NO.

EE361002 Signal and System HW4

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2.49

(a) $x[n]=\begin{cases}0 & ,\ \text{if } h[-n]=0\\ \frac{h[-n]}{|h[-n]|} & ,\ \text{if } h[-n]\neq 0\end{cases} \Rightarrow |x[n]|=\begin{cases}0 & ,\ \text{if } h[-n]=0\\ \frac{|h[-n]|}{|h[-n]|}=1 & ,\ \text{if } h[-n]\neq 0\end{cases}$

$\Rightarrow |x[n]|\leq 1 = B,\ \forall n$

$\Rightarrow x[n]$ represents a bounded input. The smallest number B is 1. #

(b) Consider $h[n]$ is <u>not absolutely summable</u> (i.e., $\sum_{k=-\infty}^{\infty}|h[k]|=\infty$) & $x[n]$ is a bounded input,

for $n=0$, $y[0]=\sum_{k=-\infty}^{\infty}x[0-k]h[k]$

$$=\sum_{k=-\infty}^{\infty}\frac{h[k]^2}{|h[k]|}$$

$$=\sum_{k=-\infty}^{\infty}|h[k]|=\infty \quad \text{(not bounded)}$$

* Note:
$P\to Q$: P is sufficient for Q
$\neg P\to\neg Q$: P is necessary for Q

not BIBO $\Rightarrow$ This LTI system is <u>not stable</u>

$\therefore$ absolute summability is also a necessary condition for stability. #

2.43

(a) $[x(t)*h(t)]*g(t)=\left[\int_{-\infty}^{\infty}x(\tau_1)h(t-\tau_1)d\tau_1\right]*g(t)$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x(\tau_1)h(\tau_2-\tau_1)g(t-\tau_2)\,d\tau_1\,d\tau_2$$

let $\tau=\tau_1$, $\sigma=\tau_2-\tau_1$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x(\tau)h(\sigma)g(t-\tau-\sigma)\,d\tau\,d\sigma$$

$x(t)*[h(t)*g(t)]=x(t)*\left[\int_{-\infty}^{\infty}h(\tau_1)g(t-\tau_1)d\tau_1\right]$

$$=\int_{-\infty}^{-\infty}\int_{\infty}^{\infty}x(t-\tau_2)h(\tau_1)g(\tau_2-\tau_1)\,d\tau_1\,d\tau_2$$

let $\tau=t-\tau_2$, $\sigma=\tau_1$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x(\tau)h(\sigma)g(t-\tau-\sigma)\,d\tau\,d\sigma$$

$\Rightarrow [x(t)*h(t)]*g(t)=x(t)*[h(t)*g(t)]$ #

(b)(i) $w[n]=u[n]*h_1[n]=\sum_{k=-\infty}^{\infty}u[k]h_1[n-k]$

$h_1[n]=(-\frac{1}{2})^n u[n]$

$\Rightarrow h_1[n-k]=(-\frac{1}{2})^{n-k}u[n-k]=\begin{cases}(-\frac{1}{2})^{n-k} & ,\ k\leq n\\ 0 & ,\ k>n\end{cases}$

$$=\sum_{k=0}^{\infty}h_1[n-k]$$

$$=\left[\sum_{k=0}^{n}(-\tfrac{1}{2})^{n-k}\right]\cdot u[n]$$

$$=\left[\sum_{k=0}^{n}(-\tfrac{1}{2})^{k}\right]\cdot u[n]$$

$$=\frac{2}{3}\left[1-(-\tfrac{1}{2})^{n+1}\right]u[n]$$

$h_2[n]=u[n]+\frac{1}{2}u[n-1]$

$\Rightarrow h_2[n-k]=u[n-k]+\frac{1}{2}u[n-k-1]=\begin{cases}0 & ,\ k>n\\ 1 & ,\ k=n\\ \frac{3}{2} & ,\ k\leq n-1\end{cases}$

$y[n]=w[n]*h_2[n]=\sum_{k=0}^{\infty}\frac{2}{3}\left[1-(-\tfrac{1}{2})^{k+1}\right]h_2[n-k]$

$$=\left\{\frac{2}{3}\left[1-(-\tfrac{1}{2})^{n+1}\right]+\sum_{k=0}^{n-1}\frac{3}{2}\cdot\frac{2}{3}\left[1-(-\tfrac{1}{2})^{k+1}\right]\right\}\cdot u[n]$$

$$=\left[\frac{2}{3}+\cancel{\frac{1}{3}(-\tfrac{1}{2})^n}+n+\frac{1}{3}-\cancel{\frac{1}{3}(-\tfrac{1}{2})^n}\right]\cdot u[n]$$

$$=(n+1)u[n]$$

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(ii) $g[n]=h_1[n]*h_2[n]=\sum_{k=-\infty}^{\infty}h_1[k]h_2[n-k]$

$$=\sum_{k=0}^{\infty}\left(-\tfrac{1}{2}\right)^k h_2[n-k]$$

$$=\left[\left(-\tfrac{1}{2}\right)^n+\tfrac{3}{2}\sum_{k=0}^{n-1}\left(-\tfrac{1}{2}\right)^k\right]\cdot u[n]$$

$$=\left\{\left(-\tfrac{1}{2}\right)^n+\cancel{\tfrac{3}{2}}\cdot\cancel{\tfrac{2}{3}}\left[1-\cancel{\left(-\tfrac{1}{2}\right)^n}\right]\right\}\cdot u[n]$$

$$=u[n]$$

$y[n]=u[n]*g[n]=\sum_{k=-\infty}^{\infty}u[k]u[n-k]$ $\qquad u[n]=\begin{cases}1, & n\geq 0\\ 0, & n<0\end{cases}$

$$=\sum_{k=0}^{\infty}u[n-k] \qquad \Rightarrow u[n-k]=\begin{cases}1, & k\leq n\\ 0, & n>n\end{cases}$$

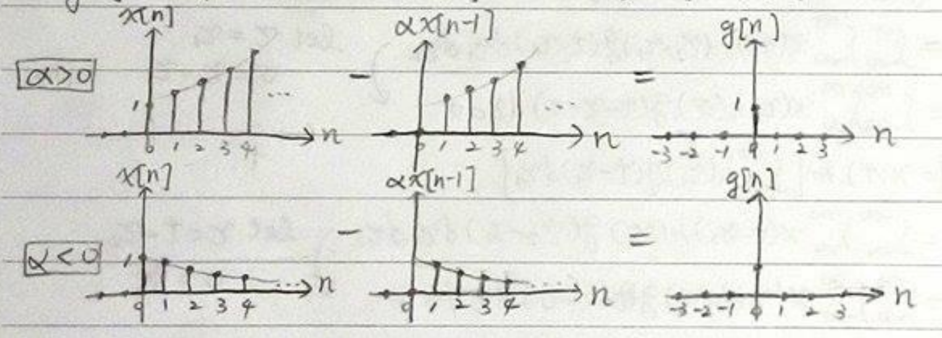
$$=\left(\sum_{k=0}^{n}1\right)\cdot u[n]$$

$$=(n+1)u[n]$$

$\Rightarrow$ The answers to (i) and (ii) is identical #

2.41

(a) $g[n]=x[n]-\alpha x[n-1]=\alpha^n u[n]-\alpha\cdot\alpha^{n-1}u[n-1]=\delta[n]$



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(b) $x[n]*h[n]=\left(\tfrac{1}{2}\right)^n\cdot\{u[n+2]-u[n-2]\}$

$$=\left(\tfrac{1}{2}\right)^n\cdot\{\delta[n+2]+\delta[n+1]+\delta[n]+\delta[n-1]\}$$

$$=4\delta[n+2]+2\delta[n+1]+\delta[n]+\tfrac{1}{2}\delta[n-1]$$

$$=4\{x[n+2]-\alpha x[n+1]\}+2\{x[n+1]-\alpha x[n]\}+\{x[n]-\alpha x[n-1]\}+\tfrac{1}{2}\{x[n-1]-\alpha x[n-2]\}$$

$$=4x[n+2]+(2-\alpha)x[n+1]+(1-2\alpha)x[n]+(\tfrac{1}{2}-\alpha)x[n-1]-\tfrac{1}{2}\alpha x[n-2]$$

$$=4\{x[n]*\delta[n+2]\}+(2-\alpha)\{x[n]*\delta[n+1]\}+(1-2\alpha)\{x[n]*\delta[n]\}$$
$$+(\tfrac{1}{2}-\alpha)\{x[n]*\delta[n-1]\}-\tfrac{1}{2}\alpha\{x[n]*\delta[n-2]\}$$

$$=x[n]*\{4\delta[n+2]+(2-\alpha)\delta[n+1]+(1-2\alpha)\delta[n]+(\tfrac{1}{2}-\alpha)\delta[n-1]-\tfrac{1}{2}\alpha\delta[n-2]\}$$

$\Rightarrow h[n]=4\delta[n+2]+(2-\alpha)\delta[n+1]+(1-2\alpha)\delta[n]+(\tfrac{1}{2}-\alpha)\delta[n-1]-\tfrac{1}{2}\alpha\delta[n-2]$ #

*Note: $x[n+k]=\sum_{m=-\infty}^{\infty}x[m]\delta[n+k-m]$ (sifting property), $k\in\mathbb{Z}$

$x[n]*\delta[n+k]=\sum_{m=-\infty}^{\infty}x[m]\delta[n+k-m]$ $\Rightarrow x[n+k]=x[n]*\delta[n+k]$, $k\in\mathbb{Z}$

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2.34

(a) $\frac{dy(t)}{dt}+2y(t)=x(t)$

(1) given $x_1(t)=Ke^{3t}u(t)$, $K\in\mathbb{R}$

(i) We hypothesize $y_h(t)=Ae^{st}$, $A,S\in\mathbb{R}$ (homogeneous solution)

$\Rightarrow \frac{dy_h(t)}{dt}+2y_h(t)=sAe^{st}+2Ae^{st}=(s+2)Ae^{st}=0 \Rightarrow s=-2$

$\Rightarrow y_h(t)=Ae^{-2t}$, $A\in\mathbb{R}$

(ii) We hypothesize $y_p(t)=Ye^{3t}u(t)$, $Y\in\mathbb{R}$ (particular solution)

$\Rightarrow \frac{dy_p(t)}{dt}+2y_p(t)=3Ye^{3t}u(t)+2Ye^{3t}u(t)=5Ye^{3t}u(t)=x_1(t)=Ke^{3t}u(t) \Rightarrow Y=\frac{K}{5}$

$\Rightarrow y_p(t)=\frac{K}{5}e^{3t}u(t)$

Then for $t>0$,
by (i), (ii) $\Rightarrow y_1(t)=y_h(t)+y_p(t)=[Ae^{-2t}+\frac{K}{5}e^{3t}]u(t)$

given auxiliary condition $y_1(1)=1 \Rightarrow y_1(1)=Ae^{-2}+\frac{K}{5}e^{3}=1 \Rightarrow A=e^{2}-\frac{K}{5}e^{5}$

$\Rightarrow y_1(t)=[(e^{2}-\frac{K}{5}e^{5})e^{-2t}+\frac{K}{5}e^{3t}]\cdot u(t)$, $K\in\mathbb{R}$

(2) given $x_2(t)=0$, then for $t>0$,

$y_2(t)=y_h(t)+y_p(t)=Ae^{-2t}u(t)$

given auxiliary condition $y_2(1)=1 \Rightarrow y_2(1)=Ae^{-2}=1 \Rightarrow A=e^{2} \Rightarrow y_2(t)=e^{-2(t-1)}u(t)$

Now, consider $x_3(t)=x_1(t)+x_2(t)=x_1(t)$. If the system is linear, then $y_3(t)$ should be $y_1(t)+y_2(t)=(2e^{-2(t-1)}-\frac{K}{5}e^{-2(t-\frac{5}{2})}+\frac{K}{5}e^{3t})u(t)$; however, $y_3(t)=y_1(t)=(e^{-2(t-1)}-\frac{K}{5}e^{-2(t-\frac{5}{2})}+\frac{K}{5}e^{3t})u(t)$

$\therefore$ The system is not linear. #

(b) given $x_1(t)=Ke^{3t}u(t)$. $K\in\mathbb{R}$. auxiliary condition $y_1(1)=1$

then for $t>0$, $y_1(t)=[e^{-2(t-1)}-\frac{K}{5}e^{-2(t-\frac{5}{2})}+\frac{K}{5}e^{3t}]\cdot u(t)$

Consider $x_2(t)=x_1(t-T)=Ke^{3(t-T)}u(t-T)$

then for $t>T$, $y_2(t)=Ae^{-2t}+\frac{K}{5}e^{3(t-T)}$

given auxiliary condition $y_2(1)=1 \Rightarrow y_2(1)=Ae^{-2}+\frac{K}{5}e^{3(1-T)}=1 \Rightarrow A=e^{2}-\frac{K}{5}e^{5-3T}$

for $t>T$ $\Rightarrow y_2(t)=e^{-2(t-1)}-\frac{K}{5}e^{-2(t+\frac{3T}{2}-\frac{5}{2})}+\frac{K}{5}e^{3(t-T)}$

$\neq y_1(t-T)=e^{-2(t-T-1)}-\frac{K}{5}e^{-2(t-T-\frac{5}{2})}+\frac{K}{5}e^{3(t-T)}$

$\therefore$ The system is not time-invariant. #