Fundamental frequency ω_0	Fundamental frequency* ω_0/m
Fundamental period $\omega_0 = 0$: undefined $\omega_0 \neq 0$: $\frac{2\pi}{\omega_0}$	Fundamental period* $\omega_0 = 0$: undefined $\omega_0 \neq 0$: $m\left(\frac{2\pi}{\omega_0}\right)$

*Assumes that m and N do not have any factors in common.

Example 1.6

$$N = m\left(\frac{2\pi}{\omega_0}\right) \qquad (1.58)$$

Suppose that we wish to determine the fundamental period of the discrete-time signal

$$x[n] = e^{j(2\pi/3)n} + e^{j(3\pi/4)n} \qquad (1.59)$$

Sketch:

1. Find each terms N; 2 compute least common multiple

1.3.3 Periodicity Properties of Discrete-Time Complex Exponentials

- Harmonically related complex exponential

$$\varphi_k[n] = e^{jk(2\pi/N)n}, \qquad k = 0, \pm 1, \ldots \tag{1.60}$$

$$\varphi_{k+N}[n] = e^{j(k+N)(2\pi/N)n}$$
$$= e^{jk(2\pi/N)n} e^{j2\pi n} = \varphi_k[n] \tag{1.61}$$

$$\varphi_0[n] = 1, \varphi_1[n] = e^{j2\pi n/N}, \varphi_2[n] = e^{j4\pi n/N}, \ldots, \varphi_{N-1}[n] = e^{j2\pi(N-1)n/N} \tag{1.62}$$

1.4.1 The Discrete-Time Unit Impulse and Unit Step Sequences

One of the simplest discrete-time signals is the unit impulse, which is defined as

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases} \tag{1.63}$$

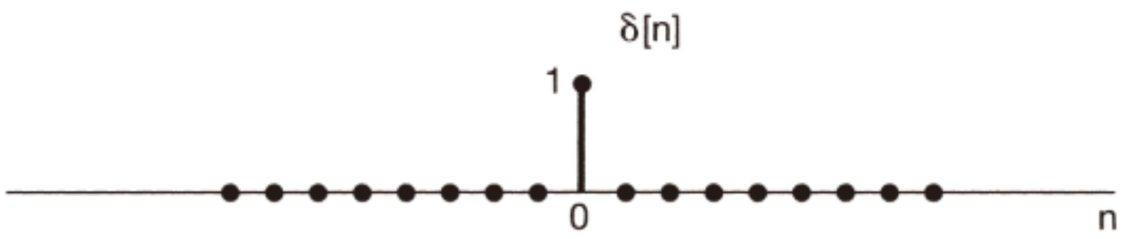


Figure 1.28 Discrete-time unit impulse (sample).

1.4.1 The Discrete-Time Unit Impulse and Unit Step Sequences

A second basic discrete-time signal is the discrete-time unit step, denoted by $u[n]$ and defined by

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \boxed{\geq} 0 \end{cases} \qquad (1.64)$$

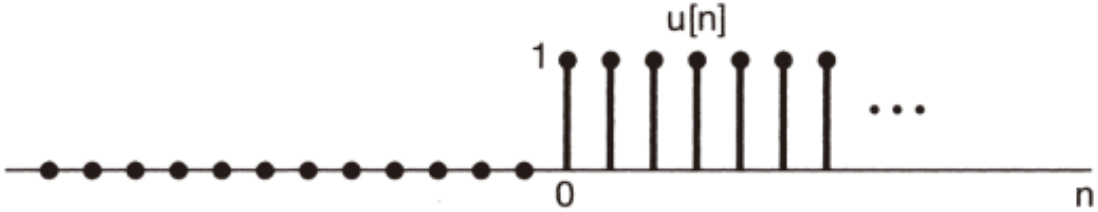


Figure 1.29 Discrete-time unit step sequence.

How are they related?

1.4.1 The Discrete-Time Unit Impulse and Unit Step Sequences

In particular, the discrete-time unit impulse is the first difference of the discrete-time step

$$\delta[n] = u[n] - u[n-1] \qquad (1.65)$$

Conversely, the discrete-time unit step is the running sum of the unit sample. That is,

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] \qquad (1.66)$$

1.4.1 The Discrete-Time Unit Impulse and Unit Step Sequences

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] \tag{1.66}$$

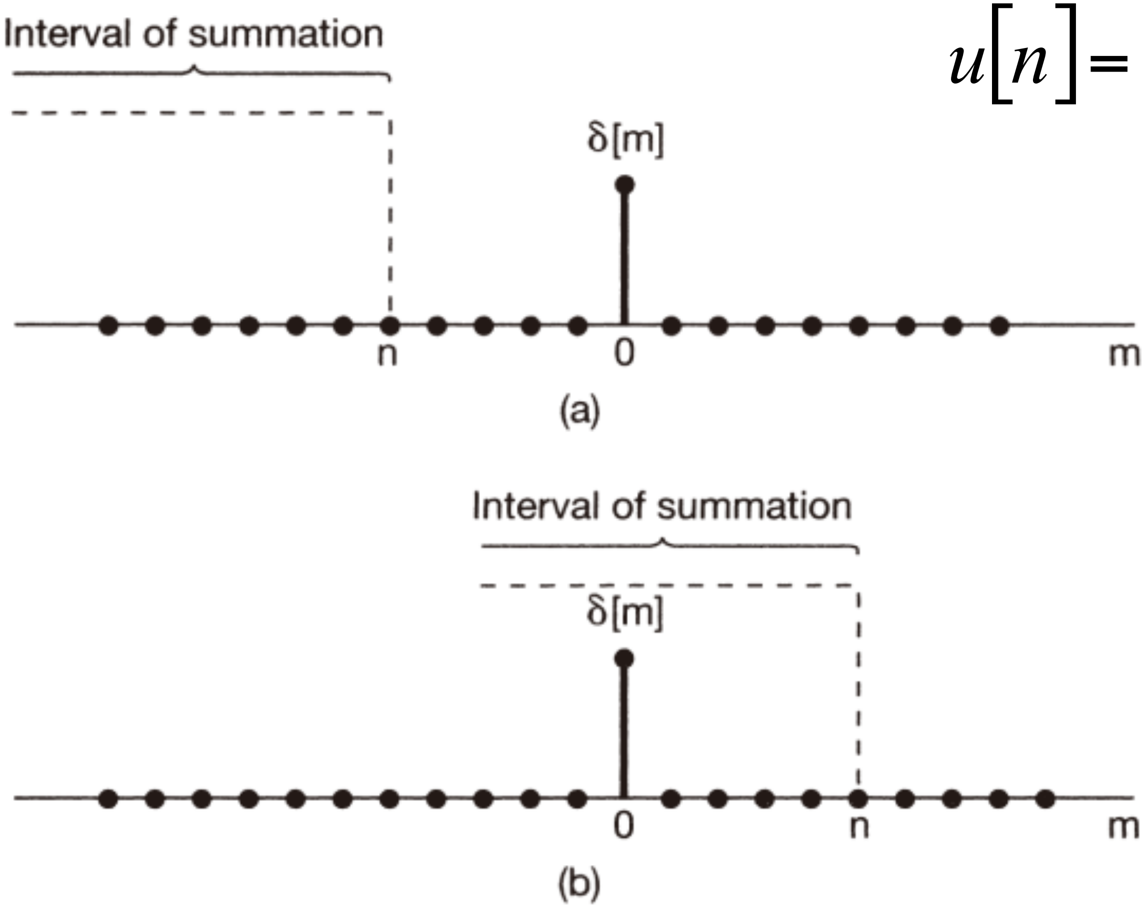


Figure 1.30 Running sum of eq. (1.66): (a) $n < 0$; (b) $n > 0$.

1.4.1 The Discrete-Time Unit Impulse and Unit Step Sequences

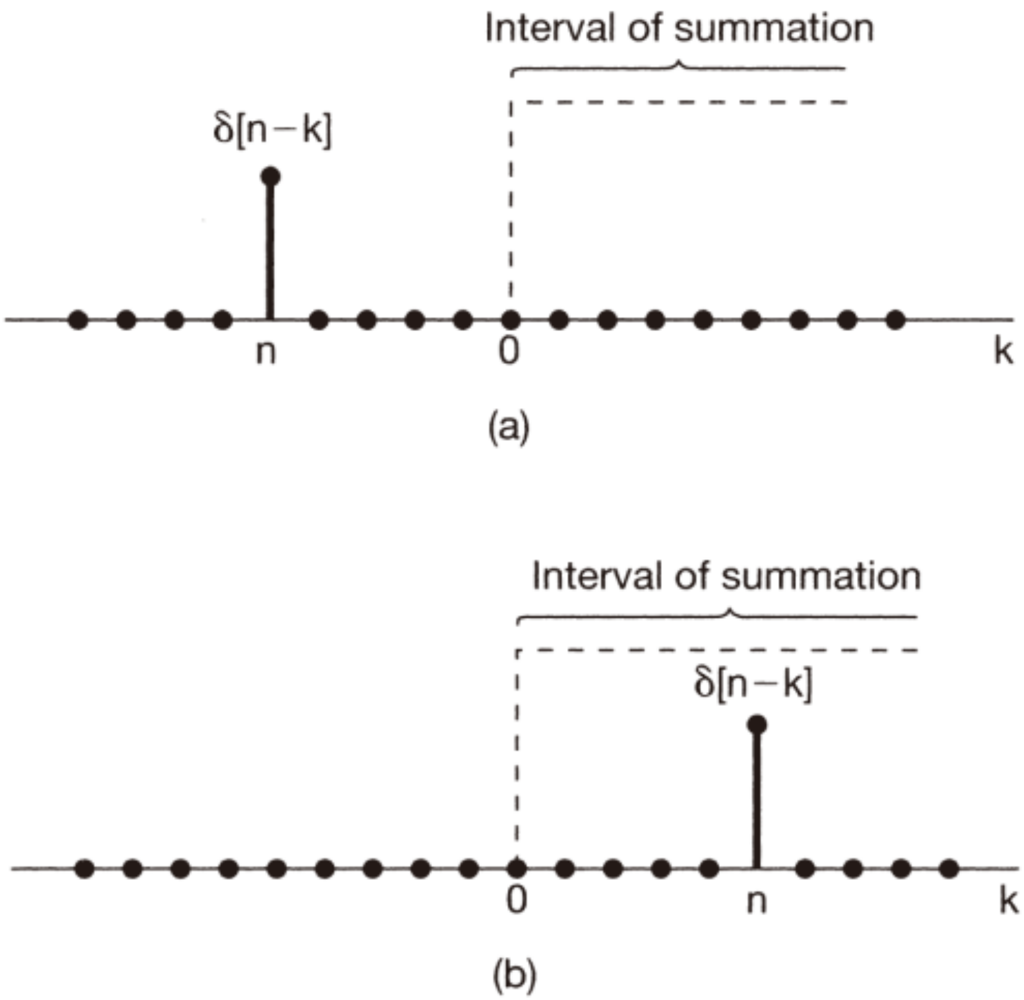
We find that the discrete-time unit step can also be written in terms of the unit sample as

$$u[n]=\sum_{m=-\infty}^{n}\delta[m] \quad \xrightarrow[m=n-k]{} \quad u[n]=\sum_{k=\infty}^{0}\delta[n-k]$$

Or equivalently,

$$u[n]=\sum_{k=0}^{\infty}\delta[n-k] \qquad (1.67)$$

1.4.1 The Discrete-Time Unit Impulse and Unit Step Sequences



$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

Figure 1.31 Relationship given in eq. (1.67): (a) $n < 0$; (b) $n > 0$.

1.4.1 The Discrete-Time Unit Impulse and Unit Step Sequences

Sampling property:

In particular, since $\delta[n]$ is nonzero only for $n = 0$, it follows that

$$x[n]\delta[n] = x[0]\delta[n] \qquad (1.68)$$

More generally, if we consider a unit impulse $\delta[n-n_0]$ at $n = n_0$, then

$$x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0] \qquad (1.69)$$

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

$$\delta[n], u[n] \Rightarrow \delta(t), u(t) \qquad ?$$

The continuous-time unit step function u(t) is defined in a manner similar to its discrete-time counterpart. Specifically,

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases} \tag{1.70}$$

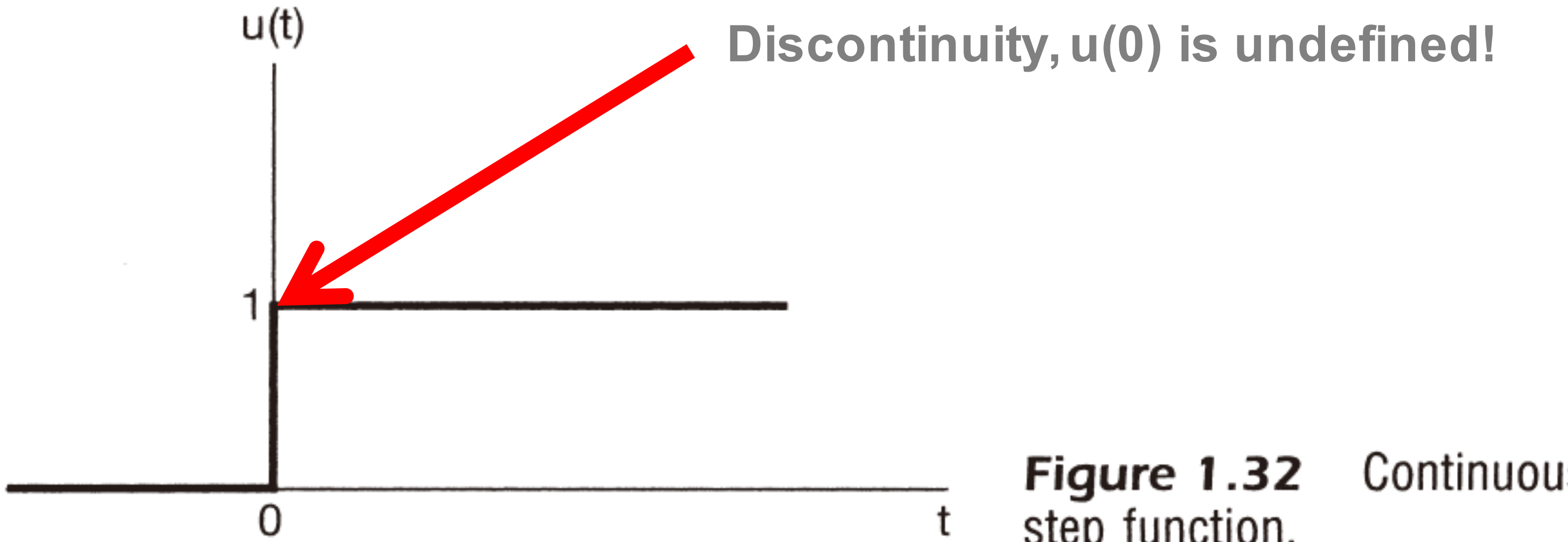


Figure 1.32 Continuous-time unit step function.

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] \qquad (1.66) \qquad\qquad u(t) = \int_{\tau=-\infty}^{t} \delta(\tau)\, d\tau \qquad (1.71)$$

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases} \qquad (1.70)$$

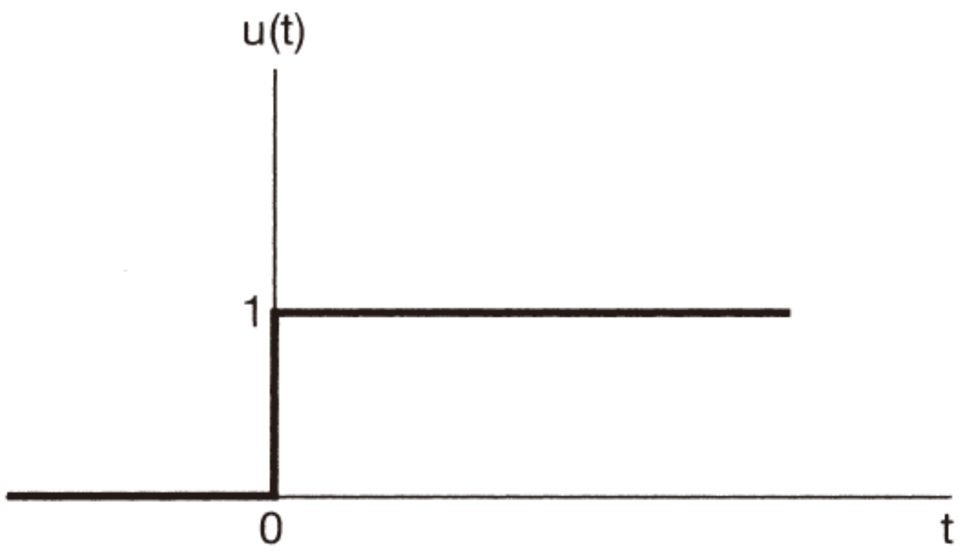


Figure 1.32 Continuous-time unit step function.

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] \qquad (1.66)$$

$$u(t) = \int_{\tau=-\infty}^{t} \delta(\tau)\, d\tau \qquad (1.71)$$

$$u(t) = \lim_{\Delta \to 0} u_{\Delta}(t) \qquad (1.74)$$

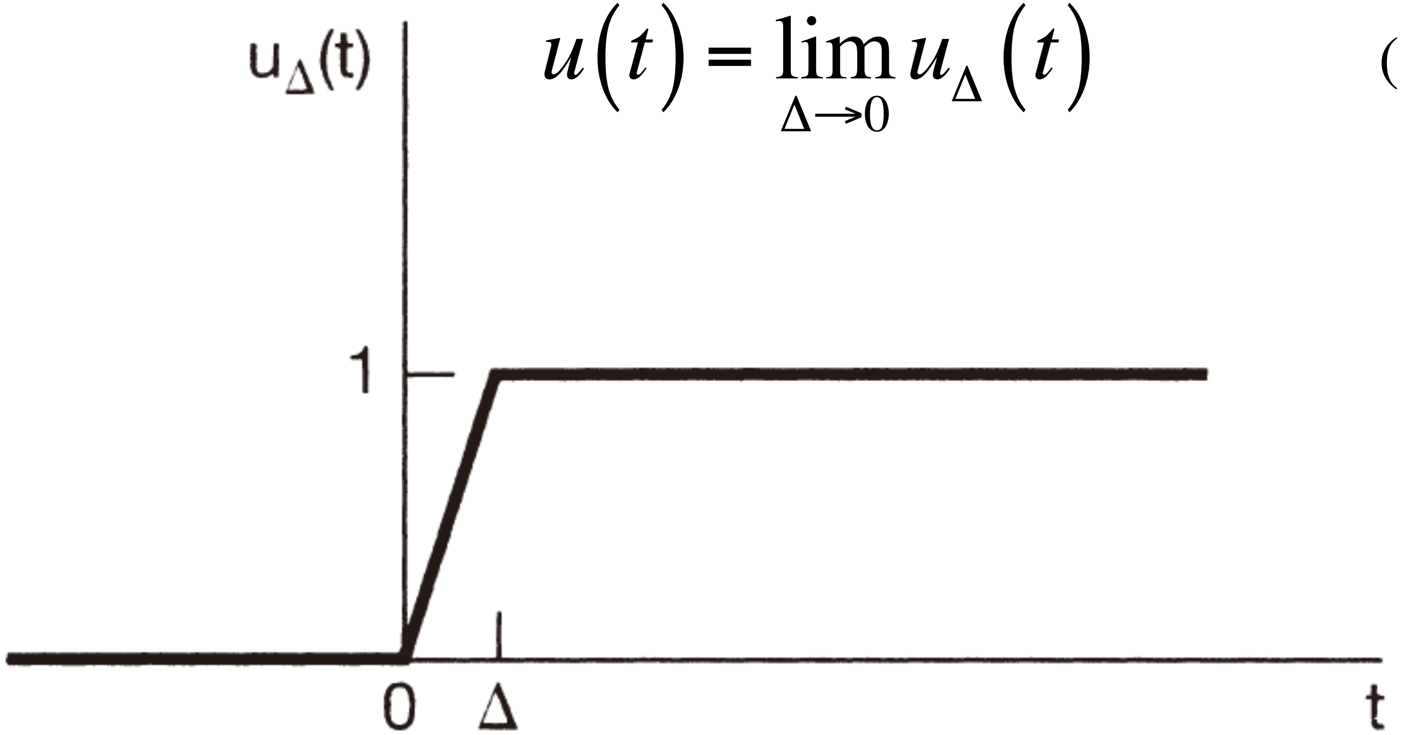


Figure 1.33 Continuous approximation to the unit step, $u_{\Delta}(t)$.

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

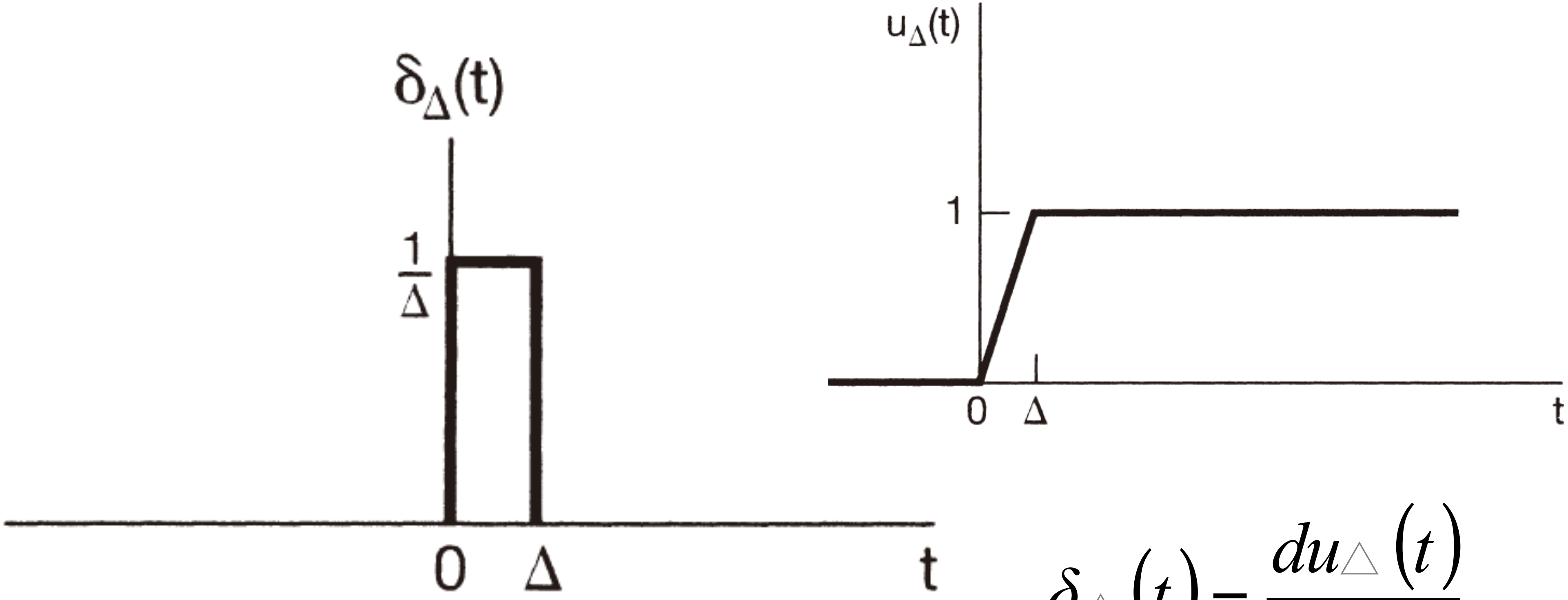
$$u[n] = \sum_{m=-\infty}^{n} \delta[m] \qquad (1.66) \qquad\qquad u(t) = \int_{\tau=-\infty}^{t} \delta(\tau)\, d\tau \qquad (1.71)$$

In particular, it follows from eq.(1.71) that the continuous-time unit impulse can be thought of as the **first derivative** of the continuous-time step:

$$\delta(t) = \frac{du(t)}{dt} \qquad (1.72)$$

$$\delta_{\triangle}(t) = \frac{du_{\triangle}(t)}{dt} \qquad (1.73)$$

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions



$$\delta_{\triangle}(t) = \frac{du_{\triangle}(t)}{dt}$$

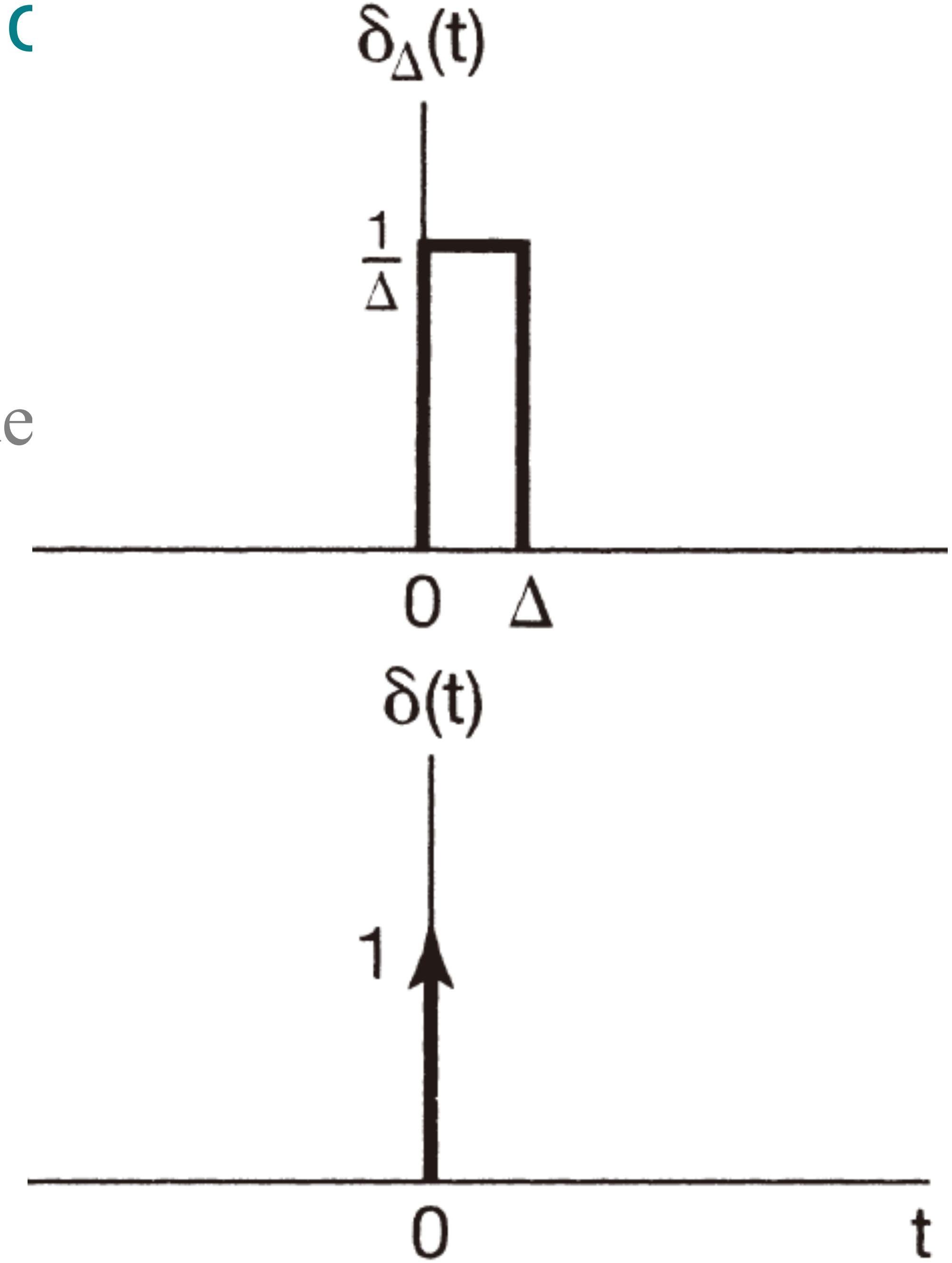
Figure 1.34 Derivative of $u_\Delta(t)$. **Area = 1 = (1/Δ)*Δ**

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functic

Note that $\delta_\Delta(t)$ is a short pulse, of duration Δ and with unit area for any value of Δ. As $\Delta \to 0$, $\delta_\Delta(t)$ becomes narrower and higher, **maintaining its unit area**.

Its limiting form,

$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t) \qquad (1.74)$$



1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

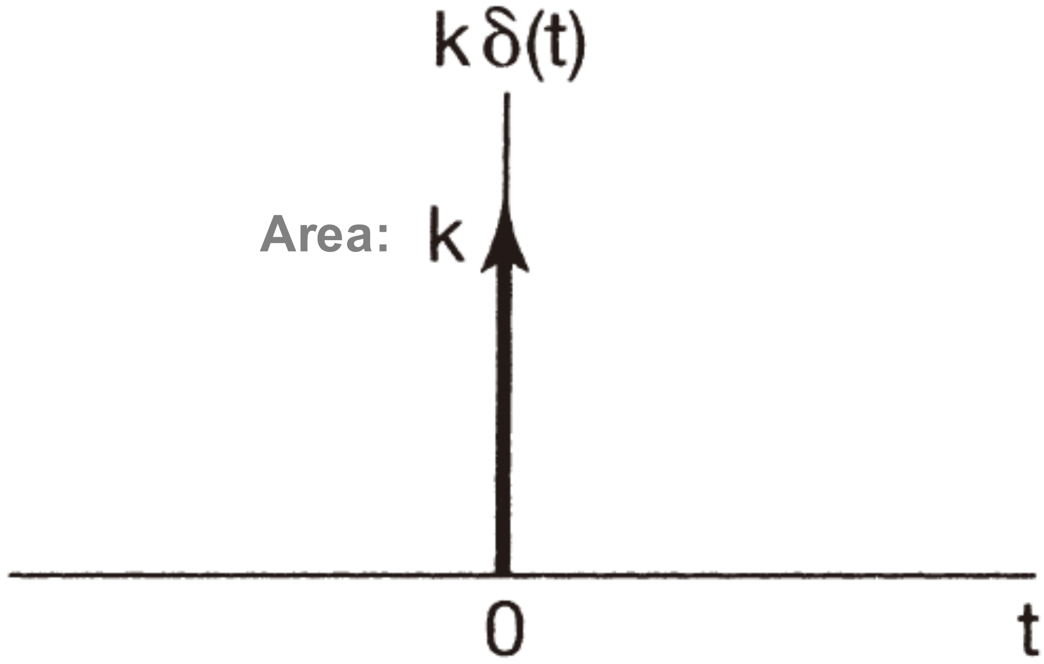


Figure 1.36 Scaled impulse.

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

The continuous-time unit step is also the running **integral** of the unit impulse.

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau \quad (1.71)$$

$$\delta(t) = \frac{du(t)}{dt} \quad (1.72)$$

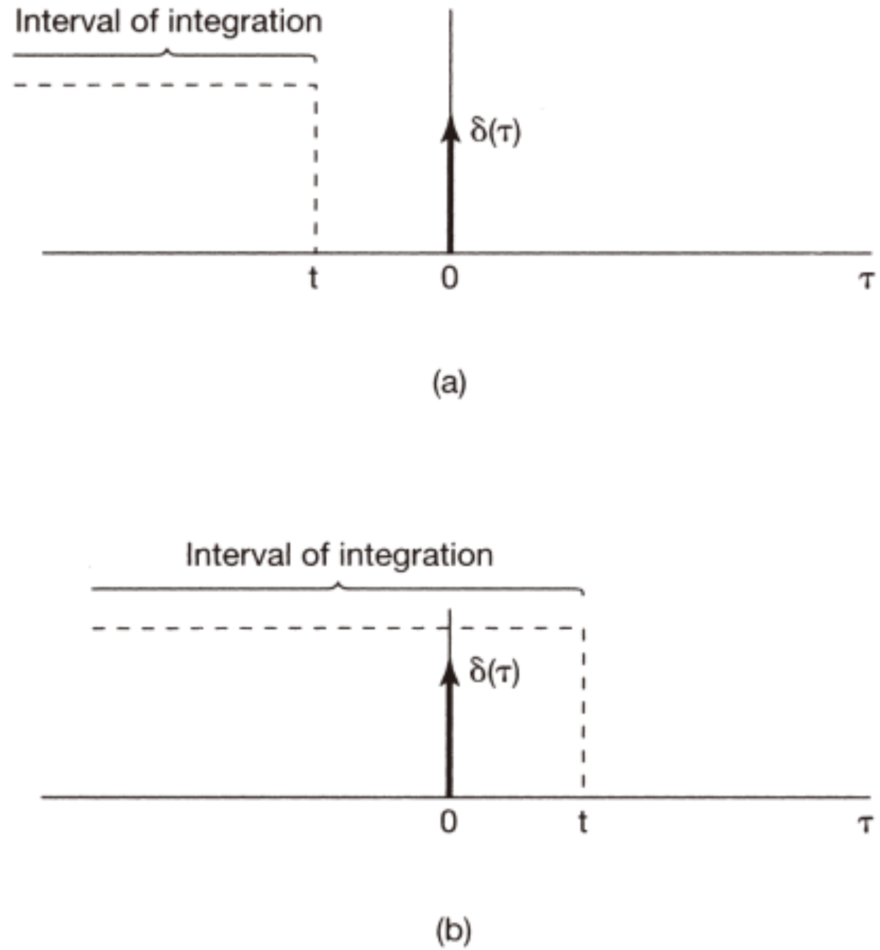


Figure 1.37 Running integral given in eq. (1.71): (a) $t < 0$; (b) $t > 0$.

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

Changing the variable of integration from τ to $\sigma = t - \tau$:

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau = \int_{\infty}^{0} \delta(t-\sigma)(-d\sigma)$$

Or equivalently,

$$u(t) = \int_{0}^{\infty} \delta(t-\sigma)\, d\sigma \qquad (1.75)$$

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

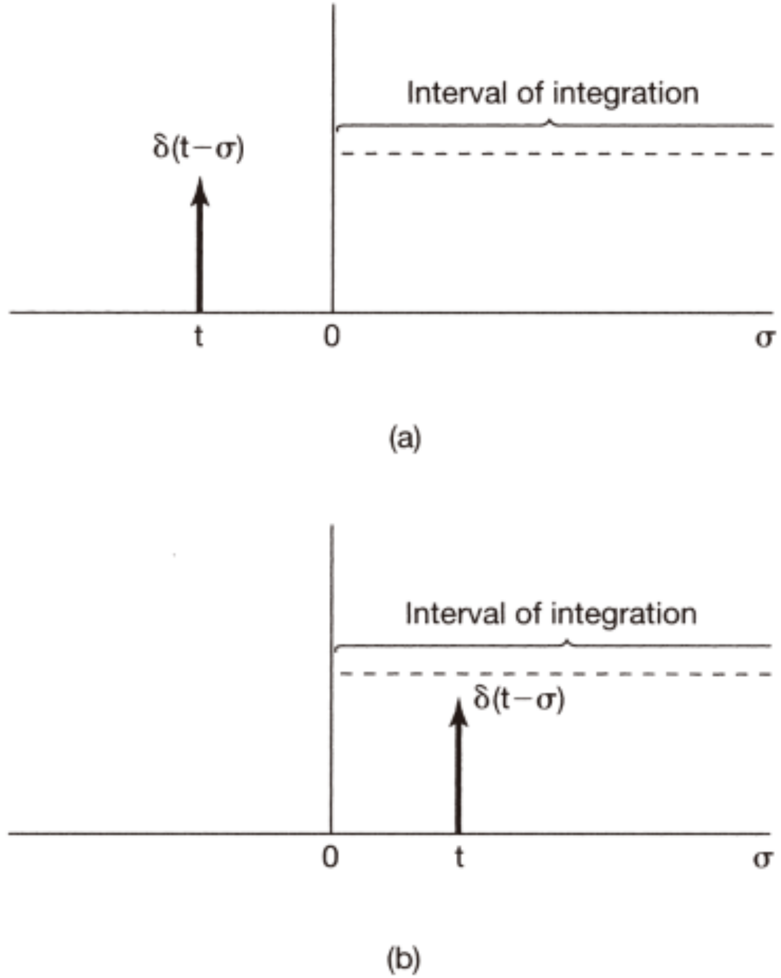


Figure 1.38 Relationship given in eq. (1.75): (a) $t < 0$; (b) $t > 0$.

$$u(t) = \int_0^\infty \delta(t - \sigma)\, d\sigma \quad (1.75)$$

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

Sampling property:

For Δ sufficiently small so that $x(t)$ is approximately constant over this interval,

$$x(t)\delta_{\Delta}(t) \approx x(0)\delta_{\Delta}(t)$$

Since $\delta(t)$ is the limit as $\Delta \to 0$ of $\delta_{\Delta}(t)$, it follows that

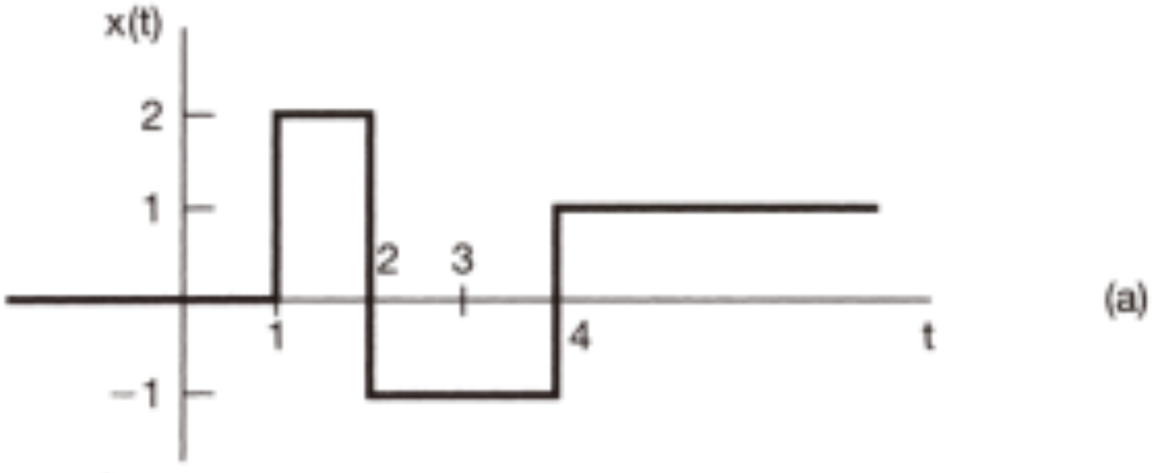
$$x(t)\delta(t) = x(0)\delta(t) \qquad (1.76)$$

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions

By the same argument, we have an analogous expression for an impulse concentrated at an arbitrary point, say, t_0. That is,

$$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$$

Example 1.7



Consider the discontinuous signal *x(t)* depicted in Figure 1.40(a). Because of the relationship between the continuous-time unit impulse and unit step, we can readily calculate and graph the derivative of this signal. Specifically, the derivative of *x(t)* is clearly 0, except at the discontinuities.

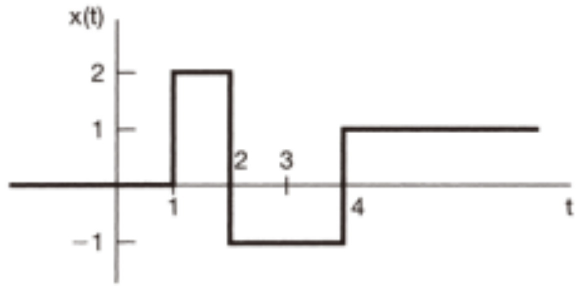
Example 1.7

$$\delta(t) = \frac{du(t)}{dt} \qquad (1.72)$$

In the case of the unit step, we have seen [eq.(1.72)] that differentiation gives rise to a unit impulse located at the point of discontinuity. Furthermore, by multiplying both sides of eq.(1.72) by any number *k*, we see that the derivative of a unit step with a discontinuity of size *k* gives rise to an impulse of area *k* at the point of discontinuity.

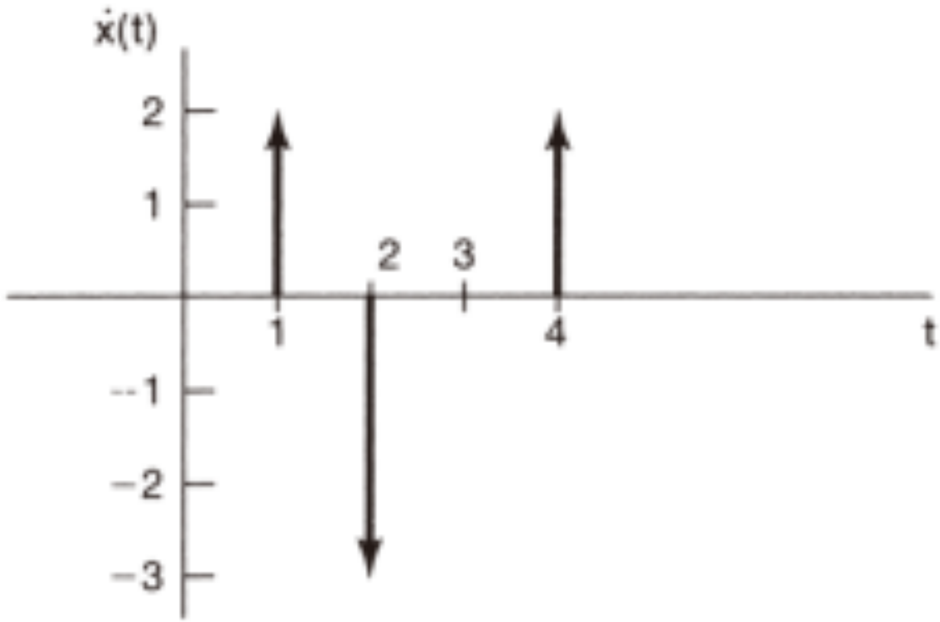
$$k\delta(t) = \frac{dku(t)}{dt} = \mathrm{k}\frac{du(t)}{dt}$$

Example 1.7



(a)

This rule also holds for any other signal with a jump discontinuity, such as *x(t)* in Figure 1.40(a). Consequently, we can sketch its derivative *x(t)*, as in Figure 1.40(b), where an impulse is placed at each discontinuity of *x(t)*, with area equal to the size of the discontinuity.

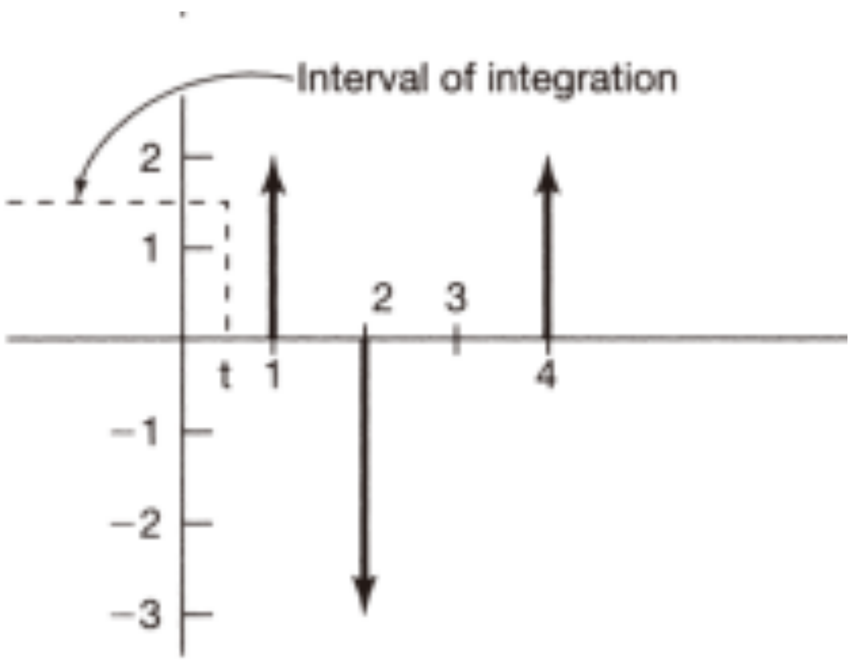


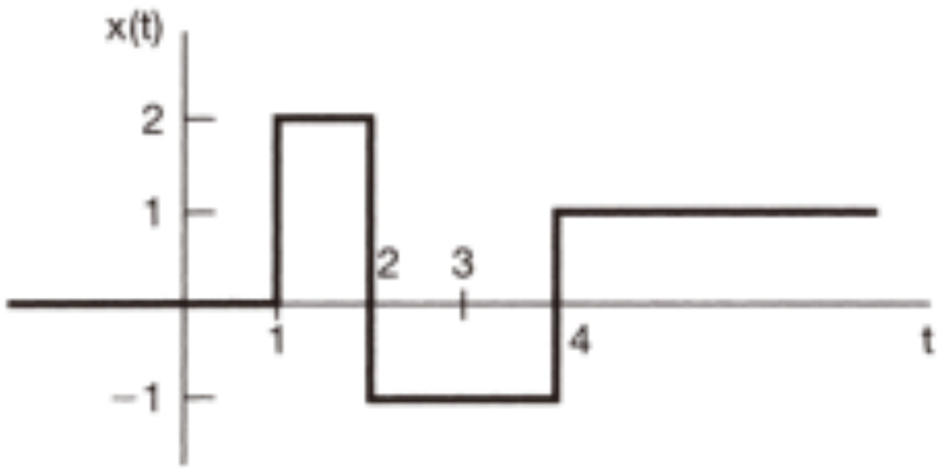
(b)

Example 1.7

As a check of our result, we can verify that we can recover $x(t)$ from $\dot{x}(t)$. Specifically, since $x(t)$ and $\dot{x}(t)$ are both zero for $t \leq 0$, we need only check that for $t > 0$,

$$x(t) = \int_0^t \dot{x}(\tau)\, d\tau. \tag{1.77}$$





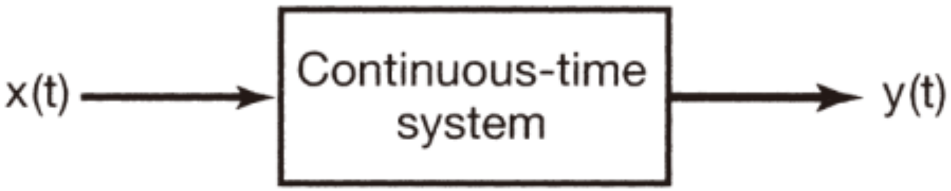
1.5 Continuous-Time and Discrete-Time System

- In signal processing and communications to electromechanical motors, automotive vehicles, and chemical-processing plants, and many more, a system can be viewed as a **process** in which input signals are transformed by the system or cause the system to respond, resulting in other signals as outputs.
  - Verbal description: blah blah blah…
  - Math description: equations
  - Graphical description: block diagram

1.5 Continuous-Time and Discrete-Time System

- A continuous-time system is a system in which continuous-time input signals are applied and result in continuous-time output signals.
- We will often represent the input-output relation of a continuous-time system by the notation.

$$x(t) \to y(t) \qquad (1.78)$$



(a)

1.5 Continuous-Time and Discrete-Time System

- A discrete-time system – that is, a system that transforms discrete-time inputs into discrete-time output – will be depicted as in Figure 1.41(b) and will sometimes by represented symbolically as

$$x[n] \rightarrow y[n] \tag{1.79}$$

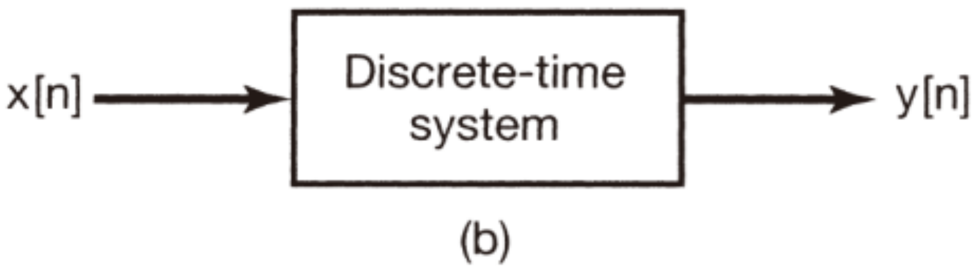


Figure 1.41 (a) Continuous-time system; (b) discrete-time system.

Example 1.8

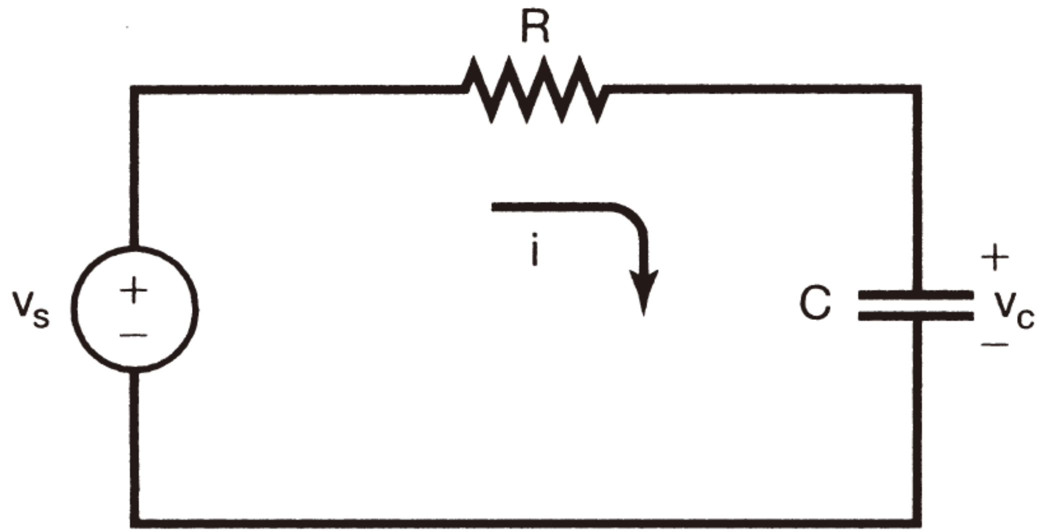


Figure 1.1 A simple *RC* circuit with source voltage v_s and capacitor voltage v_c.

We can use Ohm's law to establish the relation between *i(t)* and $v_s(t)-v_c(t)$

$$i(t)=\frac{v_s(t)-v_c(t)}{R} \qquad (1.80)$$

Example 1.8

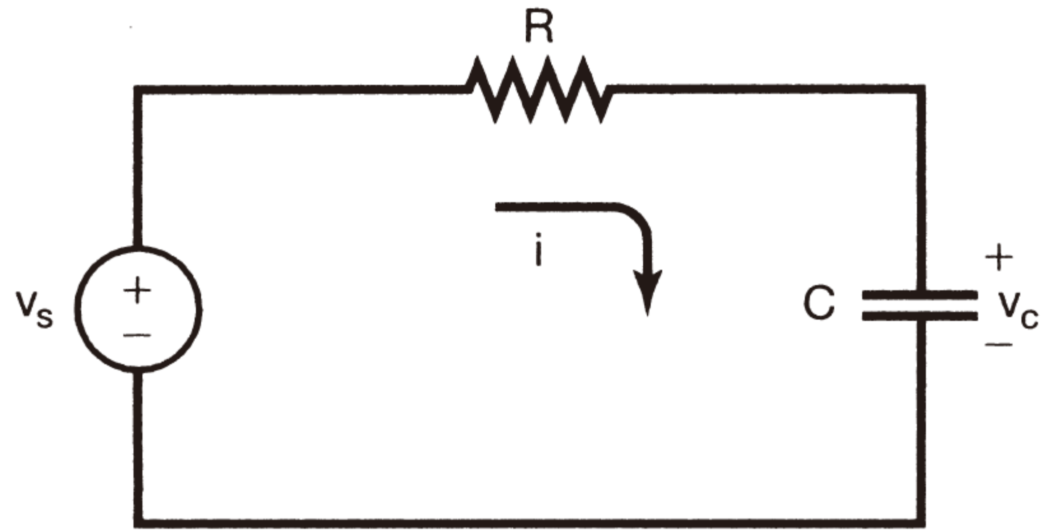


Figure 1.1 A simple *RC* circuit with source voltage v_s and capacitor voltage v_c.

We can relate *i(t)* to the rate of change with time of the voltage across the capacitor:

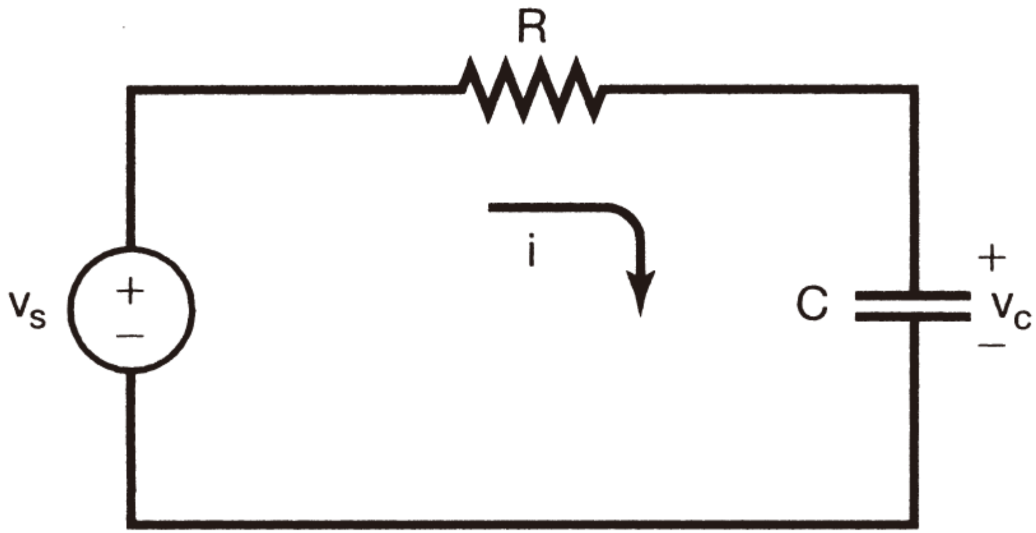
$$i(t) = C\frac{dv_c(t)}{dt} \qquad (1.81)$$

Example 1.8

$$i(t) = \frac{v_s(t) - v_c(t)}{R} \quad (1.80) \qquad i(t) = C\frac{dv_c(t)}{dt} \quad (1.81)$$

Equating the right-hand sides of eq.(1.80) and (1.81), we obtain a **differential equation** describing the relationship between the input $v_s(t)$ and the output $v_c(t)$:

$$\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}v_s(t) \qquad (1.82)$$



1.5 Continuous-Time and Discrete-Time System

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} v_s(t) \qquad (1.82)$$

- First-order linear differential equation (*one* **class** *of system*)

$$\frac{dy(t)}{dt} + ay(t) = bx(t) \qquad (1.85)$$

Where *x(t)* is the input, *y(t)* is the output, and *a* and *b* are constants.

Example 1.9 also has the same form.

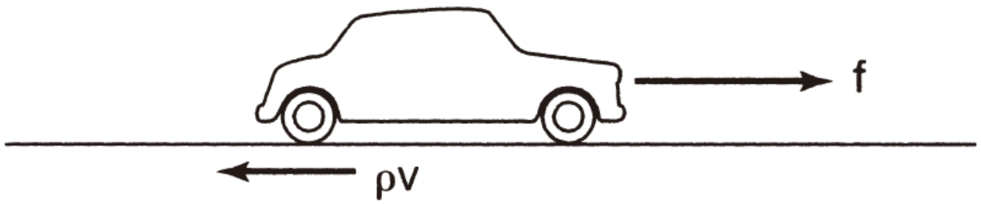


Figure 1.2 An automobile responding to an applied force f from the engine and to a retarding frictional force ρv proportional to the automobile's velocity v.

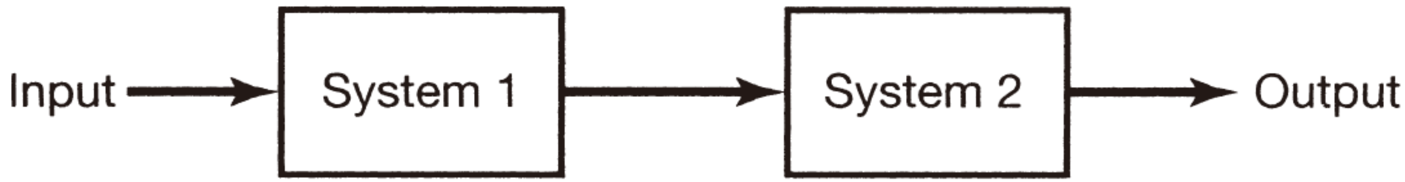
1.5 Continuous-Time and Discrete-Time System

- Identifying classes of systems that have two important characteristics:
  1. The systems in this class have properties and structures that we can exploit to **gain insight** into their behavior and to **develop effective tools** for their analysis.
  2. Many systems of practical importance can be **accurately** modeled using systems in this class.

1.5.2 Interconnections of Systems

From small system to big system

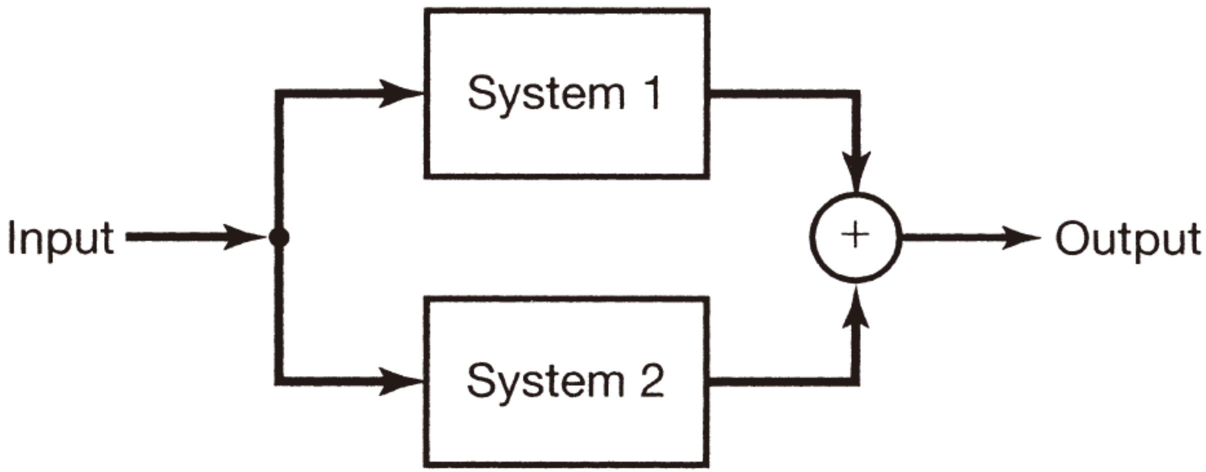
- A **series** or cascade interconnection
  - Diagrams such as this are referred to as block diagrams. Here, the output of System 1 is the input to System 2, and the overall system transforms an input by processing it first by System 1 and then by System 2.



(a)

1.5.2 Interconnections of Systems

- A **parallel** interconnection
  - The same input signal is applied to System 1 and 2. The output of the parallel interconnection is the sum of the outputs of System 1 and 2.



(b)

1.5.2 Interconnections of Systems

- Feedback interconnection
  - The output of System 1 is the input to System 2, while the output of System 2 is fed back and added to the external input to produce the actual input to System 1.

系統的回授連接圖

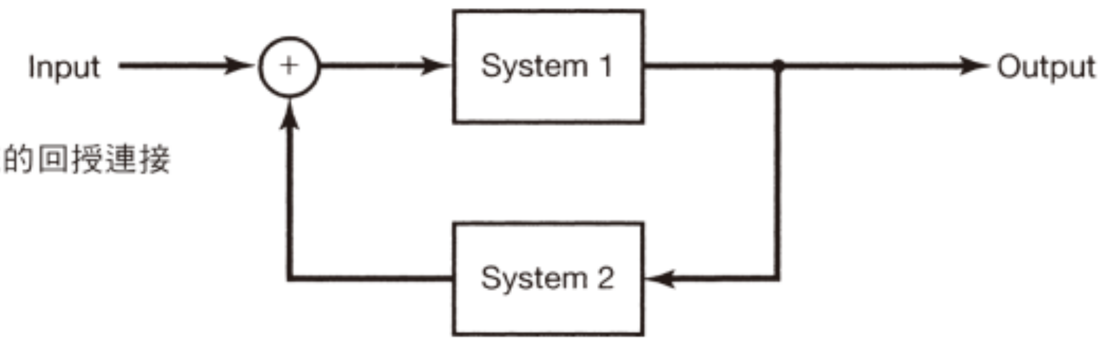


Figure 1.43 Feedback interconnection.

1.5.2 Interconnections of Systems

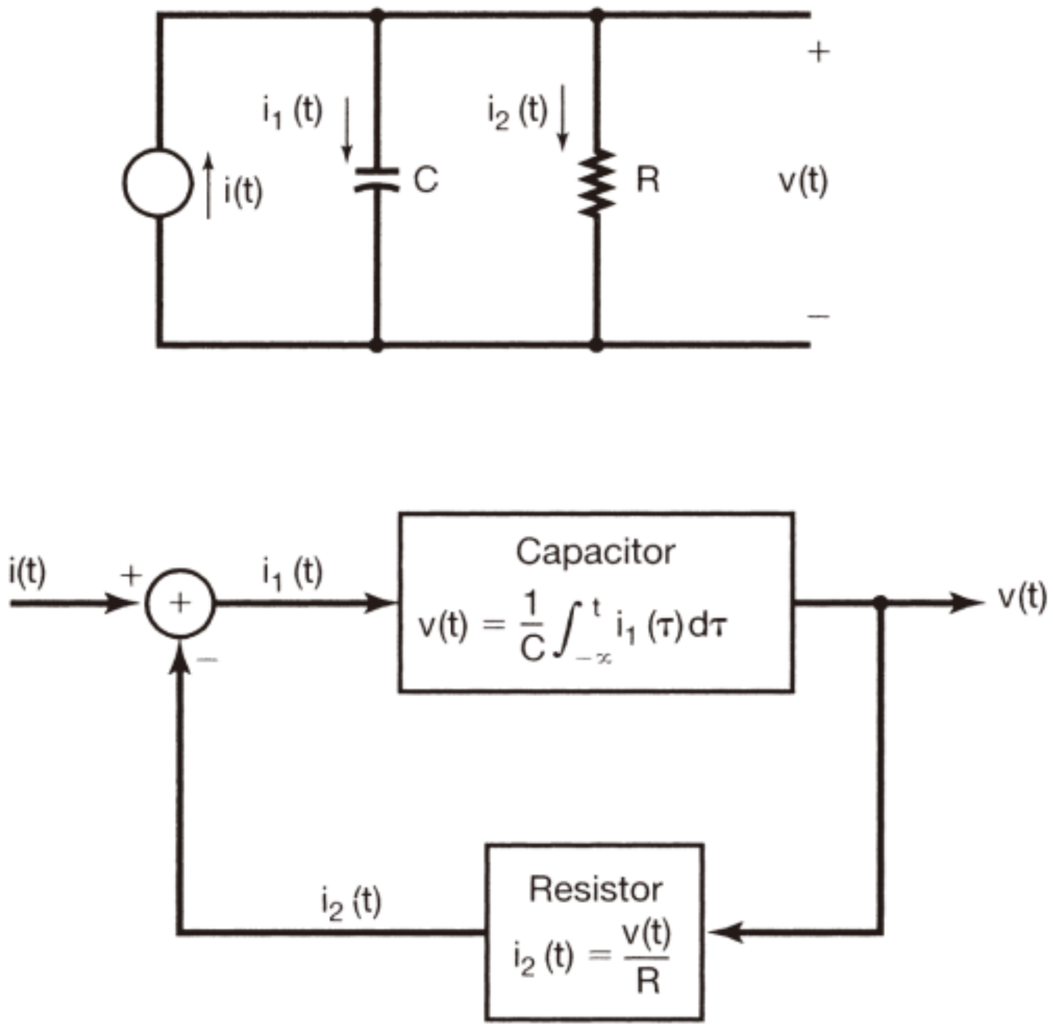


Figure 1.44 (a) Simple electrical circuit; (b) block diagram in which the circuit is depicted as the feedback interconnection of two circuit elements.

1.6.1 **System properties**: Systems With and Without Memory

- A system is said to be memoryless if its output for each value of the independent variable at a given time is dependent on the input at only that same time.

無記憶系統為系統的輸出只與當時的輸入值有關

$$y[n] = \left(2x[n] - x^2[n]\right)^2 \qquad (1.90)$$

1.6.1 Systems With and Without Memory

A resistor is a memoryless system; with the input *x(t)* taken as the **current** and with the **voltage** taken as the output *y(t)*, the input-output relationship of a resistor is

$$y(t) = Rx(t) \tag{1.91}$$

Where *R* is the resistance.

1.6.1 Systems With and Without Memory

An example of discrete-time system **with memory** is an accumulator or summer

$$y[n] = \sum_{k=-\infty}^{n} x[k] \qquad (1.92)$$

and a second example is a delay

$$y[n] = x[n-1] \qquad (1.93)$$

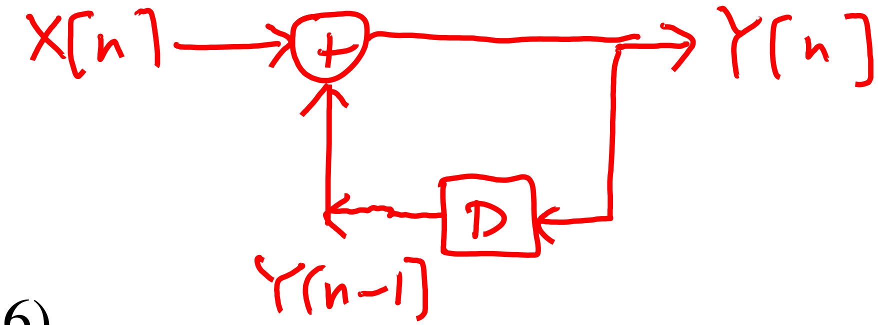
1.6.1 Systems With and Without Memory

$$y[n] = \sum_{k=-\infty}^{n} x[k] \qquad (1.92)$$

- The relationship between the input and output of an **accumulator** can be described as

$$y[n] = \sum_{k=-\infty}^{n-1} x[k] + x[n] \qquad (1.95)$$

feedback system



or **equivalently**,

$$y[n] = y[n-1] + x[n] \qquad (1.96)$$

1.6.1 Systems With and Without Memory

A capacitor is an example of a continuous-time system with memory, since if the input is taken to be the **current** and the output is the **voltage**, then

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)\,d\tau \qquad (1.94)$$

where C is the capacitance.

1.6.2 Invertibility and Inverse System

- An example of an invertible continuous-time system is

$$y(t) = 2x(t) \qquad (1.97)$$

for which the inverse system is

$$w(t) = \frac{1}{2}y(t) \qquad (1.98)$$

1.6.2 Invertibility and Inverse System

The inverse system of

$$y[n] = \sum_{k=-\infty}^{n} x[k] \Rightarrow y[n] = y[n-1] + x[n]$$

is

$$w[n] = y[n] - y[n-1] \qquad (1.99)$$

1.6.2 Invertibility and Inverse System

Examples of noninvertible systems are

$$y[n] = 0 \qquad (1.100)$$

That is, the system that produces the zero output sequence for any input sequence, and

$$y(t) = x^2(t) \qquad (1.101)$$

If y(t)=1, who knows x(t)=1 or -1?

1.6.3 Causality

- A system is causal if the output at any time depends on values of the input at only **the present and past** times.

  若一系統的輸出只與當時和過去的輸入有關，則稱為「因果系統」

1.6.3 Causal system

$$y[n] = \sum_{k=-\infty}^{n} x[k] \tag{1.92}$$

$$y[n] = x[n-1] \tag{1.93}$$

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau \tag{1.94}$$

1.6.3 None causal system

$$y[n] = x[n] - x[n+1] \qquad (1.102)$$

$$y(t) = x(t+1) \qquad (1.103)$$

$$y[n] = \frac{1}{2M+1} \sum_{k=-M}^{+M} x[n-k] \qquad (1.104)$$

Example 1.12

The first system is defined by

$$y[n] = x[-n] \qquad (1.105)$$

In particular, for $n < 0$, e.g. $n = -4$, we see that $y[-4] = x[4]$, so that the output at this time depends on a future value of the input.

Not causal

Example 1.12

$$y(t) = x(t)\cos(t+1) \qquad (1.106)$$

In this system, the output at any time t equals the input at that same multiplied by a number that varies with time.

$$y(t) = x(t)g(t)$$

Where g(t) is a time-varying function, namely g(t) = cos(t+1).

Causal

1.6.4 Stability

- The preceding examples provide us with an intuitive understanding of the concept of stability. More formally, if the input to a stable system is bounded, then the output must also be bounded and therefore cannot diverge. Bounded Input, Bounded Output (BIBO)

  「穩定性」的定義為當一個穩定系統的輸入為有界時，其輸出亦必為有界(不發散)

1.6.4 Stability

Example: $$y[n] = \sum_{k=-\infty}^{n} x[k] \qquad (1.92)$$

If x[k] is u[k]

$$y[n] = \sum_{k=-\infty}^{n} u[k] = (n+1)u[n]$$

y[n] grows without bound while u[n] is a bounded input

Example 1.13

$$S_1 : y(t) = tx(t) \qquad (1.109)$$

For system S_1 in eq.(1.109), a constant input $x(t) = 1$ yields $y(t) = t$, which is unbounded, since no matter what finite constant we pick, $|y(t)|$ will exceed that constant for some T. We conclude that system S_1 is unstable.

Example 1.13

$$S_2\colon\ y(t) = e^{x(t)} \tag{1.110}$$

For system S_2, which happens to be stable, we would be unable to find a bounded input that results in an unbounded output. Specifically, let B be an arbitrary positive number, and let $x(t)$ be an arbitrary signal bounded by B; that is, we are making no assumption about $x(t)$, except that

$$|x(t)| < B \tag{1.111}$$

$$-B < x(t) < B, \tag{1.112}$$

Example 1.13

For all t. Using the definition of S_2 in eq. (1.110), we then see that if *x(t)* satisfies eq.(1.111), then *y(t)* must satisfy

$$e^{-B} < |y(t)| < e^{B} \tag{1.113}$$

We conclude that if any input to S_2 is bounded by an arbitrary positive number B, the corresponding output is guaranteed to be bounded by e^B. Thus, S_2 is stable.

1.6.5 Time Invariance

- [Concept] a system is time invariant if the behavior and characteristics of the system are fixed over time.
  若系統的表現和特性在時間上是固定不變的，則稱為「非時變系統」
- [Signal & system] a system is time invariant if a time shift in the input signal results in an identical time shift in the output signal
  若系統的輸入訊號有時間移位時，其輸出訊號亦有相同的時間移位，則系統為非時變

1.6.5 Time Invariance

- If $y[n]$ is the output of a discrete-time, time-invariant system when $x[n]$ is the input, then $y[n-n_0]$ is the output when $x[n-n_0]$ is applied. In continuous time with $y(t)$ the output corresponding to the input $x(t)$, a time-invariant system will have $y(t-t_0)$ as the output when $x(t-t_0)$ is the input.

Example 1.14

$$y(t) = \sin[x(t)] \qquad (1.114)$$

let $x_1(t)$ be an arbitrary input to this system , and let

$$y_1(t) = \sin[x_1(t)] \qquad (1.115)$$

Be the corresponding output. Then consider a second input obtained by shifting $x_1(t)$ in time:

$$x_2(t) = x_1(t - t_0) \qquad (1.116)$$

Example 1.14

$$y_1(t) = \sin[x_1(t)] \tag{1.115}$$

The output corresponding to this input is

$$y_2(t) = \sin[x_2(t)] = \sin[x_1(t - t_0)] \tag{1.117}$$

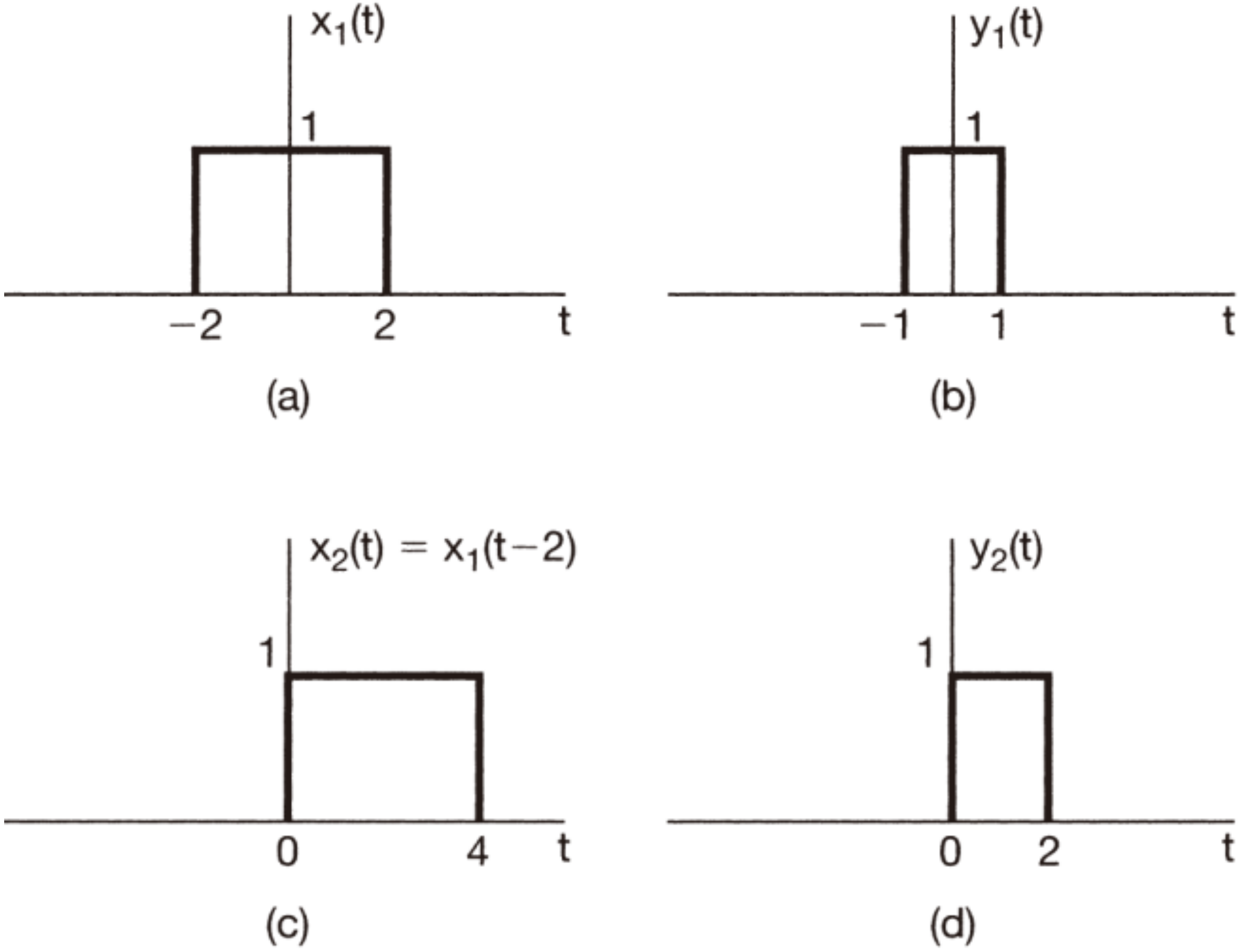
Similarly, from eq.(1.115)

$$y_1(t - t_0) = \sin[x_1(t - t_0)] \tag{1.118}$$

Comparing eqs. (1.117) and (1.118), we see that $y_2(t) = y_1(t - t_0)$, and therefore, this system is time invariant

Example 1.16

$$y(t) = x(2t)$$



Example 1.16

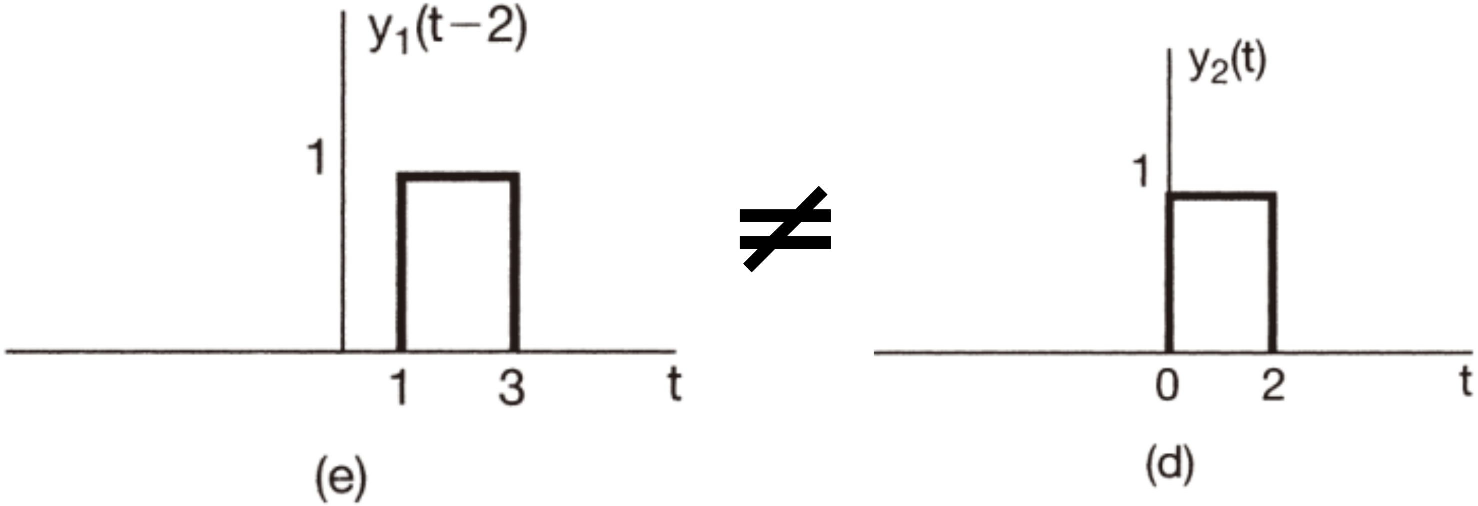


Figure 1.47 (a) The input $x_1(t)$ to the system in Example 1.16; (b) the output $y_1(t)$ corresponding to $x_1(t)$; (c) the shifted input $x_2(t) = x_1(t-2)$; (d) the output $y_2(t)$ corresponding to $x_2(t)$; (e) the shifted signal $y_1(t-2)$. Note that $y_2(t) \neq y_1(t-2)$, showing that the system is not time invariant.

1.6.6 Linearity

- A linear system, in continuous time or discrete time, is a system that possesses the important property of **superposition**: If an input consists of the weighted sum of several signals, then the output is the superposition – that is, the weighted sum – of the responses of the system to each of those signals.

1.6.6 Linearity

- The system is linear if
  1. (Additivity) The response to $x_1(t)+x_2(t)$ is $y_1(t)+y_2(t)$.
  2. (Homogeneity) The response to $ax_1(t)$ is $ay_1(t)$, where a is any complex constant.
- The two properties defining a linear system can be combined into a single statement;

$$\text{continous time}: ax_1(t)+bx_2(t) \rightarrow ay_1(t)+by_2(t) \tag{1.121}$$

$$\text{discrete time}: ax_1[n]+bx_2[n] \rightarrow ay_1[n]+by_2[n] \tag{1.122}$$

1.6.6 Linearity

$$x[n] = \sum_k a_k x_k[n] = a_1 x_1[n] + a_2 x_2[n] + a_3 x_3[n] + \dots \qquad (1.123)$$

Is

$$y[n] = \sum_k a_k y_k[n] = a_1 y_1[n] + a_2 y_2[n] + a_3 y_3[n] + \dots \qquad (1.124)$$

if $x[n] \rightarrow y[n]$, then the homogeneity property tells us that

$$a \cdot x[n] \rightarrow a \cdot y[n] \qquad \text{homogeneity}$$

$$0 = 0 \cdot x[n] \rightarrow 0 \cdot y[n] = 0 \qquad (1.125)$$

Example 1.17

Consider a system S whose input $x(t)$ and output $y(t)$ are related by

$$y(t) = tx(t)$$

To determine whether or not S is linear, we consider two arbitrary inputs $x_1(t)$ and $x_2(t)$.

$$x_1(t) \rightarrow y_1(t) = tx_1(t)$$
$$x_2(t) \rightarrow y_2(t) = tx_2(t)$$

Example 1.17

Let $x_3(t)$ be a linear combination of $x_1(t)$ and $x_2(t)$. That is

$$x_3(t) = ax_1(t) + bx_2(t)$$

Where a and b are arbitrary scalars. If $x_3(t)$ is the input to S, then the corresponding output may be expressed as

$$\begin{aligned} y_3(t) &= tx_3(t) \\ &= t(ax_1(t) + bx_2(t)) \\ &= atx_1(t) + btx_2(t) \\ &= ay_1(t) + by_2(t) \end{aligned}$$

So S is linear.

1.7 Summary

- Signal:
  - Notation of continuous and discrete signals
  - Transformation
  - Periodic signal
  - Even and odd signal
  - Energy and power
  - Complex exponential & sinusoidal signals
  - Unit and step functions

1.7 Summary

- System:
  - Block diagram of system
  - Interconnection of systems
  - Feed-forward and Feed-back
  - Memory & memoryless
  - Invertibility and inverse systems
  - Causality
  - Stability
  - Time invariant
  - Linearity