EE 361002 Signal and System HW5 Answer

3.23

(b) First let us consider a signal $y(t)$ with FS coefficients

$$b_k = \frac{\sin(k\pi/8)}{2k\pi}.$$

From Example 3.5, we know that $y(t)$ must be a periodic square wave which over one period is

$$y(t) = \begin{cases} 1/2, & |t| < 1/4 \\ 0, & 1/4 < |t| < 2 \end{cases}.$$

Now note that $a_k = b_k e^{j\pi k}$. Therefore, the signal $x(t) = y(t+2)$ which is as shown in Figure S2.23(b).

(c) The only nonzero FS coefficients are $a_1 = a_{-1}^* = j$ and $a_2 = a_{-2}^* = 2j$. Using the FS synthesis equation, we get

$$\begin{aligned} x(t) &= a_1 e^{j(2\pi/T)t} + a_{-1}e^{-j(2\pi/T)t} + a_2 e^{j2(2\pi/T)t} + a_{-2}e^{-j2(2\pi/T)t} \\ &= je^{j(2\pi/4)t} - je^{-j(2\pi/4)t} + 2je^{j2(2\pi/4)t} - 2je^{-j2(2\pi/4)t} \\ &= -2\sin(\frac{\pi}{2}t) - 4\sin(\pi t) \end{aligned}$$

3.26. (a) If $x(t)$ is real, then $x(t) = x^*(t)$. This implies that for $x(t)$ real $a_k = a_{-k}^*$. Since this is not true in this case problem, $x(t)$ is not real.

(b) If $x(t)$ is even, then $x(t) = x(-t)$ and $a_k = a_{-k}$. Since this is true for this case, $x(t)$ is even.

(c) We have

$$g(t) = \frac{dx(t)}{dt} \overset{FS}{\longleftrightarrow} b_k = jk\frac{2\pi}{T_0}a_k.$$

Therefore,

$$b_k = \begin{cases} 0, & k = 0 \\ -k(1/2)^{|k|}(2\pi/T_0), & \text{otherwise} \end{cases}.$$

Since b_k is not even, $g(t)$ is not even.

3.31. (a) $g[n]$ is as shown in Figure S3.31. Clearly, $g[n]$ has a fundamental period of 10.

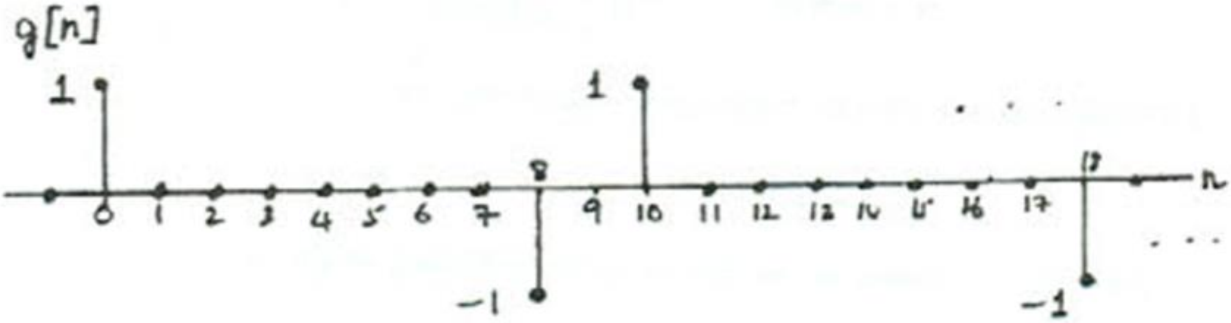

Figure S3.31

(b) The Fourier series coefficiennts of $g[n]$ are $b_k = (1/10)[1 - e^{-j(2\pi/10)8k}]$.

(c) Since $g[n] = x[n] - x[n-1]$, the FS coeffcients a_k and b_k must be related as

$$b_k = a_k - e^{-j(2\pi/10)k} a_k.$$

Therefore,

$$a_k = \frac{b_k}{1 - e^{-j(2\pi/10)k}} = \frac{(1/10)[1 - e^{-j(2\pi/10)8k}]}{1 - e^{-j(2\pi/10)k}}.$$

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The only unknown FS coefficients are a_1, a_{-1}, a_2, and a_{-2}. Since $x(t)$ is real, $a_1 = a_{-1}^*$, and $a_2 = a_{-2}^*$. Since a_1 is real, $a_1 = a_{-1}$. Now, $x(t)$ is of the form

$$x(t) = A_1 \cos(\omega_0 t) + A_2 \cos(2\omega_0 t + \theta),$$

where $\omega_0 = 2\pi/6$. From this we get

$$x(t-3) = A_1 \cos(\omega_0 t - 3\omega_0) + A_2 \cos(2\omega_0 t + \theta - 6\omega_0).$$

Now if we need $x(t) = -x(t-3)$, then $3\omega_0$ and $6\omega_0$ should both be odd multiples of π. Clearly, this is impossible. Therefore, $a_2 = a_{-2} = 0$ and

$$x(t) = A_1 \cos(\omega_0 t).$$

Now, using Parseval's relation on Clue 5, we get

$$\sum_{k=-\infty}^{\infty} |a_k|^2 = |a_1|^2 + |a_{-1}|^2 = \frac{1}{2}.$$

Therefore, $|a_1| = 1/2$. Since a_1 is positive, we have $a_1 = a_{-1} = 1/2$. Therefore, $x(t) = \cos(\pi t/3)$.

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(d) We have

$$\int_{t_0}^{t_0+T} e^{jm\omega_0\tau} e^{-jn\omega_0\tau} d\tau = e^{j(m-n)\omega_0 t_0} \frac{[e^{j(m-n)2\pi} - 1]}{(m-n)\omega_0}$$

This evaluates to 0 when $m \neq n$ and to jT when $m = n$. Therefore, the functions are orthogonal but not orthonormal.

(e) We have

$$\begin{aligned}
\int_{-T}^{T} x_e(t)x_o(t)dt &= \frac{1}{4}\int_{-T}^{T}[x(t) + x(-t)][x(t) - x(-t)]dt \\
&= \frac{1}{4}\int_{-T}^{T} x^2(t)dt - \frac{1}{4}\int_{-T}^{T} x^2(-t)dt \\
&= 0.
\end{aligned}$$