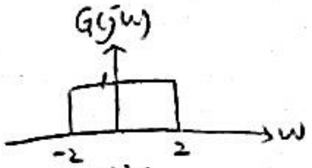
EE 361002 Signal and System HW8 Answer

4.30

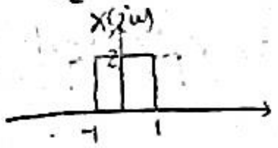
(a) $g(t)=x(t)\cos t \overset{\mathcal{F}}{\longleftrightarrow} G(j\omega)=\frac{1}{2\pi}\left[X(j\omega) * \{\pi[\delta(\omega+1)+\delta(\omega-1)]\}\right]$

$$=\frac{1}{2}\left[X(j(\omega+1))+X(j(\omega-1))\right]$$

$$=\begin{cases}1, & |\omega|\le 2\\ 0, & \text{otherwise}\end{cases}$$



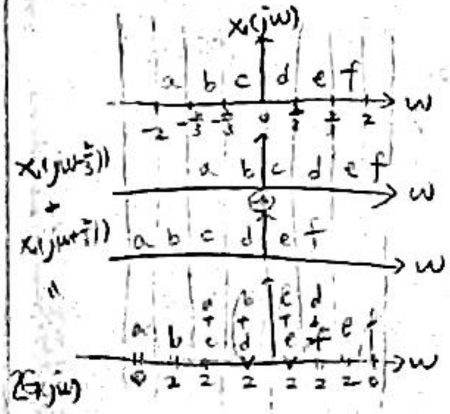
$$\Rightarrow X(j\omega)=\begin{cases}2, & |\omega|\le 1\\ 0, & \text{otherwise}\end{cases} \overset{\mathcal{F}}{\longleftrightarrow} x(t)=\frac{2\sin t}{\pi t}$$ #



(b)

$g(t)=x_1(t)\cos(\frac{2}{3}t) \overset{\mathcal{F}}{\longleftrightarrow} G(j\omega)=\frac{1}{2\pi}\left[X_1(j\omega) * \{\pi[\delta(\omega-\frac{2}{3})+\delta(\omega+\frac{2}{3})]\}\right]$

$$=\frac{1}{2}\left[X_1(j(\omega+\tfrac{2}{3}))+X_1(j(\omega-\tfrac{2}{3}))\right]$$ #



∴ $X_1(j\omega)$疊不出來

4.32

$h(t) = \frac{\sin(4(t-1))}{\pi(t-1)} \Rightarrow H(j\omega) = \begin{cases} e^{-j\omega}, & |\omega| < 4 \\ 0, & \text{o.w.} \end{cases}$

(a) $x_1(t) = \cos\left(6t + \frac{\pi}{2}\right) = \cos\left(6\left(t + \frac{\pi}{12}\right)\right)$

$\Rightarrow X_1(j\omega) = \pi e^{j\frac{\pi}{12}\omega}\left[\delta(\omega-6) + \delta(\omega+6)\right]$

$\Rightarrow Y_1(j\omega) = H(j\omega) \cdot X_1(j\omega) = 0$

$\Rightarrow y_1(t) = 0$

(b) $x_2(t) = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k \sin(3kt)$

$\Rightarrow X_2(j\omega) = \frac{\pi}{j}\left\{ \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k \left[\delta(\omega-3k) - \delta(\omega+3k)\right]\right\}$

$= \frac{\pi}{j}\left\{ \cancel{\left[\delta(\omega) - \delta(\omega)\right]} + \left(\frac{1}{2}\right)\left[\delta(\omega-3) - \delta(\omega+3)\right] + \left(\frac{1}{2}\right)^2\left[\delta(\omega-6) - \delta(\omega+6)\right] + \dots\right\}$

$\Rightarrow Y_2(j\omega) = H(j\omega) \cdot X_2(j\omega) = \frac{\pi}{j} e^{-j\omega} \frac{1}{2}\left[\delta(\omega-3) - \delta(\omega+3)\right]$

$\Rightarrow y_2(t) = \frac{1}{2}\sin(3(t-1))$

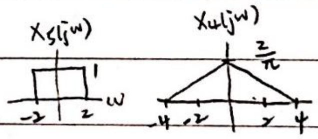
(c) $x_3(t) = \frac{\sin(4(t+1))}{\pi(t+1)} \Rightarrow X_3(j\omega) = \begin{cases} e^{j\omega}, & |\omega| < 4 \\ 0, & \text{o.w.} \end{cases}$

$\Rightarrow Y_3(j\omega) = H(j\omega) \cdot X_3(j\omega) = \begin{cases} 1, & |\omega| < 4 \\ 0, & \text{o.w.} \end{cases}$

$\Rightarrow y_3(t) = \frac{\sin(4t)}{\pi t}$

(d) $x_4(t) = \left(\frac{\sin 2t}{\pi t}\right)^2$

let $x_5(t) = \frac{\sin 2t}{\pi t} \Rightarrow X_5(j\omega) = \begin{cases} 1, & |\omega| < 2 \\ 0, & \text{o.w.} \end{cases}$



$X_4(j\omega) = \frac{1}{2\pi} X_5(j\omega) * X_5(j\omega) = \frac{-1}{2\pi}|\omega| + \frac{2}{\pi}$ $\Rightarrow$ within $|\omega| < 4$, $H(j\omega)$ causes time-shift in time domain

$\Rightarrow y_4(t) = \left(\frac{\sin 2(t-1)}{\pi(t-1)}\right)^2$

4.33

(a)

$$H(jw)=\frac{Y(jw)}{X(jw)}=\frac{2}{(jw)^2+6jw+8}=\frac{2}{(jw+2)(jw+4)}=\frac{1}{jw+2}-\frac{1}{jw+4}$$

$$\Rightarrow h(t)=e^{-2t}u(t)-e^{-4t}u(t)$$

(b) $x(t)=te^{-2t}u(t)\Rightarrow X(jw)=\frac{1}{(jw+2)^2}$

$$Y(jw)=X(jw)H(jw)=\frac{1}{(2+jw)^2}\left(\frac{1}{(2+jw)}-\frac{1}{4+jw}\right)$$

$$=\frac{1}{(2+jw)^3}-\frac{1}{(2+jw)^2(4+jw)}$$

$$=\frac{1}{(2+jw)^3}+\frac{\frac{1}{4}}{jw+2}-\frac{\frac{1}{2}}{(jw+2)^2}-\frac{\frac{1}{4}}{(jw+4)}$$

$$y(t)=\frac{t^2}{2}e^{-2t}u(t)+\frac{1}{4}e^{-2t}u(t)-\frac{1}{2}te^{-2t}u(t)-\frac{1}{4}e^{-4t}u(t)$$

(c) $$H(jw)=\frac{-2w^2-2}{-w^2+\sqrt{2}jw+1}=2+\frac{-2\sqrt{2}jw-4}{-w^2-\sqrt{2}jw+1}$$

$$=2+\frac{-\sqrt{2}+\sqrt{2}j}{\left(jw-\frac{-\sqrt{2}+\sqrt{2}j}{2}\right)}+\frac{-\sqrt{2}-\sqrt{2}j}{\left(jw-\frac{-\sqrt{2}-\sqrt{2}j}{2}\right)}$$

$$\Rightarrow h(t)=2\delta(t)+(-\sqrt{2}+\sqrt{2}j)e^{\left(\frac{-\sqrt{2}+\sqrt{2}j}{2}\right)t}u(t)+(-\sqrt{2}-\sqrt{2}j)e^{\left(\frac{-\sqrt{2}-\sqrt{2}j}{2}\right)t}u(t)$$

4.41

$g(t)$ denotes the inverse Fourier Transform of $\frac{1}{2\pi}\{X(j\omega) * Y(j\omega)\}$

(a) Using $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{2\pi}\{X(j\omega) * Y(j\omega)\} e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{2\pi}\left[\int_{-\infty}^{\infty} X(j\theta) \cdot Y(j(\omega-\theta))\, d\theta\right] e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} Y(j(\omega-\theta))\, e^{j\omega t} d\omega\right] d\theta \quad \text{——} \quad ①$$

(b)

Freq-shift $e^{j\omega_0 t} x(t) \xleftrightarrow{\mathcal{F}} X(j(\omega-\omega_0))$

$$\Rightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(j(\omega-\theta)) e^{j\omega t} d\omega = e^{j\theta t} y(t) \quad \text{——} \quad ②$$

(c)

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)\, e^{j\theta t} y(t)\, d\theta \quad (② 代入 ①)$$

$$= \underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)\, e^{j\theta t} d\theta}_{\mathcal{F}^{-1}} \cdot y(t)$$

$$= x(t) \cdot y(t)$$

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