EE 361002 Signal and System HW2 Answer

1.27 (a) $y(t) = x(t-2) + x(2-t)$

(1) not memoryless.

Output at t is dependent on input at $t-2$ and $2-t$.

(2) not time-invariant

Let $y_1(t) = x_1(t-2) + x_1(2-t)$

$x_2(t) = x_1(t-t_0)$

$y_1(t-t_0) = x_1(t-t_0-2) + x_1(2-t+t_0)$

$y_2(t) = x_2(t-2) + x_2(2-t)$

$= x_1(t-t_0-2) + x_1(2-t-t_0)$

$y_1(t-t_0) \neq y_2(t)$

$\therefore$ This system is not time-invariant.

(3) linear

Let $x_1(t) \to y_1(t) = x_1(t-2) + x_1(2-t)$

$x_2(t) \to y_2(t) = x_2(t-2) + x_2(2-t)$

$a x_1(t) + b x_2(t) \to a x_1(t-2) + b x_2(t-2)$

$+ a x_1(2-t) + b x_2(2-t)$

$= a y_1(t) + b y_2(t)$

(4) not causal

Output depends on input at future time when $t < 1$.

(5) stable

If $|x(t)| < B \ \forall t$

then $|y(t)| = |x(t-2) + x(2-t)| < |x(t-2)| + |x(2-t)| < 2B$

$\therefore$ The system is stable.

(b) $y(t) = [\cos(3t)]x(t)$

(1) memoryless

Output is dependent on input only at current time.

(2) not time-invarient

Let $y_1(t) = [\cos(3t)]x_1(t)$

$x_2(t) = x_1(t-t_0)$

$y_1(t-t_0) = [\cos(3t-3t_0)]x_1(t-t_0)$

$y_2(t) = [\cos(3t)]x_2(t)$

$= [\cos(3t)]x_1(t-t_0)$

$y_1(t-t_0) \neq y_2(t)$

$\therefore$ This system is not time-invariant.

(3) linear

Let $x_1(t) \to y_1(t) = [\cos(3t)]x_1(t)$

$x_2(t) \to y_2(t) = [\cos(3t)]x_2(t)$

$a x_1(t) + b x_2(t) \to a[\cos(3t)]x_1(t) + b[\cos(3t)]x_2(t)$

$= a y_1(t) + b y_2(t)$

(4) causal

Output depends on input at present time.

(5) stable

If $|x(t)| < B \ \forall t$.

then $|y(t)| = |[\cos(3t)]x(t)|$

$= |\cos(3t)||x(t)| \quad (|\cos(3t)| < 1)$

$< B$

(d) $y(t) = \begin{cases} 0 & , t<0 \\ x(t)+x(t-2) & , t\geq 0 \end{cases}$

(1) not memoryless.

Output at t is dependent on input at t-2 when $t \geq 0$.

(2) not time-invariant

Let $y_1(t) = \begin{cases} 0 & , t<0 \\ x_1(t)+x_1(t-2) & , t\geq 0 \end{cases}$

$x_2(t) = x_1(t-t_0)$

$y_1(t-t_0) = \begin{cases} 0 & , t<t_0 \\ x_1(t-t_0)+x_1(t-t_0-2) & , t\geq t_0 \end{cases}$

$y_2(t) = \begin{cases} 0 & , t<0 \\ x_2(t)+x_2(t-2) & , t\geq 0 \end{cases}$

$= \begin{cases} 0 & , t<0 \\ x_1(t-t_0)+x_1(t-2-t_0) & , t\geq 0 \end{cases}$

$y_1(t-t_0) \neq y_2(t)$

(3) linear

Let $x_1(t) \to y_1(t) = \begin{cases} 0 & , t<0 \\ x_1(t)+x_1(t-2) & , t\geq 0 \end{cases}$

$x_2(t) \to y_2(t) = \begin{cases} 0 & , t<0 \\ x_2(t)+x_2(t-2) & , t\geq 0 \end{cases}$

$ax_1(t)+bx_2(t) \to \begin{cases} 0 & , t<0 \\ ax_1(t)+bx_1(t) \\ +ax_1(t-2)+bx_2(t-2) & , t\geq 0 \end{cases}$

$= ay_1(t)+by_2(t)$

(4) causal

Output depends on input at present and past times.

(5) Stable

Same as (a).

(e) $y(t) = \begin{cases} 0 & , x(t)<0 \\ x(t)+x(t-2) & , x(t)\geq 0 \end{cases}$

(1) not memoryless

Output at t is dependent on input at t-2 when $x(t) \geq 0$

(2) time-invariant

Let $y_1(t) = \begin{cases} 0 & , x_1(t)<0 \\ x_1(t)+x_1(t-2) & , x_1(t)\geq 0 \end{cases}$

$x_2(t) = x_1(t-t_0)$

$y_1(t-t_0) = \begin{cases} 0 & , x_1(t-t_0)<0 \\ x_1(t-t_0)+x_1(t-t_0-2) & , x_1(t-t_0)\geq 0 \end{cases}$

$y_2(t) = \begin{cases} 0 & , x_2(t)<0 \\ x_2(t)+x_2(t-2) & , x_2(t)\geq 0 \end{cases}$

$= \begin{cases} 0 & , x_1(t-t_0)<0 \\ x_1(t-t_0)+x_1(t-2-t_0) & , x_1(t-t_0)\geq 0 \end{cases}$

$y_1(t-t_0) = y_2(t)$, $\therefore$ The system is time-invariant.

(3) not linear

Let $x_1(t) \to y_1(t) = \begin{cases} 0 & , x_1(t)<0 \\ x_1(t)+x_1(t-2) & , x_1(t)\geq 0 \end{cases}$

$x_2(t) \to y_2(t) = \begin{cases} 0 & , x_2(t)<0 \\ x_2(t)+x_2(t-2) & , x_2(t)\geq 0 \end{cases}$

$ax_1(t)+bx_2(t) \to \begin{cases} 0 & , ax_1(t)+bx_2(t)<0 \\ ax_1(t)+bx_2(t) \\ +ax_1(t-2)+bx_2(t-2) & , ax_1(t)+bx_2(t)\geq 0 \end{cases}$

$\neq ay_1(t)+by_2(t)$

(4) causal

Output depends on input at present and past times.

(5) Stable

Same as (a).

(f) $y(t) = x(\frac{t}{3})$

(1) not memoryless.

Output at t is dependent on input at $\frac{t}{3}$.

(2) not time-invariant

Let $y_1(t) = x_1(\frac{t}{3})$

$x_2(t) = x_1(t-t_0)$

$y_1(t-t_0) = x_1(\frac{t-t_0}{3})$.

$y_2(t) = x_2(\frac{t}{3}) = x_1(\frac{t}{3}-t_0)$

$y_1(t-t_0) \neq y_2(t)$.

(3) linear

Let $x_1(t) \to y_1(t) = x_1(\frac{t}{3})$

$x_2(t) \to y_2(t) = x_2(\frac{t}{3})$

$ax(t)+bx_2(t) \to ax_1(\frac{t}{3})+bx_2(\frac{t}{3})$

$= ay_1(t)+by_2(t)$

(4) not causal

Output depends on input at future time when $t<0$.

(5) stable

If $|x(t)| < B$ $\forall t$.

then $|y(t)| = |x(\frac{t}{3})| < B$

1.28 (b) $y[n] = x[n-2] - 2x[n-8]$

(1) not memoryless

Output at n depends on input at n-2 and n-8.

(2) time-invariant

Let $y_1[n] = x_1[n-2] - 2x_1[n-8]$

$x_2[n] = x_1[n-n_0]$

$y_1[n-n_0] = x_1[n-n_0-2] - 2x_1[n-n_0-8]$

$y_2[n] = x_2[n-2] - 2x_2[n-8]$

$= x_1[n-2-n_0] - 2x_2[n-8-n_0]$

$\Rightarrow y_1[n-n_0] = y_2[n]$.

(3) linear

Let $x_1[n] \rightarrow y_1[n] = x_1[n-2] - 2x_1[n-8]$

$x_2[n] \rightarrow y_2[n] = x_2[n-2] - 2x_2[n-8]$

$ax_1[n] + bx_2[n] \rightarrow ax_1[n-2] + bx_2[n-2]$

$-2ax_1[n-8] - 2bx_2[n-8]$

$= ay_1[n] + by_2[n]$

(4) causal

Output depends on input at past time.

(5) stable

If $|x[n]| < B \ \forall n$

then $|y[n]| = |x[n-2] - 2x[n-8]|$

$< |x[n-2]| + |2x[n-8]|$

$< 3B$

(c) $y[n] = n\,x[n]$

(1) memoryless

Output at n is dependent on input at n.

(2) not time-invariant

Let $y_1[n] = n\,x_1[n]$

$x_2[n] = x_1[n-n_0]$

$y_1[n-n_0] = (n-n_0)\,x_1[n-n_0]$ $\quad y_1[n-n_0] \neq y_2[n]$

$y_2[n] = n\,x_2[n] = n\,x_1[n-n_0]$

(3) linear

Let $x_1[n] \rightarrow y_1[n] = n x_1[n]$

$x_2[n] \rightarrow y_2[n] = n x_2[n]$

$ax_1[n] + bx_2[n] \rightarrow n(ax_1[n] + bx_2[n])$

$= ay_1[n] + by_2[n]$

(4) causal

Output depends on input at present time.

(5) not stable.

If $|x[n]| < B \ \forall n$.

then $|y[n]| = |n|\,|x[n]| \rightarrow \infty$ when $n \rightarrow \infty$.

(d) $y[n] = \mathcal{E}v\{x[n-1]\} = \frac{1}{2}\{x[n-1] + x[-n+1]\}$

(1) not memoryless.

Output at n depends at input at n-1 and -n+1.

(2) not time-invariant

Let $y_1[n] = \frac{1}{2}\{x_1[n-1] + x_1[-n+1]\}$

$x_2[n] = x_1[n-n_0]$

$y_1[n-n_0] = \frac{1}{2}\{x_1[n-n_0-1] + x_1[-n+n_0+1]\}$.

$y_2[n] = \frac{1}{2}\{x_2[n-1] + x_2[-n+1]\}$

$= \frac{1}{2}\{x_1[n-1-n_0] + x_1[-n+1-n_0]\}$

$\rightarrow y_1[n-n_0] \neq y_2[n]$

(3) linear

Let $x_1[n] \rightarrow y_1[n] = \frac{1}{2}\{x_1[n-1] + x_1[-n+1]\}$

$x_2[n] \rightarrow y_2[n] = \frac{1}{2}\{x_2[n-1] + x_2[-n+1]\}$.

$ax_1[n] + bx_2[n] \rightarrow \frac{1}{2}\{ax_1[n-1] + bx_2[n-1]$

$+ ax_1[-n+1] + bx_2[-n+1]\}$

$= ay_1[n] + by_2[n]$

(4) not causal

Output depends on input at future when $n < \frac{1}{2}$.

(5) stable

If $|x[n]| < B \ \forall n$

then $|y[n]| = \frac{1}{2}|x[n-1] + x[-n+1]|$

$< \frac{1}{2}|x[n-1]| + \frac{1}{2}|x[-n+1]|$

$< \frac{1}{2}B + \frac{1}{2}B = B$.

(e) $y[n] = \begin{cases} x[n] & , n \geq 1 \\ 0 & , n = 0 \\ x[n+1] & , n \leq -1 \end{cases}$

(1) not memoryless

Output at n depends on input at $n+1$ when $n \leq -1$.

(2) not time-invariant

Let $y_1[n] = \begin{cases} x_1[n] & , n \geq 1 \\ 0 & , n = 0 \\ x_1[n+1] & , n \leq -1 \end{cases}$

$x_2[n] = x_1[n-n_0]$

$y_1[n-n_0] = \begin{cases} x_1[n-n_0] & , n-n_0 \geq 1 \\ 0 & , n-n_0 = 0 \\ x_1[n-n_0+1] & , n-n_0 \leq -1 \end{cases}$

$y_2[n] = \begin{cases} x_2[n] & , n \geq 1 \\ 0 & , n = 0 \\ x_2[n+1] & , n \leq -1 \end{cases}$

$= \begin{cases} x_1[n-n_0] & , n \geq 1 \\ 0 & , n = 0 \\ x_1[n+1-n_0] & , n \leq -1 \end{cases}$

$\Rightarrow y_1[n-n_0] \neq y_2[n]$

(3) linear

Let $x_1[n] \rightarrow y_1[n] = \begin{cases} x_1[n] & , n \geq 1 \\ 0 & , n = 0 \\ x_1[n+1] & , n \leq -1 \end{cases}$

$x_2[n] \rightarrow y_2[n] = \begin{cases} x_2[n] & , n \geq 1 \\ 0 & , n = 0 \\ x_2[n+1] & , n \leq -1 \end{cases}$

$ax_1[n] + bx_2[n] \rightarrow \begin{cases} ax_1[n] + bx_2[n] & , n \geq 1 \\ 0 & , n = 0 \\ ax_1[n+1] + bx_2[n+1] & , n \leq -1 \end{cases}$

$= ay_1[n] + by_2[n]$.

(4) not causal

Output depends on input at future time when $n \leq -1$.

(5) stable

If $|x[n]| < B \ \forall n$

then $|y[n]| = \begin{cases} |x[n]| & , n \geq 1 \\ 0 & , n = 0 \\ |x[n+1]| & , n \leq -1 \end{cases} < B$.

(g) $y[n] = x[4n+1]$

(1) not memoryless

Output at n depends on input at $4n+1$.

(2) not time-invariant

Let $y_1[n] = x_1[4n+1]$

$x_2[n] = x_1[n-n_0]$

$y_1[n-n_0] = x_1[4n-4n_0+1]$

$y_2[n] = x_2[4n+1] = x_1[4n+1-n_0]$

$\Rightarrow y_1[n-n_0] \neq y_2[n]$.

(3) linear

Let $x_1[n] \rightarrow y_1[n] = x_1[4n+1]$

$x_2[n] \rightarrow y_2[n] = x_2[4n+1]$

$ax_1[n] + bx_2[n] \rightarrow ax_1[4n+1] + bx_2[4n+1]$

$= ay_1[n] + by_2[n]$

(4) not causal

Output depends on input at future time when $n \geq 0$.

(5) stable

If $|x[n]| < B \ \forall n$

then $|y[n]| = |x[4n+1]| < B$.

#HW2 1.30 (cefilmn)

(c) $y[n] = n x[n]$

No.

ex. $x[n] = \delta[n]$, $2\delta[n]$

(e) $y[n] = \begin{cases} x[n-1], & n \geq 1 \\ 0, & n = 0 \\ x[n], & n \leq 1 \end{cases}$

$\underset{n\geq 1}{x[n]} \rightarrow \boxed{sys} \rightarrow y[n] = x[n-1] \rightarrow \boxed{inv} \rightarrow z[n]$

$z[n] = \underset{n\geq 0}{y[n+1]} = x[n+1-1]$

$= x[n]$

同理 $\Rightarrow$ $x[n] \longleftrightarrow y[n]$, $n \leq -1$

Yes, $y[n] = \begin{cases} x[n+1], & n \geq 0 \\ x[n], & n \leq -1 \end{cases}$

(f) $y[n] = x[n]x[n-1]$

No.

ex. $x[n]$ & $-x[n]$

(i) $y[n] = \sum_{k=-\infty}^{n} (\frac{1}{2})^{n-k} x[k]$

$\left(x[n] \rightarrow sys \rightarrow y[n] \rightarrow inv \rightarrow z[n] \right.$

$\left. x[n] \leftarrow ?" \right)$

$y[n] = (\frac{1}{2})^0 x[n] + \frac{1}{2}x[n-1] + (\frac{1}{2})^2 x[n-2] + \cdots$

$\frac{1}{2} y[n-1] = \frac{1}{2}x[n-1] + (\frac{1}{2})^2 x[n-2] + \cdots$

$\Rightarrow y[n] - \frac{1}{2}y[n-1] = x[n]$

Yes, $y[n] = x[n] - \frac{1}{2}x[n-1]$

(l) $y(t) = x(2t)$

$x(t) \rightarrow sys \rightarrow y(t) = x(2t) \longrightarrow inv \rightarrow z(t)$

$z(t) = y(\frac{t}{2}) = x(\frac{2t}{2}) = x(t)$

Yes, $y(t) = x(\frac{t}{2})$

(m) $y[n] = x[2n]$

No.

$x_1[n] = \delta[n] + \delta[n-1]$

$\rightarrow y_1[n] = \delta[2n] + \delta[2n-1] = \delta[2n]$

$(\because n \in \mathbb{Z})$

$x_2[n] = \delta[n]$

$\rightarrow y_2[n] = \delta[2n]$

(n) $y[n] = \begin{cases} x[\frac{n}{2}] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$

$x[n] \rightarrow sys \rightarrow y[n] = x[\frac{n}{2}] \rightarrow inv \rightarrow z[n]$

$z[n] = y[2n] = x[\frac{2n}{2}] = x[n]$

Yes, $y[n] = x[2n]$

1.42. **(a)** Consider two systems S_1 and S_2 connected in series. Assume that if $x_1(t)$ and $x_2(t)$ are the inputs to S_1, then $y_1(t)$ and $y_2(t)$ are the outputs, respectively. Also, assume that if $y_1(t)$ and $y_2(t)$ are the inputs to S_2, then $z_1(t)$ and $z_2(t)$ are the outputs, respectively. Since S_1 is linear, we may write

$$ax_1(t) + bx_2(t) \xrightarrow{S_1} ay_1(t) + by_2(t),$$

where a and b are constants. Since S_2 is also linear, we may write

$$ay_1(t) + by_2(t) \xrightarrow{S_2} az_1(t) + bz_2(t),$$

We may therefore conclude that

$$ax_1(t) + bx_2(t) \xrightarrow{S_1,S_2} az_1(t) + bz_2(t).$$

Therefore, the series combination of S_1 and S_2 is linear.

Since S_1 is time invariant, we may write

$$x_1(t - T_0) \xrightarrow{S_1} y_1(t - T_0)$$

and

$$y_1(t - T_0) \xrightarrow{S_2} z_1(t - T_0).$$

Therefore,

$$x_1(t - T_0) \xrightarrow{S_1,S_2} z_1(t - T_0).$$

Therefore, the series combination of S_1 and S_2 is time invariant.

(b) False. Let $y(t) = x(t) + 1$ and $z(t) = y(t) - 1$. These correspond to two nonlinear systems. If these systems are connected in series, then $z(t) = x(t)$ which is a linear system.

(c) system 1: $y_1[n] = \begin{cases} x[\frac{n}{2}], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

system 2: $y_2[n] = x_2[n] + \frac{1}{2}x_2[n-1] + \frac{1}{4}x_2[n-2]$

system 3: $y[n] = x_3[2n]$

$$x \longrightarrow \boxed{S1} \xrightarrow{y_1\ \ x_2} \boxed{S2} \xrightarrow{y_2\ \ x_3} \boxed{S3} \longrightarrow y$$

$$\begin{aligned} y[n] = x_3[2n] = y_2[2n] &= x_2[2n] + \tfrac{1}{2}x_2[2n-1] + \tfrac{1}{4}x_2[2n-2] \\ &= \underbrace{y_1[2n]}_{\text{even}} + \tfrac{1}{2}\underbrace{y_1[2n-1]}_{\text{odd}} + \tfrac{1}{4}\underbrace{y_1[2n-2]}_{\text{even}} \\ &= x[n] + 0 + \tfrac{1}{4}x[n-1] \end{aligned}$$

$\Rightarrow y[n] = x[n] + \frac{1}{4}x[n-1]$

(i) Consider $x_3[n] = ax_1[n] + bx_2[n]$

then we get $y_3[n] = (ax_1[n] + bx_2[n]) + (\frac{1}{4}ax_1[n-1] + \frac{1}{4}bx_2[n-1])$

$= a(x_1[n] + \frac{1}{4}x_1[n-1]) + b(x_2[n] + \frac{1}{4}x_2[n-1])$

$= ay_1[n] + by_2[n]$

$\therefore$ The overall interconnected system is linear.

(ii) Consider $x_2[n] = x_1[n-n_0]$, $y_1[n-n_0] = x_1[n-n_0] + \frac{1}{4}x_1[n-1-n_0]$

then we get $y_2[n] = x_2[n] + \frac{1}{4}x_2[n-1]$

$= x_1[n-n_0] + \frac{1}{4}x_1[n-n_0-1]$

$= y_1[n-n_0]$

$\therefore$ The overall interconnected system is time-invariant. #

1.47

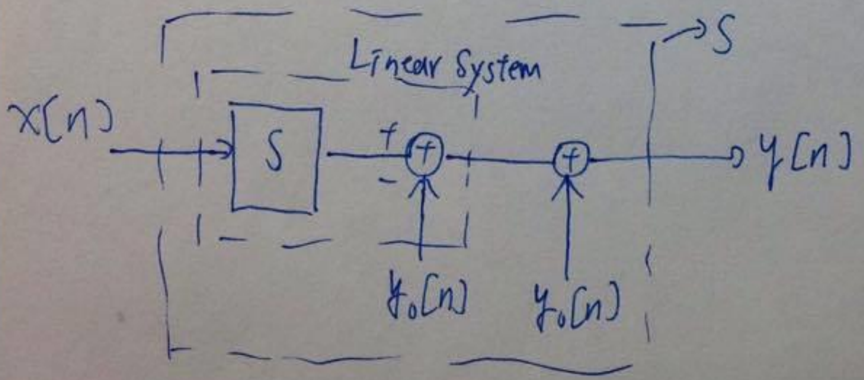
(a) $y[n] = S\{x[n] + x_1[n]\} - y_1[n]$

$= S\{x[n]\} + S\{x_1[n]\} - S\{x_1[n]\}$

$= S\{x[n]\}$

$\Rightarrow$ not depend on the particular choice of $x_1[n]$

(b) If $x_1[n] = 0$ for all n, then $y_1[n]$ will be the zero-input reponse $y_0[n]$



the same figure as 課本 p57 Figure 1.48.