EE 361002 Signal and System HW15 Answer

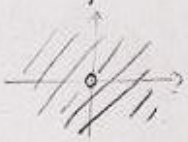
10.21

10.21

(b) $\delta[n-5]$. By $\delta[n-m] \xleftrightarrow{z} z^{-m}$

$X(z) = z^5$, for all z except 0

FT. exist



(d)

$x[n] = (\frac{1}{2})^{n+1} u[n+3]$

$X(z) = \sum_{n=-3}^{\infty} (\frac{1}{2})^{n+1} \cdot z^{-n}$

$= \frac{(\frac{1}{2})^{-2} \cdot z^3}{1 - \frac{1}{2} z^{-1}}$

$= \frac{4z^3}{1 - \frac{1}{2z}}$, $|\frac{1}{2z}| < 1$

$\Rightarrow |z| > \frac{1}{2}$ FT exist.

(e)

$x[n] = (-\frac{1}{3})^n u[-n-2]$

$X(z) = \sum_{n=-\infty}^{-2} x[n] z^{-n}$

$= \frac{(-\frac{1}{3})^{-2} z^2}{1 - (-\frac{1}{3})^{-1} z}$

$= \frac{9z^2}{1 + 3z}$, $|z| < \frac{1}{3}$

FT NOT exist.

(g)

$x[n] = 2^n u[-n] + (\frac{1}{4})^n u[n-1]$

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$X(z) = \sum_{n=0}^{-\infty} 2^n z^{-n} + \sum_{n=1}^{\infty} (\frac{1}{4})^n z^{-n}$

$= \frac{1}{1 - \frac{1}{2} z} + \frac{\frac{1}{4} z^{-1}}{1 - \frac{1}{4} z^{-1}}$

$|z| < 2$ $|z| > \frac{1}{4}$ $\Rightarrow \frac{1}{4} < |z| < 2$, FT exist.

(h)

$x[n] = (\frac{1}{3})^{n-2} u[n-2]$

$X(z) = \sum_{n=2}^{\infty} x[n] z^{-n}$

$= \frac{(\frac{1}{3})^0 z^{-2}}{1 - (\frac{1}{3}) z^{-1}}$

$= \frac{\frac{1}{3z^2}}{1 - \frac{1}{3z}}$

$|z| > \frac{1}{3}$ FT exist.

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10.22.

(b)

$x[n] = n\left(\frac{1}{2}\right)^{|n|}$

令 $x_1[n] = \left(\frac{1}{2}\right)^{|n|}$

$= \left(\frac{1}{2}\right)^n \cdot u[n] + \left(\frac{1}{2}\right)^{-n} u[-n-1]$

$\Rightarrow X_1(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{\frac{1}{2}z}{1-\frac{1}{2}z}$,

$\Rightarrow \frac{1}{2} < |z| < 2.$

$x[n] = n\,x_1[n]$

$X(z) = -z\frac{dX_1(z)}{dz}$

$= -z\left[\frac{2(2z)^{-2}}{(1-\frac{1}{2z})^2} - \frac{\frac{2}{z^2}}{(1-\frac{2}{z})^2}\right]$

$= \frac{-\frac{1}{2z}}{(1-\frac{1}{2z})^2} + \frac{\frac{2}{z}}{(1-\frac{2}{z})^2}$

$\frac{1}{2} < |z| < 2$, FT exist.

(d)

$x[n] = 4^n \cos\left[\frac{2\pi}{6}n + \frac{\pi}{4}\right] u[-n-1]$

$= 4^n \left\{\frac{e^{-j(\frac{2\pi}{6}n+\frac{\pi}{4})} + e^{j(\frac{2\pi}{6}n+\frac{\pi}{4})}}{2}\right\} u[-n-1]$

$= \frac{e^{j\frac{\pi}{4}}}{2} \cdot \left[4^n u[-n-1] \cdot e^{j\frac{2\pi}{6}n}\right]$

$+ \frac{e^{-j\frac{\pi}{4}}}{2} \cdot \left[4^n u[-n-1] \cdot e^{-j\frac{2\pi}{6}n}\right]$

By $e^{j\omega_0 n}x[n] \xleftrightarrow{z} X(e^{-j\omega_0}z)$

$4^n u[-n-1] \xleftrightarrow{z} \frac{z}{1-\frac{z}{4}} = \frac{1}{\frac{4}{z}-1}$

$X(z) = \frac{e^{j\frac{\pi}{4}}}{2} \times \frac{1}{4(e^{-j\frac{2\pi}{6}}z)^{-1}-1} + \frac{e^{-j\frac{\pi}{4}}}{2} \times \frac{1}{4(e^{j\frac{2\pi}{6}}z)^{-1}-1}$

$|z| < 4$, FT exist.

10.25

(a)

$$X(z)=\frac{1}{(1-\frac{1}{2}z^{-1})(1-z^{-1})}=\frac{a}{1-\frac{1}{2}z^{-1}}+\frac{b}{1-z^{-1}}$$

$$a=\left.\frac{1}{1-z^{-1}}\right|_{z=2}=-1,\quad b=\left.\frac{1}{1-\frac{1}{2}z^{-1}}\right|_{z=1}=2$$

$$\Rightarrow X(z)=\frac{-1}{1-\frac{1}{2}z^{-1}}+\frac{2}{1-z^{-1}}$$

z ↕

$$x[n]=-\left(\frac{1}{2}\right)^n u[n]+2u[n]$$ #

(b)

$$X(z)=\frac{z^2}{(z-\frac{1}{2})(z-1)}=z\left(\frac{a}{z-\frac{1}{2}}+\frac{b}{z-1}\right)$$

$$a=\left.\frac{1}{z-1}\right|_{z=\frac{1}{2}}=-2,\quad b=\left.\frac{1}{z-\frac{1}{2}}\right|_{z=1}=2$$

$$\Rightarrow X(z)=2z\left(\frac{-z}{z-\frac{1}{2}}+\frac{z}{z-1}\right)$$

$$\frac{-z}{z-\frac{1}{2}}+\frac{z}{z-1}\overset{\mathcal{Z}}{\longleftrightarrow}-\left(\frac{1}{2}\right)^n u[n]+u[n]$$

$$\Rightarrow X(z)\overset{\mathcal{Z}}{\longleftrightarrow}-2\left(\frac{1}{2}\right)^{n+1}u[n+1]+2u[n+1]$$

$$=-\left(\frac{1}{2}\right)^n u[n+1]+2u[n+1]$$

And $x[-1]=0 \Rightarrow x[n]=-\left(\frac{1}{2}\right)^n u[n]+2u[n]$

$x[n]$ is identical to that obtained in part (a) #

10.29

10.29. The plots are as shown in Figure S10.29.

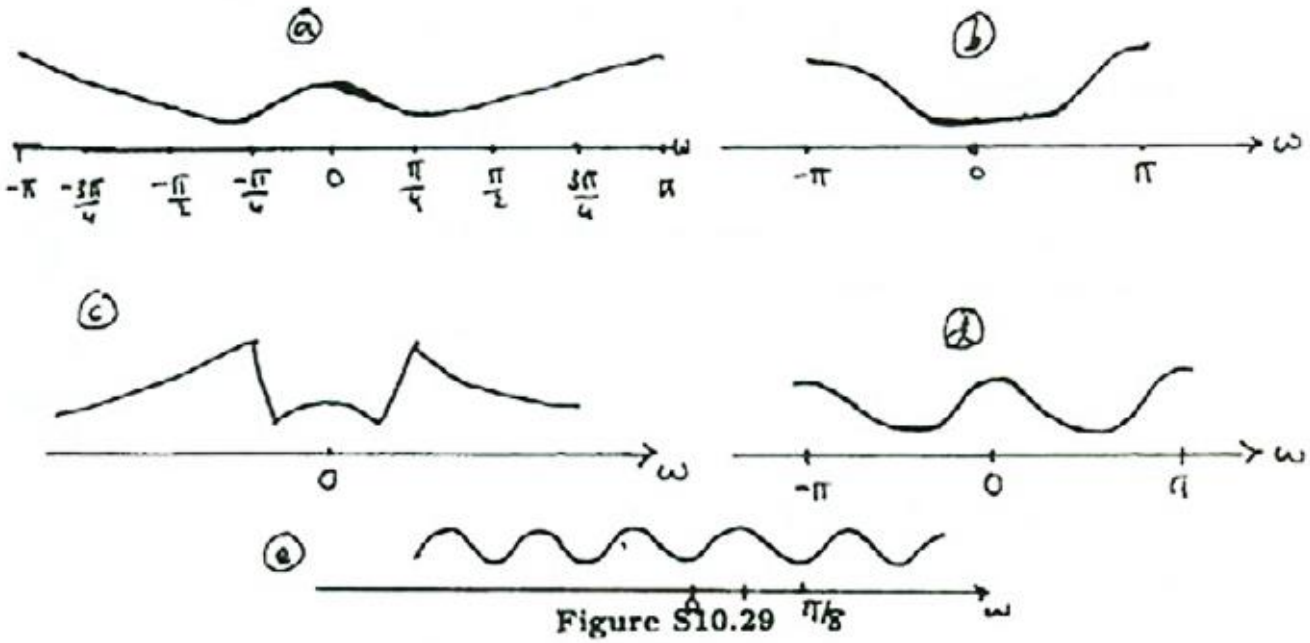

Figure S10.29