EE361002 Signals and Systems - Midterm 1

1. (25%)

(a) (9%) Given a signal $x(t) = \begin{cases} -\frac{1}{2}t + \frac{3}{2}, & \text{if } -1 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$

Plot the following signals

(1) x(-t) (2) even part of x(t) (3) odd part of x(t) (4) 3x(-2t + 1)

(b) (8%) Given a system

$$y(t)=x(t-5) + x(5-t)$$

Determine whether the system is

(1) Memoryless (2) Time invariant (3) Linear (4) Causal (5) Stable

(c) (8%) If the system is $y(t) = \begin{cases} 0, & x(t) < 0 \\ x(t) + x(t-5), & x(t) \ge 0 \end{cases}$

Repeat (b)(Note that you have to explain the details to get full credits.)

2. (25%)

Consider an LTI system:

(a) (8%) If the system input and output is x(t) and y(t) respectively, and the relation between the input and the output can be described by:

$y(t) = \int_{t-T}^{t} x(\tau)\sin(\tau)d\tau$, where T > 0

Please show that the system is indeed an LTI system.

(b) (7%) Please find the unit impulse response of the system in (a) and determine whether or not the system is causal, memoryless, and stable.

(c) (10%) Consider another LTI system, where the system input x(t) and the unit impulse response h(t) is given by

$$x(t) = \begin{cases} 2, 1 < t < 3 \\ 1, 0 < t \le 1 \\ 0, otherwise \end{cases}, \text{and } h(t) = \begin{cases} e^{-t}, & t \ge 0 \\ 0, otherwise \end{cases}$$

Derive the output y(t) of the system.

3. (25%)

Consider the CT/DT LTI system with the unit impulse response h(t) or h[n]

(a) (5%) If the DT input is $x[n] = e^{j\omega n}$, show that the output can be expressed as $y[n] = e^{j\omega n}H(e^{j\omega})$ and derive the expression of $H(e^{j\omega})$.

(b) (5%) If the CT input is $x(t) = e^{j\omega t}$, show that the output can be expressed as $y(t) = e^{j\omega t}H(j\omega)$ and derive the expression of $H(j\omega)$.

(c) (7%) For a periodic signal $x[n] = 3+2\cos((2\pi/N)n)$, determine the output when the unit impulse response is $h[n] = \alpha^n u[n]$, where $-1 < \alpha < 1$.

(d) (8%) For a periodic signal $x(t) = 3 + \cos^3(\pi t)$, determine the output when the frequency response is $H(j\omega) = 1 / (1+j\omega)$

4. (25%)

Consider two discrete-time signals with period 8 and their Fourier series coefficients

$$x[n] \overset{FS}{\leftrightarrow} a_k$$

$$y[n] = u[n] - u[n-3] \overset{FS}{\leftrightarrow} b_k$$

(a) (7%) Suppose $a_k = -a_{k-4}$, show that $x[0]=x[\pm 2]= x[\pm 4]=\ldots=0$. [Hint: frequency shifting]

(b) (6%) According to (a), let $x[2n+1]=(-1)^n$. Sketch one period of x[n].

(c) (6%) Find b_k, the Fourier coefficients of y[n].

(d) (6%) Find Fourier series coefficients c_k of $\sum_{r=<8>} x[r]y[n-r]$.