EE 361002 Signal and System HW16 Answer

10.32

(a) We are given that $h[n] = a^n u[n]$ and $x[n] = u[n] - u[n-N]$. Therefore,

$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= \sum_{k=-\infty}^{\infty} h[n-k]x[k] \\ &= \sum_{k=0}^{N-1} a^{n-k}u[n-k] \end{aligned}$$

Now, $y[n]$ may be evaluated to be

$$y[n] = \begin{cases} 0, & n < 0 \\ \sum_{k=0}^{n} a^n a^{-k}, & 0 \le n \le N-1 \\ \sum_{k=0}^{N-1} a^n a^{-k}, & n > N-1 \end{cases}$$

Simplifying,

$$y[n] = \begin{cases} 0, & n < 0 \\ (a^n - a^{-1})/(1 - a^{-1}), & 0 \le n \le N-1 \\ a^n(1 - a^{-N})/(1 - a^{-1}), & n > N-1 \end{cases}$$

(b) Using Table 10.2, we get

$$H(z) = \frac{1}{1 - az^{-1}}, \qquad |z| > |a|$$

and

$$X(z) = \frac{1 - z^{-N}}{1 - z^{-1}}, \qquad \text{All } z.$$

Therefore,

$$Y(z) = X(z)H(z) = \frac{1}{(1 - z^{-1})(1 - az^{-1})} - \frac{z^{-N}}{(1 - z^{-1})(1 - az^{-1})}$$

The ROC is $|z| > |a|$. Consider

$$P(z) = \frac{1}{(1 - z^{-1})(1 - az^{-1})}$$

with ROC $|z| > |a|$. The partial fraction expansion of $P(z)$ is

$$P(z) = \frac{1/(1-a)}{1 - z^{-1}} + \frac{1/(1 - a^{-1})}{1 - az^{-1}}.$$

Therefore,

$$p[n] = \frac{1}{1-a}u[n] + \frac{1}{1 - a^{-1}}a^n u[n].$$

Now, note that

$$Y(z) = P(z)[1 - z^{-N}].$$

Therefore,

$$y[n] = p[n] - p[n-N] = \frac{1}{1-a}\{u[n] - u[n-N]\} + \frac{1}{1-a^{-1}}\{a^n u[n] - a^{n-N}u[n-N]\}.$$

This may be written as

$$y[n] = \begin{cases} 0, & n < 0 \\ (a^n - a^{-1})/(1 - a^{-1}), & 0 \le n \le N-1 \\ a^n(1 - a^{-N})/(1 - a^{-1}), & n > N-1 \end{cases}$$

This is the same as the result of part (a).

10.35

Taking the z-transform of both sides of the given difference equation and simplifying, we get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{z - \frac{5}{2} + z^{-1}} = \frac{z^{-1}}{1 - \frac{5}{2}z^{-1} + z^{-2}}.$$

The partial fraction expansion of $H(z)$ is

$$H(z) = \frac{-2/3}{1 - \frac{1}{2}z^{-1}} + \frac{2/3}{1 - 2z^{-1}}.$$

If the ROC is $|z| > 2$, then

$$h_1[n] = -\frac{2}{3}\left(\frac{1}{2}\right)^n u[n] + \frac{2}{3}(2)^n u[n].$$

If the ROC is $1/2 < |z| < 2$, then

$$h_2[n] = -\frac{2}{3}\left(\frac{1}{2}\right)^n u[n] - \frac{2}{3}(2)^n u[-n-1].$$

If the ROC is $|z| < 1/2$, then

$$h_3[n] = \frac{2}{3}\left(\frac{1}{2}\right)^n u[-n-1] - \frac{2}{3}(2)^n u[-n-1].$$

For each $h_i[n]$, we now need to show that if $y[n] = h_i[n]$ in the difference equation, then $x[n] = \delta[n]$. Consider substituting $h_1[n]$ into the difference equation. This yields

$$\begin{aligned} \frac{2}{3} \quad & \left(\tfrac{1}{2}\right)^{n-1} u[n-1] - \tfrac{2}{3}(2)^{n-1}u[n-1] - \tfrac{5}{3}\left(\tfrac{1}{2}\right)^n u[n] \\ & + \tfrac{5}{3}(2)^n u[n] + \tfrac{2}{3}\left(\tfrac{1}{2}\right)^{n+1} u[n+1] - \tfrac{2}{3}(2)^{n+1}u[n+1] = x[n] \end{aligned}$$

Then,

$$\begin{aligned} x[n] &= 0, \qquad \text{for } n < -1, \\ x[-1] &= 2/3 - 2/3 = 0, \\ x[n] &= 0, \qquad \text{for } n > 0. \end{aligned}$$

It follows that $x[n] = \delta[n]$. It can similarly be shown that $h_2[n]$ and $h_3[n]$ satisfy the difference equation.

10.38

(a) $e_1[n] = f_1[n]$.

(b) $e_2[n] = f_2[n]$.

(c) Using the results of parts (a) and (b), we may redraw the block-diagram as shown in Figure S10.38.

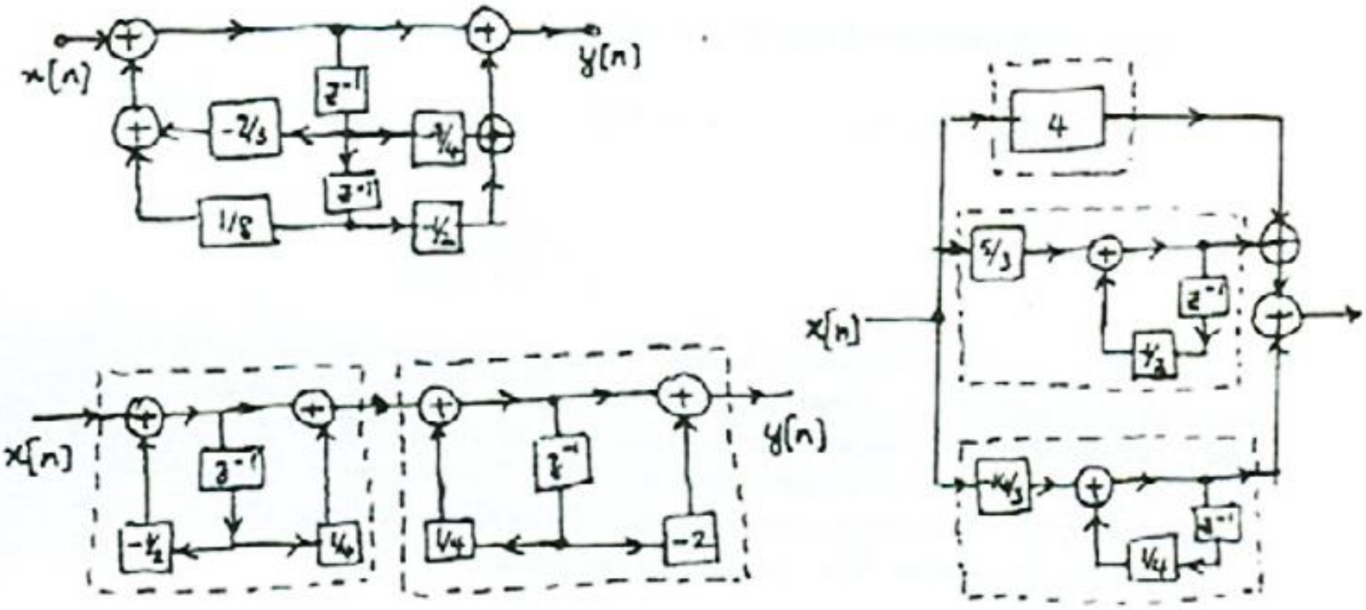

Figure S10.38

(d) Using the approach shown in the examples in the textbook we may draw the block-diagram of $H_1(z) = [1 + (1/4)z^{-1}]/[1 + (1/2)z^{-1}]$ and $H_2(z) = [1 - 2z^{-1}]/[1 - (1/4)z^{-1}]$ as shown in the dotted boxes in the figure below. $H(z)$ is the cascade of these two systems.

(e) Using the approach shown in the examples shown in the textbook, we may draw the block-diagram of $H_1(z) = 4$, $H_2(z) = [5/3]/[1 + (1/2)z^{-1}]$ and $H_3(z) = [-14/3]/[1 - (1/4)z^{-1}]$ as shown in the dotted boxes in the figure below. $H(z)$ is the parallel combination of $H_1(z)$, $H_2(z)$, and $H_3(z)$.

10.42

(a) Taking the unilateral z-transform of both sides of the given difference equation, we get

$$\mathcal{Y}(z) + 3z^{-1}\mathcal{Y}(z) + 3y[-1] = \mathcal{X}(z).$$

Setting $\mathcal{X}(z) = 0$, we get

$$\mathcal{Y}(z) = \frac{-3}{1 + 3z^{-1}}.$$

The inverse unilateral z-transform gives the zero-input response

$$y_{zi}[n] = -3(-3)^n u[n] = (-3)^{n+1} u[n].$$

Now, since it is given that $x[n] = (1/2)^n u[n]$, we have

$$\mathcal{X}(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}, \qquad |z| > 1/2.$$

Setting $y[-1]$ to be zero, we get

$$\mathcal{Y}(z) + 3z^{-1}\mathcal{Y}(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}.$$

Therefore,

$$\mathcal{Y}(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})(1 + 3z^{-1})}.$$

The partial fraction expansion of $\mathcal{Y}(z)$ is

$$\mathcal{Y}(z) = \frac{1/7}{1 - \frac{1}{2}z^{-1}} + \frac{6/7}{1 + 3z^{-1}}.$$

The inverse unilateral z-transform gives the zero-state response

$$y_{zs}[n] = \frac{1}{7}\left(\frac{1}{2}\right)^n u[n] + \frac{6}{7}(-3)^n u[n].$$

(b) Taking the unilateral z-transform of both sides of the given difference equation, we get

$$\mathcal{Y}(z) - \frac{1}{2}z^{-1}\mathcal{Y}(z) - \frac{1}{2}y[-1] = \mathcal{X}(z) - \frac{1}{2}z^{-1}\mathcal{X}(z).$$

Setting $\mathcal{X}(z) = 0$, we get

$$\mathcal{Y}(z) = 0.$$

The inverse unilateral z-transform gives the zero-input response

$$y_{zi}[n] = 0.$$

Now, since it is given that $x[n] = u[n]$, we have

$$\mathcal{X}(z) = \frac{1}{1 - z^{-1}}, \qquad |z| > 1.$$

Setting $y[-1]$ to be zero, we get

$$\mathcal{Y}(z) - \frac{1}{2}z^{-1}\mathcal{Y}(z) = \frac{1}{1 - z^{-1}} - \frac{(1/2)z^{-1}}{1 - z^{-1}}.$$

Therefore,

$$\mathcal{Y}(z) = \frac{1}{1 - z^{-1}}.$$

The inverse unilateral z-transform gives the zero-state response

$$y_{zs}[n] = u[n].$$

(c) Taking the unilateral z-transform of both sides of the given difference equation, we get

$$\mathcal{Y}(z) - \frac{1}{2}z^{-1}\mathcal{Y}(z) - \frac{1}{2}y[-1] = \mathcal{X}(z) - \frac{1}{2}z^{-1}\mathcal{X}(z).$$

Setting $\mathcal{X}(z) = 0$, we get

$$\mathcal{Y}(z) = \frac{1/2}{1 - \frac{1}{2}z^{-1}}.$$

The inverse unilateral z-transform gives the zero-input response

$$y_{zi}[n] = \left(\frac{1}{2}\right)^{n+1} u[n].$$

Since the input $x[n]$ is the same as the one used in the part (b), the zero-state response is still

$$y_{zs}[n] = u[n].$$