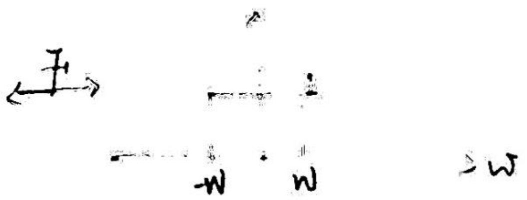
EE 361002 Signal and System HW10 Answer

5.30

5.30

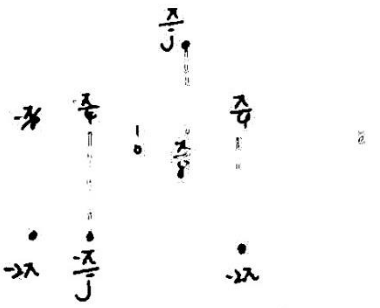
(a)

sinc $\overset{\mathcal{F}}{\longleftrightarrow}$



(b)

$X(e^{j\omega})$



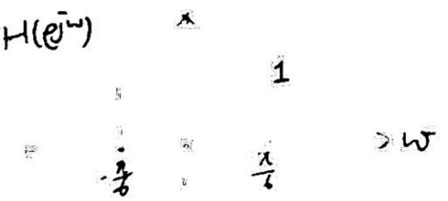
$x[n] = \sin\left(\frac{\pi n}{8}\right) - 2\cos\left(\frac{\pi n}{4}\right)$

$\updownarrow \mathcal{F}$

$X(e^{j\omega})$

(i)

$H(e^{j\omega})$



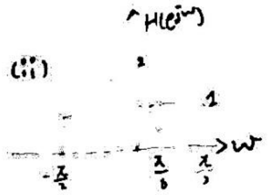
$Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega})$

只有在 $-\frac{\pi}{8} \sim \frac{\pi}{8}$ 會被保留)

$\updownarrow \mathcal{F}$

$y[n] = \sin\left[\frac{\pi n}{8}\right]$

(ii)



$y[n] = 2\sin\left[\frac{\pi n}{8}\right] - 2\cos\left[\frac{\pi n}{4}\right]$

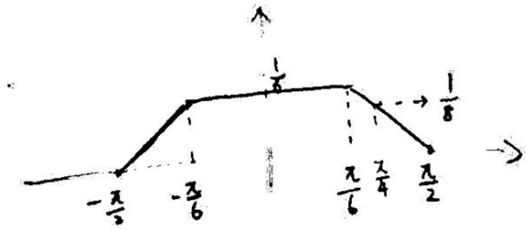
(iii)

$\frac{\sin\left(\frac{\pi n}{6}\right)\ \sin\left(\frac{\pi n}{3}\right)}{\pi n \cdot \pi n}$

$H(e^{j\omega}) = H_1(e^{j\omega}) * H_2(e^{j\omega})$

$= \frac{1}{2\pi}\int H_1(e^{j\theta})\, H_2(e^{j(\omega-\theta)})\, d\omega$

(方波 convolution)



$y[n] = \frac{1}{6}\sin\left[\frac{\pi n}{8}\right] - 2 \times \frac{1}{8}\cos\left[\frac{\pi n}{4}\right]$

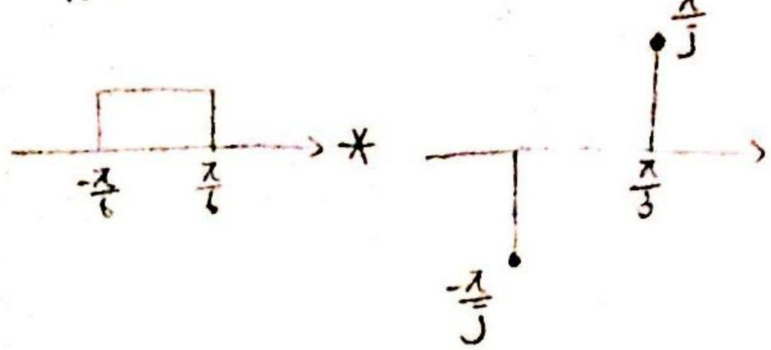
$= \frac{1}{6}\sin\left[\frac{\pi n}{8}\right] - \frac{1}{4}\cos\left[\frac{\pi n}{4}\right]$

(d)

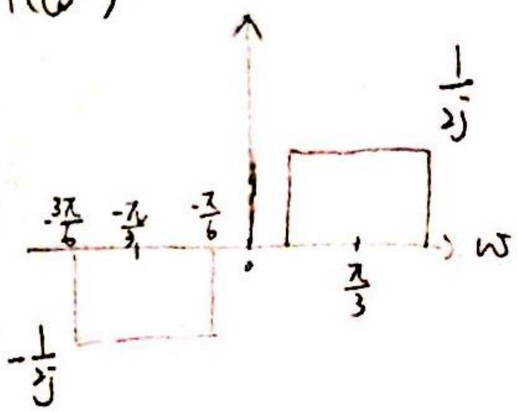
(iv).

$$h[n] = \frac{\sin\left(\frac{\pi n}{6}\right)}{\pi n} \times \sin\left(\frac{\pi n}{3}\right)$$

=



$H(e^{jw})$



$$Y(e^{jw}) = H(e^{jw}) \cdot X(e^{jw})$$

$$= -\frac{1}{2} \times 2 \sin\left[\frac{\pi n}{4}\right] = -\sin\left[\frac{\pi n}{4}\right]$$

HW10

5.30 (c) $h[n] = \frac{\sin(\frac{\pi}{3}n)}{\pi n} = \frac{1}{3}\,\mathrm{sinc}(\frac{n}{3}) \xleftrightarrow{\mathcal{F}} H(e^{jw}) = \begin{cases} 1, & 0 \le |w| \le \frac{\pi}{3} \\ 0, & \frac{\pi}{3} < |w| \le \pi \end{cases}$

(i) $x[n] = \sum_{k=\langle 8 \rangle} a_k e^{jk(\frac{2\pi}{8})n}$, $a_k = \frac{1}{8}\sum_{n=0}^{7} x[n] e^{-jk\frac{2\pi}{8}n}$

$\xleftrightarrow{\mathcal{F}} X(e^{jw}) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(w - \frac{2\pi k}{8})$

$y[n] = x[n] * h[n] \xleftrightarrow{\mathcal{F}} Y(e^{jw}) = X(e^{jw})H(e^{jw})$

$= 2\pi[a_0\delta(w) + a_1\delta(w-\frac{\pi}{4}) + a_{-1}\delta(w+\frac{\pi}{4})]$

$\Rightarrow y[n] = a_0 + a_1 e^{j\frac{\pi}{4}n} + a_{-1} e^{-j\frac{\pi}{4}n}$

$$\begin{cases} a_0 = \frac{1}{8}\sum_{n=0}^{7} x[n]\cdot 1 = \frac{5}{8} \\ a_1 = \frac{1}{8}\sum_{n=0}^{7} x[n] e^{-j\frac{2\pi}{8}n} = \frac{1}{8}(1 + e^{-j\frac{2\pi}{8}\cdot 1} + e^{-j\frac{2\pi}{8}\cdot 2} + e^{-j\frac{2\pi}{8}\cdot 6} + e^{-j\frac{2\pi}{8}\cdot 7}) = \frac{1}{8}(1+\sqrt{2}) \\ a_{-1} = \frac{1}{8}\sum_{n=0}^{7} x[n] e^{j\frac{2\pi}{8}n} = \frac{1}{8}(1 + e^{j\frac{2\pi}{8}\cdot 1} + e^{j\frac{2\pi}{8}\cdot 2} + e^{j\frac{2\pi}{8}\cdot 6} + e^{j\frac{2\pi}{8}\cdot 7}) = \frac{1}{8}(1+\sqrt{2}) \end{cases}$$

$\Rightarrow y[n] = \frac{5}{8} + \frac{1}{8}(1+\sqrt{2})\cdot 2\cos(\frac{\pi}{4}n) = \underline{\frac{5}{8} + \frac{1+\sqrt{2}}{4}\cos(\frac{\pi}{4}n)}$ #

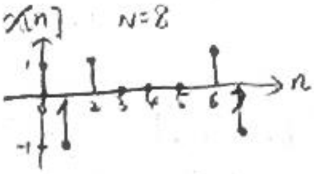
(ii) $x[n]$ period $= 8$

$y[n] = a_0 + a_1 e^{j\frac{\pi}{4}n} + a_{-1} e^{-j\frac{\pi}{4}n}$

$$\begin{cases} a_0 = \frac{1}{8}\sum_{n=0}^{7} x[n]\cdot 1 = \frac{1}{8} \\ a_1 = \frac{1}{8}\sum_{n=0}^{7} x[n] e^{-j\frac{2\pi}{8}n} = \frac{1}{8} \\ a_{-1} = \frac{1}{8}\sum_{n=0}^{7} x[n] e^{j\frac{2\pi}{8}n} = \frac{1}{8} \end{cases}$$

$\Rightarrow y[n] = \frac{1}{8} + \frac{1}{8}\cdot 2\cos(\frac{\pi}{4}n) = \underline{\frac{1}{8} + \frac{1}{4}\cos(\frac{\pi}{4}n)}$ #

(iii)



$$\begin{cases} a_0 = \frac{1}{8} \\ a_1 = \frac{1}{8}(1 - e^{-j\frac{\pi}{4}} + e^{-j\frac{\pi}{2}} + e^{-j\frac{3}{2}\pi} - e^{-j\frac{7}{4}\pi}) = \frac{1}{8}(1-\sqrt{2}) \\ a_{-1} = \frac{1}{8}(1 - e^{j\frac{\pi}{4}} + e^{j\frac{\pi}{2}} + e^{j\frac{3}{2}\pi} - e^{j\frac{7}{4}\pi}) = \frac{1}{8}(1-\sqrt{2}) \end{cases}$$

$\Rightarrow y[n] = a_0 + a_1 e^{j\frac{\pi}{4}n} + a_{-1} e^{-j\frac{\pi}{4}n} = \frac{1}{8} + \frac{1}{8}(1-\sqrt{2})\cdot 2\cos(\frac{\pi}{4}n) = \underline{\frac{1}{8} + \frac{1-\sqrt{2}}{4}\cos(\frac{\pi}{4}n)}$ #

(iv) $x[n] = \delta[n+1] + \delta[n-1]$

$y[n] = x[n] * h[n] = (\delta[n+1] + \delta[n-1]) * \frac{\sin(\frac{\pi}{3}n)}{\pi n} = \underline{\frac{\sin(\frac{\pi}{3}(n+1))}{\pi(n+1)} + \frac{\sin(\frac{\pi}{3}(n-1))}{\pi(n-1)}}$ #

5.33

(a) $y[n]+\frac{1}{2}y[n-1]=x[n]$

$\Rightarrow Y(e^{jw})+\frac{1}{2}e^{-jw}Y(e^{jw})=X(e^{jw})$

$\Rightarrow H(e^{jw})=\frac{Y(e^{jw})}{X(e^{jw})}=\frac{1}{1+\frac{1}{2}e^{-jw}}$ #

(b) $H(e^{jw})=\frac{1}{1+\frac{1}{2}e^{-jw}}$

(i) $x[n]=\left(\frac{1}{2}\right)^n u[n] \Rightarrow X(e^{jw})=\frac{1}{1-\frac{1}{2}e^{-jw}}$

$Y(e^{jw})=\frac{1}{1+\frac{1}{2}e^{-jw}}\cdot\frac{1}{1-\frac{1}{2}e^{-jw}}=\frac{\frac{1}{2}}{1+\frac{1}{2}e^{-jw}}+\frac{\frac{1}{2}}{1-\frac{1}{2}e^{-jw}}$

$y[n]=\frac{1}{2}\left(\frac{-1}{2}\right)^n u[n]+\frac{1}{2}\left(\frac{1}{2}\right)^n u[n]$

(ii) $x[n]=\left(\frac{-1}{2}\right)^n u[n] \Rightarrow X(e^{jw})=\frac{1}{1+\frac{1}{2}e^{-jw}}$

$Y(e^{jw})=\left(\frac{1}{1+\frac{1}{2}e^{-jw}}\right)^2 \Rightarrow y[n]=(n+1)\left(\frac{-1}{2}\right)^n u[n]$

(iii) $x[n]=\delta[n]+\frac{1}{2}\delta[n-1] \Rightarrow X(e^{jw})=1+\frac{1}{2}e^{-jw}$

$Y(e^{jw})=1 \Rightarrow y[n]=\delta[n]$

(iv) $x[n]=\delta[n]-\frac{1}{2}\delta[n-1] \Rightarrow X(e^{jw})=1-\frac{1}{2}e^{-jw}$

$Y(e^{jw})=\frac{1-\frac{1}{2}e^{-jw}}{1+\frac{1}{2}e^{-jw}}=\frac{-(1+\frac{1}{2}e^{-jw})+2}{1+\frac{1}{2}e^{-jw}}=-1+\frac{2}{1+\frac{1}{2}e^{-jw}}$

$y[n]=-\delta[n]+2\left(\frac{-1}{2}\right)^n u[n]$

5-33

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1+\frac{1}{2}e^{-j\omega}}$$

(c) (i)

$$Y(e^{j\omega}) = \frac{1-\frac{1}{4}e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}} \cdot \frac{1}{1+\frac{1}{2}e^{-j\omega}} = \frac{1}{(1+\frac{1}{2}e^{-j\omega})^2} - \frac{\frac{1}{4}e^{-j\omega}}{(1+\frac{1}{2}e^{-j\omega})^2}$$

$$\Rightarrow y[n] = (n+1)\left(-\frac{1}{2}\right)^n u[n] - \frac{1}{4}n\left(-\frac{1}{2}\right)^{n-1}u[n-1]$$

(ii)

$$Y(e^{j\omega}) = \frac{1+\frac{1}{2}e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}} \cdot \frac{1}{1+\frac{1}{2}e^{-j\omega}} = \frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

$$\Rightarrow y[n] = \left(\frac{1}{4}\right)^n u[n]$$

(iii)

$$Y(e^{j\omega}) = \frac{1}{(1-\frac{1}{4}e^{-j\omega})(1+\frac{1}{2}e^{-j\omega})} \cdot \frac{1}{(1+\frac{1}{2}e^{-j\omega})}$$

$$= \frac{a}{(1+\frac{1}{2}e^{-j\omega})^2} + \frac{b}{(1+\frac{1}{2}e^{-j\omega})} + \frac{c}{(1-\frac{1}{4}e^{-j\omega})}$$

$$= \frac{\frac{2}{3}}{(1+\frac{1}{2}e^{-j\omega})^2} + \frac{\frac{2}{9}}{(1+\frac{1}{2}e^{-j\omega})} + \frac{\frac{1}{9}}{(1-\frac{1}{4}e^{-j\omega})}$$

$$\Rightarrow y[n] = \left(\frac{2}{3}\right)(n+1)\left(\frac{-1}{2}\right)^n u[n] + \frac{2}{9}\left(-\frac{1}{2}\right)^n u[n] + \frac{1}{9}\left(\frac{1}{4}\right)^n u[n]$$

5-33 (iii)

$$a(1-\frac{1}{4}t) \Rightarrow a-\frac{a}{4}t$$

$$b(1+\frac{1}{2}t)(1-\frac{1}{4}t) \Rightarrow b+\frac{b}{4}t-\frac{b}{8}t^2$$

$$c(1+\frac{1}{2}t)^2 \Rightarrow c+ct+\frac{c}{4}t^2$$

$$\begin{cases} a+b+c=1 \\ -\frac{a}{4}+\frac{b}{4}+c=0 \\ -\frac{b}{8}+\frac{c}{4}=0 \Rightarrow b=2c \end{cases}$$

$$\Rightarrow \begin{cases} a+3c=1 \\ -\frac{a}{4}+\frac{3}{2}c=0 \end{cases} \Rightarrow \begin{cases} a=\frac{2}{3} \\ c=\frac{1}{9} \\ \Rightarrow b=\frac{2}{9} \end{cases}$$

(IV)

$$Y(e^{j\omega}) = 1+2e^{-3j\omega}\cdot\left(\frac{1}{1+\frac{1}{2}e^{j\omega}}\right)$$

$$= \frac{1}{1+\frac{1}{2}e^{-j\omega}} + \frac{2e^{-3j\omega}}{1+\frac{1}{2}e^{j\omega}}$$

$$\Rightarrow y[n] = (-\frac{1}{2})^n u[n] + 2\cdot(-\frac{1}{2})^{n-3}u[n-3]$$

5.34

(a) $H(e^{jw}) = H_1(e^{jw})H_2(e^{jw}) = \frac{2-e^{-jw}}{1+\frac{1}{2}e^{-jw}} \cdot \frac{1}{1-\frac{1}{2}e^{-jw}+\frac{1}{4}e^{-j2w}} = \frac{2-e^{-jw}}{1+\frac{1}{8}e^{-j3w}} = \frac{Y(e^{jw})}{X(e^{jw})}$

$\Rightarrow Y(e^{jw}) + \frac{1}{8}e^{-j3w}Y(e^{jw}) = 2X(e^{jw}) - e^{-jw}X(e^{jw})$

$\Rightarrow y[n] + \frac{1}{8}y[n-3] = 2x[n] - x[n-1]$

(b)

$$H(e^{jw}) = \frac{2-e^{-jw}}{1+\frac{1}{8}e^{-j3w}}$$

$$= \frac{2-e^{-jw}}{(1+\frac{1}{2}e^{-jw})(1-\frac{1}{2}e^{-jw}+\frac{1}{4}e^{-j2w})}$$

$$= \frac{a}{1+\frac{1}{2}e^{-jw}} + \frac{be^{-jw}+c}{\frac{1}{4}e^{-j2w}-\frac{1}{2}e^{-jw}+1}$$

$$\Rightarrow a(\tfrac{1}{4}e^{-j2w}-\tfrac{1}{2}e^{-jw}+1)+(1+\tfrac{1}{2}e^{-jw})(be^{-jw}+c) = 2-e^{-jw}$$

$$\Rightarrow \begin{cases} a = \frac{4}{3} \\ b = -\frac{2}{3} \\ c = \frac{2}{3} \end{cases}$$

$$H(e^{jw}) = \frac{\frac{4}{3}}{1+\frac{1}{2}e^{-jw}} + \frac{-\frac{2}{3}e^{-jw}+\frac{2}{3}}{1-\frac{1}{2}e^{-jw}+\frac{1}{4}e^{-j2w}} \Rightarrow \frac{-\frac{8}{3}e^{-jw}+\frac{8}{3}}{e^{-j2w}-2e^{-jw}+4} = \frac{A}{C-1-\sqrt{3}j} + \frac{B}{C-1+\sqrt{3}j}$$

$$\Rightarrow A(e^{-jw}-1+\sqrt{3}j)+B(e^{-jw}-1-\sqrt{3}j) = -\frac{8}{3}e^{-jw}+\frac{8}{3} \Rightarrow \begin{cases} A = -\frac{4}{3} \\ B = -\frac{4}{3} \end{cases}$$

$$H(e^{jw}) = \frac{4}{3}\left(\frac{1}{1+\frac{1}{2}e^{-jw}}\right) + \left(\frac{1+\sqrt{3}j}{3}\right)\left(\frac{1}{1-\frac{1}{2}e^{j60}e^{-jw}}\right) + \left(\frac{1-\sqrt{3}j}{3}\right)\left(\frac{1}{1-\frac{1}{2}e^{-j60}e^{-jw}}\right)$$

$$\Rightarrow h[n] = \frac{4}{3}(-\tfrac{1}{2})^n u[n] + \left(\frac{1+\sqrt{3}j}{3}\right)(\tfrac{1}{2}e^{j60})^n u[n] + \left(\frac{1-\sqrt{3}j}{3}\right)(\tfrac{1}{2}e^{-j60})^n u(n)$$

(b) 細節

$$\frac{-4}{3}\left(\frac{1}{e^{-jw}-1-\sqrt{3}j}\right)+\frac{-4}{3}\left(\frac{1}{e^{-jw}-1+\sqrt{3}j}\right)$$

$$=-\frac{4}{3}\left(\frac{1}{e^{-jw}-(2e^{60j})}\right)+\frac{4}{3}\left(\frac{1}{e^{-jw}-(2e^{-60j}}\right)$$

$$=-\frac{4}{3}\cdot\frac{e^{-60j}}{(e^{-jw}-(2e^{60j}))e^{-60j}}+\frac{4}{3}\left(\frac{e^{60j}}{(e^{-jw}-(2e^{-60j}))e^{60j}}\right)$$

$$=-\frac{4}{3}\cdot\left(\frac{1-\sqrt{3}j}{2}\right)\left(\frac{1}{e^{-jw}e^{-60j}-2}\right)+\frac{4}{3}\cdot\left(\frac{1+\sqrt{3}j}{2}\right)\left(\frac{1}{e^{-jw}e^{60j}-2}\right)$$

$$=\frac{1-\sqrt{3}j}{3}\left(\frac{1}{1-\frac{1}{2}e^{-60j}e^{-jw}}\right)+\frac{1+\sqrt{3}j}{3}\left(\frac{1}{1-\frac{1}{2}e^{60j}e^{-jw}}\right)$$