EE 361002 Signal and System HW11 Answer

6.21

(a) $Y(jw) = H(jw)X(jw) \Rightarrow -2jw\,X(jw) \Rightarrow y(t) = -2\dfrac{dx(t)}{dt}$

$$H(jw) = \begin{cases} -2jw, & |w|<1 \\ 0, & \text{else} \end{cases}$$

$$x(t) = e^{jt} \Rightarrow y(t) = -2\frac{de^{jt}}{dt} = -2j\,e^{jt}$$

(b) $x(t) = (\sin w_0 t)u(t) \Rightarrow y(t) = -2\dfrac{de^{jt}}{dt}$

$$= -2(w_0 \cos(w_0 t)u(t) + \sin w_0 t\,\delta(t))$$

$$= -2w_0 \cos w_0 t\, u(t)$$

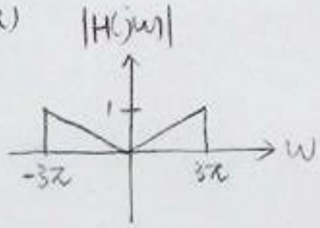
$Y(jw) = -2jw\,X(jw) \Rightarrow y(t) = -2\dfrac{dx(t)}{dt}$

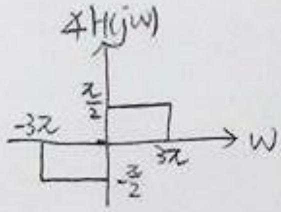
(c) $X(jw) = \dfrac{1}{(jw)(6+jw)} \Rightarrow Y(jw) = \dfrac{-2}{6+jw} \Rightarrow y(t) = -2e^{-6t}u(t)$

(d) $X(jw) = \dfrac{1}{2+jw} \Rightarrow x(t) = e^{-2t}u(t) \Rightarrow y(t) = -2\dfrac{dx(t)}{dt} = 4e^{-2t}u(t) - 2\delta(t)$

6.22

(a)

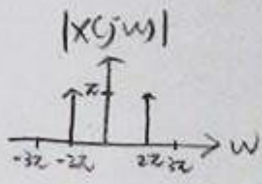




$$H(j\omega)=|H(j\omega)|\,e^{j\measuredangle H(j\omega)}$$

$$=\begin{cases} -\frac{1}{3\pi}\omega e^{j(-\frac{\pi}{2})}, & -3\pi\le\omega\le 0 \\ \frac{1}{3\pi}\omega e^{j\frac{\pi}{2}}, & 0\le\omega\le 3\pi \\ 0, & \text{otherwise} \end{cases} = \begin{cases} \frac{j\omega}{3\pi}, & 0\le|\omega|\le 3\pi \\ 0, & \text{otherwise} \end{cases}$$

$$x(t)=\cos(2\pi t+\theta) \xleftrightarrow{\mathcal{F}} X(j\omega)=\pi\left[e^{j\theta}\delta(\omega-2\pi)+e^{-j\theta}\delta(\omega+2\pi)\right]$$



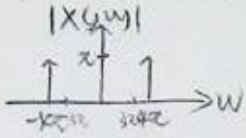
$$\Rightarrow Y(j\omega)=H(j\omega)X(j\omega)=\begin{cases} \frac{j\omega}{3\pi}X(j\omega), & 0\le|\omega|\le 3\pi \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow y(t)=\frac{1}{3\pi}\cdot\frac{dx(t)}{dt}$$

$$=\frac{1}{3\pi}\cdot 2\pi\cdot(-\sin(2\pi t+\theta))$$

$$=-\frac{2}{3}\sin(2\pi t+\theta)$$ *

(b) $x(t)=\cos(4\pi t+\theta) \xleftrightarrow{\mathcal{F}} X(j\omega)=\pi\left[e^{j\theta}\delta(\omega-4\pi)+e^{-j\theta}\delta(\omega+4\pi)\right]$



$$\Rightarrow Y(j\omega)=H(j\omega)X(j\omega)=0$$

$$\Rightarrow y(t)=0$$ *

(c) $H(j\omega) = |H(j\omega)| \measuredangle H(j\omega)$

$$= \begin{cases} -\frac{1}{3\pi}\omega \cdot (e^{j(-\frac{\pi}{2})}), & -3\pi \le \omega \le 0 \\ \frac{1}{3\pi}\omega\,(e^{j\frac{\pi}{2}}), & 0 \le \omega \le 3\pi \\ 0, & \text{otherwise} \end{cases} = \begin{cases} \frac{j\omega}{3\pi}, & -3\pi \le \omega \le 3\pi \\ 0, & \text{otherwise.} \end{cases}$$

$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t} dt.$

$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$, In (c), $\omega_0 = 2\pi$

$a_0 = \int_0^{1/2} \sin \omega_0 t\, dt = \frac{1}{\pi}$ (Average)

$a_1 = a^*_{-1} = \int_0^{1/2} \sin \omega_0 t \times e^{-j\omega_0 t} dt$

$= \int_0^{1/2} \sin 2\pi t\, (\cos 2\pi t - i \sin 2\pi t)$

積化和差 $= \frac{-1}{2}\int_0^{1/2} \sin(4\pi t) + \cancel{\sin 0} - i\cos(4\pi t) + i\cos 0$

$= \frac{-1}{2}\left(-\frac{\cos 4\pi t}{4\pi} - \frac{i \sin 4\pi t}{4\pi} + i\, t\right)\Big|_0^{1/2} = \frac{1}{4j}$

$\Rightarrow a_{-1} = \frac{-1}{4j}$

$[\frac{d}{dt}x(t)] \xleftrightarrow{\mathcal{F}} j\omega X(j\omega))$

$\Rightarrow y(t) = \frac{1}{3\pi}\frac{dx(t)}{dt} = \frac{1}{3\pi} \times \frac{d\left(\frac{1}{\pi} + \frac{1}{4j}e^{j\omega_0 t} - \frac{1}{4j}e^{-j\omega_0 t}\right)}{dt}$

$= \frac{1}{3\pi}\frac{d\left(\frac{1}{\pi} + \frac{1}{2} \times \sin(\omega_0 t)\right)}{dt}$

$= \frac{1}{3\pi} \times \frac{1}{2} \times 2\pi \cos 2\pi t = \frac{1}{3}\cos 2\pi t$

6.25. (a) We may write $H_a(j\omega)$ as

$$H_a(j\omega) = \frac{(1-j\omega)}{(1+j\omega)(1-j\omega)} = \frac{1-j\omega}{2}.$$

Therefore,

$$\sphericalangle H_a(j\omega) = \tan^{-1}[-\omega].$$

and

$$\tau_a(\omega) = -\frac{d\sphericalangle H_a(j\omega)}{d\omega} = \frac{1}{1+\omega^2}.$$

Since $\tau_a(0) = 1 \neq 2 = \tau_a(1)$, $\tau_a(\omega)$ is not a constant for all ω. Therefore, the frequency response has nonlinear phase.

(b) In this case, $H_b(j\omega)$ is the frequency response of a system which is a cascade combination of two systems, each of which has a frequency response $H_a(j\omega)$. Therefore,

$$\sphericalangle H_b(j\omega) = \sphericalangle H_a(j\omega) + \sphericalangle H_a(j\omega)$$

and

$$\tau_b(\omega) = -2\frac{d\sphericalangle H_a(j\omega)}{d\omega} = \frac{2}{1+\omega^2}.$$

Since $\tau_b(0) = 2 \neq 4 = \tau_b(1)$, $\tau_b(\omega)$ is not a constant for all ω. Therefore, the frequency response has nonlinear phase.

(c) In this case, $H_c(j\omega)$ is again the frequency response of a system which is a cascade combination of two systems. The first system has a frequency response $H_a(j\omega)$, while the second system has a frequency response $H_0(j\omega) = 1/(2+j\omega)$. Therefore,

$$\sphericalangle H_b(j\omega) = \sphericalangle H_a(j\omega) + \sphericalangle H_0(j\omega)$$

and

$$\tau_c(\omega) = -\frac{d\sphericalangle H_a(j\omega)}{d\omega} - \frac{d\sphericalangle H_0(j\omega)}{d\omega} = \frac{1}{1+\omega^2} + \frac{2}{4+\omega^2}.$$

Since $\tau_c(0) = (3/2) \neq (3/5) = \tau_c(1)$, $\tau_b(\omega)$ is not a constant for all ω. Therefore, the frequency response has nonlinear phase.

6.27. (a) Taking the Fourier transform of both sides of the given differential equation, we obtain

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{2+j\omega}.$$

The Bode plot is as shown in Figure S6.27.

(b) From the expression for $H(j\omega)$ we obtain

$$\sphericalangle H(j\omega) = -\tan^{-1}(\omega/2).$$

Therefore,

$$\tau(\omega) = -\frac{d\sphericalangle H(j\omega)}{d\omega} = \frac{2}{4+\omega^2}.$$

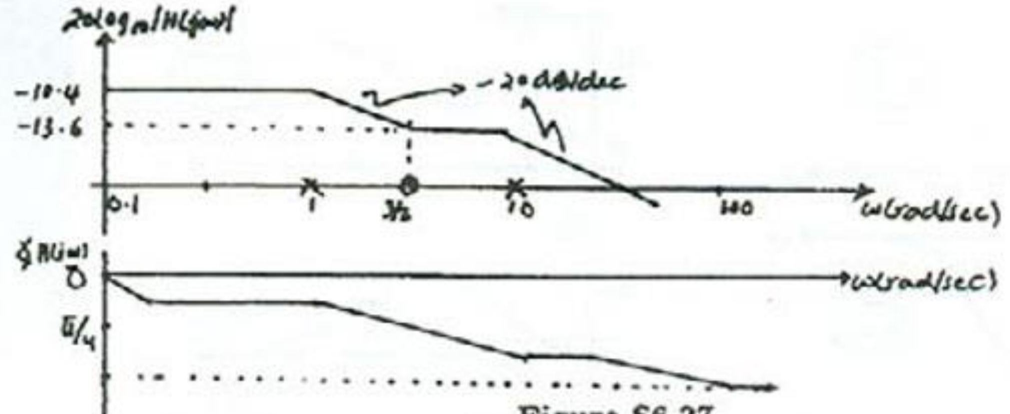


Figure S6.27

(c) Since $x(t) = e^{-t}u(t)$,

$$X(j\omega) = \frac{1}{1+j\omega}.$$

Therefore,

$$Y(j\omega) = X(j\omega)H(j\omega) = \frac{1}{(1+j\omega)(2+j\omega)}.$$

(d) Taking the inverse Fourier transform of the partial fraction expansion of $Y(j\omega)$, we obtain

$$y(t) = e^{-t}u(t) - e^{-2t}u(t).$$

(e) (i) Here,

$$Y(j\omega) = \frac{1+j\omega}{(2+j\omega)^2}.$$

Taking the inverse Fourier transform of the partial fraction expansion of $Y(j\omega)$, we obtain

$$y(t) = e^{-2t}u(t) - te^{-2t}u(t).$$

(ii) Here,

$$Y(j\omega) = \frac{1}{(1+j\omega)}.$$

Taking the inverse Fourier transform of $Y(j\omega)$, we obtain

$$y(t) = e^{-t}u(t).$$

(iii) Here,

$$Y(j\omega) = \frac{1}{(1+j\omega)(2+j\omega)^2}.$$

Taking the inverse Fourier transform of the partial fraction expansion of $Y(j\omega)$, we obtain

$$y(t) = e^{-t}u(t) + \frac{1}{2}e^{-2t}u(t) - te^{-2t}u(t).$$