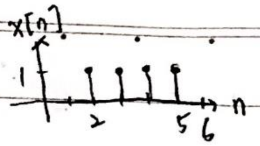
EE 361002 Signal and System HW9 Answer

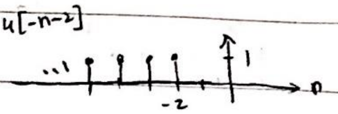
5.21

(a) $x[n] = u[n-2] - u[n-6] = \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5]$



$$X(e^{j\omega}) = e^{-2j\omega} + e^{-3j\omega} + e^{-4j\omega} + e^{-5j\omega}$$

(c) $x[n] = \left(\frac{1}{3}\right)^{|n|} u[-n-2]$



$$\Rightarrow X(e^{j\omega}) = \sum_{n=-\infty}^{-2} \left(\frac{1}{3}\right)^{|n|} e^{-j\omega n} = \sum_{n=-\infty}^{-2} \left(\frac{1}{3}\right)^{-n} e^{-j\omega n} \quad (n' = -n).$$

$$= \sum_{n'=2}^{\infty} \left(\frac{1}{3} e^{j\omega}\right)^{n'} = \frac{\frac{1}{9} e^{2j\omega}}{1 - \frac{1}{3} e^{j\omega}}$$

(g) $x[n] = \sin\left(\frac{\pi}{2} n\right) + \cos(n)$

$$\Rightarrow X(e^{j\omega}) = \sum_{l=-\infty}^{\infty} \left\{ \frac{\pi}{j} \left[\delta\left(\omega - \frac{\pi}{2} - 2\pi l\right) - \delta\left(\omega + \frac{\pi}{2} - 2\pi l\right) \right] + \pi \left[\delta(\omega - 1 - 2\pi l) + \delta(\omega + 1 - 2\pi l) \right] \right.$$

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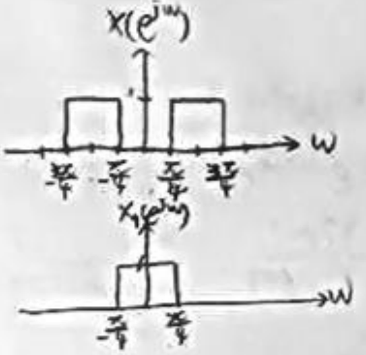

5.22

(a) $X(e^{j\omega}) = \begin{cases} 1, & \frac{\pi}{4} \le |\omega| \le \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} \le |\omega| < \pi, \ 0 \le |\omega| < \frac{\pi}{4} \end{cases}$

let $X_1(e^{j\omega}) = \begin{cases} 1, & 0 \le |\omega| \le \frac{\pi}{4} \\ 0, & \frac{\pi}{4} < |\omega| < \pi \end{cases}$,

then $X(e^{j\omega}) = X_1(e^{j(\omega-\frac{\pi}{2})}) + X_1(e^{j(\omega+\frac{\pi}{2})})$

$X_1(e^{j\omega}) \xleftrightarrow{\mathcal{F}} x_1[n] = \frac{\sin\frac{\pi}{4}n}{\pi n} = \frac{e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}}{2j\pi n}$

$$\Rightarrow x[n] = e^{j\frac{\pi}{2}n} x_1[n] + e^{-j\frac{\pi}{2}n} x_1[n]$$
$$= (e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}) \cdot \frac{(e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n})}{2j\pi n}$$
$$= \frac{e^{j\frac{3}{4}\pi n} - e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n} - e^{-j\frac{3}{4}\pi n}}{2j\pi n}$$
$$= \frac{1}{\pi n}\left(\sin(\tfrac{3}{4}\pi n) - \sin(\tfrac{\pi}{4}n)\right) \ \#$$

(c) $X(e^{j\omega}) = e^{-j\frac{1}{2}\omega}, \quad |\omega| \le \pi$

$\mathcal{F} \updownarrow$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\frac{1}{2}\omega} e^{j\omega n} d\omega$$
$$= \frac{(-1)^{n+1}}{\pi(n-\frac{1}{2})} \ \#$$

(f) $$X(e^{j\omega}) = \frac{e^{-j\omega} - \frac{1}{5}}{1 - \frac{1}{5}e^{-j\omega}} = (e^{-j\omega} - \tfrac{1}{5})\sum_{n=0}^{\infty}(\tfrac{1}{5}e^{-j\omega})^n$$
$$= \sum_{n=0}^{\infty}(\tfrac{1}{5})^n e^{-j\omega(n+1)} - \tfrac{1}{5}\sum_{n=0}^{\infty}(\tfrac{1}{5})^n e^{-j\omega n}$$
$$= 5\sum_{n=1}^{\infty}(\tfrac{1}{5})^n e^{-j\omega n} - \tfrac{1}{5}\sum_{n=0}^{\infty}(\tfrac{1}{5})^n e^{-j\omega n}$$

$$\xleftrightarrow{\mathcal{F}} x[n] = 5\cdot(\tfrac{1}{5})^n u[n-1] - \tfrac{1}{5}\cdot(\tfrac{1}{5})^n u[n]$$
$$= (\tfrac{1}{5})^{n-1} u[n-1] - (\tfrac{1}{5})^{n+1} u[n] \ \#$$

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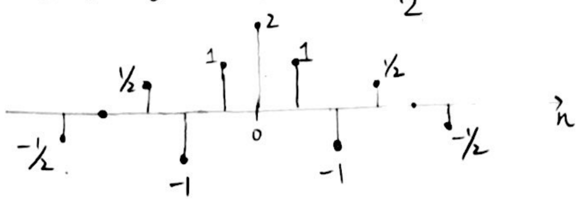
$X(e^{j\omega}) = A(\omega) + jB(\omega)$, sketch $Y(e^{j\omega}) = [B(\omega) + A(\omega)e^{j\omega}]$

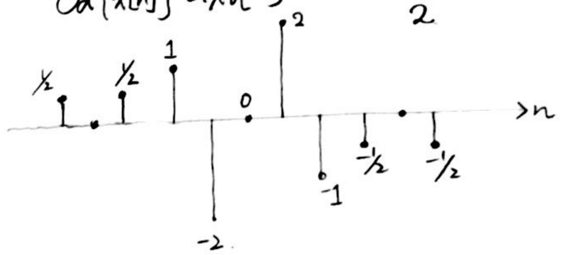
$Ev\{x[n]\} \xleftrightarrow{\mathcal{F}} Re\{X(e^{j\omega})\}$

$Od\{x[n]\} \xleftrightarrow{\mathcal{F}} Im\{X(e^{j\omega})\}\cdot j$

$Ev\{x[n]\} = x_e[n] = \dfrac{x[n]+x[-n]}{2}$

$Od\{x[n]\} = x_o[n] = \dfrac{x[n]-x[-n]}{2}$





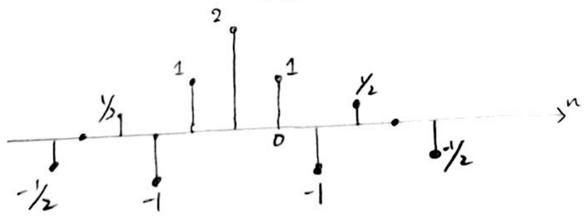
We want to find $Y(e^{j\omega}) = [B(\omega) + A(\omega)e^{j\omega}]$

$\mathcal{F}\{x_e[n]\} = A(\omega) \Rightarrow \mathcal{F}\{x_e[n+1]\} = A(\omega)e^{j\omega}$

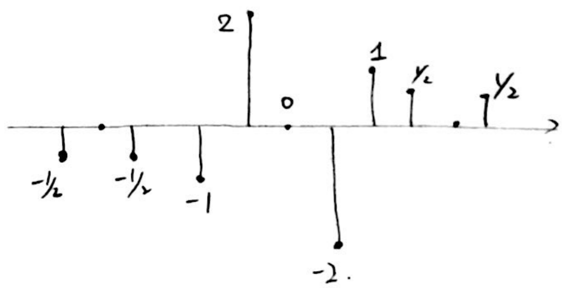
$\mathcal{F}\{x_o[n] = jB(\omega) \Rightarrow \mathcal{F}\{-j\,x_o[n]\} = B(\omega)$

$y[n] = x_e[n+1] - j\,x_o[n]$

$Re\,(x_e[n+1])$



$Im\,(-j(x_o[n]))$



5.28

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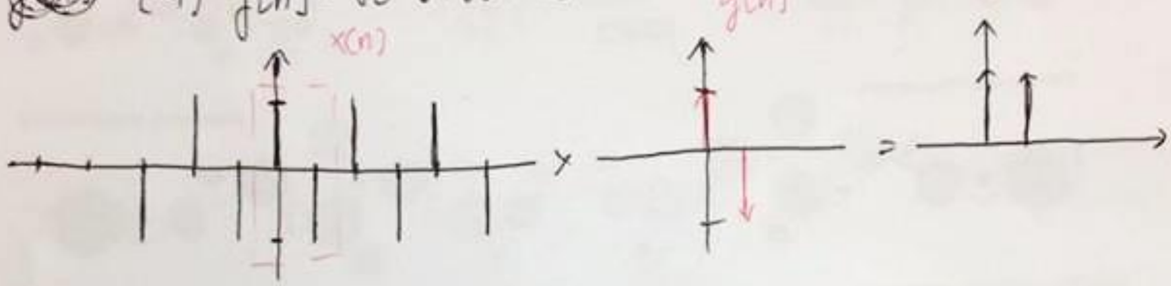
$$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})\,G(e^{j(\omega-\theta)})\,d\theta = 1+e^{-j\omega} \Rightarrow X(e^{j\theta}) * G(e^{j\theta})$$

Inverse F.T

$$\Rightarrow x[n] \times g[n] = \delta[n]+\delta[n-1]$$

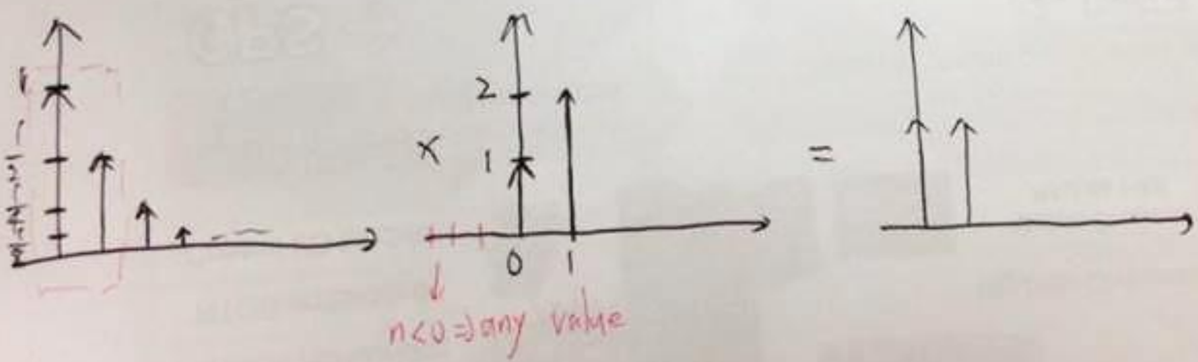
(a) $x[n] = (-1)^n$

$$(-1)^n g[n] = \delta[n]+\delta[n-1]$$



$$g[n] = \delta[n] - \delta[n-1]$$

(b) $x[n] = (\frac{1}{2})^n u[n]$



$$g[n] = \begin{cases} 1, & n=0 \\ 2, & n=1 \\ 0, & n>1 \\ \text{any value}, & n<0 \end{cases}$$