

EE 361002 Signal and System HW14 Answer

9.28

- (a) The possible ROCs are
- (i) $\mathcal{R}e\{s\} < -2$.
 - (ii) $-2 < \mathcal{R}e\{s\} < -1$.
 - (iii) $-1 < \mathcal{R}e\{s\} < 1$.
 - (iv) $\mathcal{R}e\{s\} > 1$.
- (b) (i) Unstable and anticausal.
(ii) Unstable and non causal.
(iii) Stable and non causal.
(iv) Unstable and causal.

9.31

- (a) Taking the Laplace transform of both sides of the given differential equation and simplifying, we obtain

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 - s - 2}$$

The pole-zero plot for $H(s)$ is as shown in Figure S9.31.

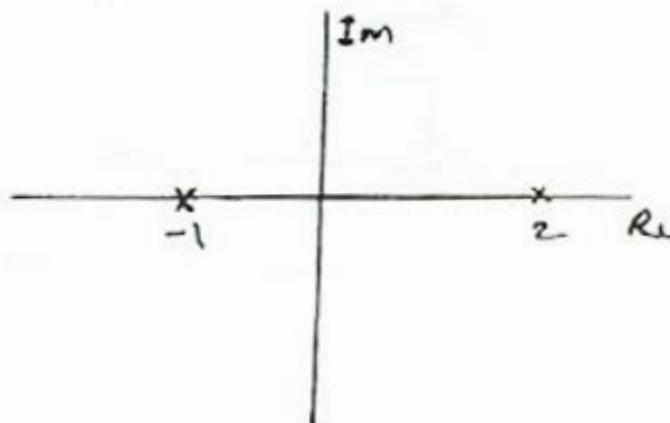


Figure S9.31

- (b) The partial fraction expansion of $H(s)$ is

$$H(s) = \frac{1/3}{s-2} - \frac{1/3}{s+1}$$

- (i) If the system is stable, the ROC for $H(s)$ has to be $-1 < \mathcal{R}e\{s\} < 2$. Therefore,

$$h(t) = -\frac{1}{3}e^{2t}u(-t) - \frac{1}{3}e^{-t}u(t).$$

- (ii) If the system is causal, the ROC for $H(s)$ has to be $\mathcal{R}e\{s\} > 2$. Therefore,

$$h(t) = \frac{1}{3}e^{2t}u(t) - \frac{1}{3}e^{-t}u(t).$$

- (iii) If the system is neither stable nor causal, the ROC for $H(s)$ has to be $\mathcal{R}e\{s\} < -1$. Therefore,

$$h(t) = -\frac{1}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(-t).$$

We know that

$$x_1(t) = u(t) \xrightarrow{\mathcal{L}} X_1(s) = \frac{1}{s}, \quad \operatorname{Re}\{s\} > 0.$$

Therefore, $X_1(s)$ has a pole at $s = 0$. Now, the Laplace transform of the output $y_1(t)$ of the system with $x_1(t)$ as the input is

$$Y_1(s) = H(s)X_1(s).$$

Since in clue 2, $Y_1(s)$ is given to be absolutely integrable, $H(s)$ must have a zero at $s = 0$ which cancels out the pole of $X_1(s)$ at $s = 0$.

We also know that

$$x_2(t) = tu(t) \xrightarrow{\mathcal{L}} X_2(s) = \frac{1}{s^2}, \quad \operatorname{Re}\{s\} > 0.$$

Therefore, $X_2(s)$ has two poles at $s = 0$. Now, the Laplace transform of the output $y_2(t)$ of the system with $x_2(t)$ as the input is

$$Y_2(s) = H(s)X_2(s).$$

Since in clue 3, $Y_2(s)$ is given to be *not* absolutely integrable, $H(s)$ does not have two zeros at $s = 0$. Therefore, we conclude that $H(s)$ has exactly one zero at $s = 0$.

From Clue 4 we know that the signal

$$p(t) = \frac{d^2h(t)}{dt^2} + 2\frac{dh(t)}{dt} + 2h(t)$$

is finite duration. Taking the Laplace transform of both sides of the above equation, we get

$$P(s) = s^2H(s) + 2sH(s) + 2H(s).$$

Therefore,

$$H(s) = \frac{P(s)}{s^2 + 2s + 2}.$$

Since $p(t)$ is of finite duration, we know that $P(s)$ will have no poles in the finite s -plane. Therefore, $H(s)$ is of the form

$$H(s) = \frac{A \prod_{i=1}^N (s - z_i)}{s^2 + 2s + 2},$$

where z_i , $i = 1, 2, \dots, N$ represent the zeros of $P(s)$. Here, A is some constant.

From Clue 5 we know that the denominator polynomial of $H(s)$ has to have a degree which is *exactly* one greater than the degree of the numerator polynomial. Therefore,

$$H(s) = \frac{A(s - s_1)}{s^2 + 2s + 2}.$$

Since we already know that $H(s)$ has a zero at $s = 0$, we may rewrite this as

$$H(s) = \frac{As}{s^2 + 2s + 2}.$$

From Clue 1 we know that $H(1)$ is 0.2. From this, we may easily show that $A = 1$. Therefore,

$$H(s) = \frac{s}{s^2 + 2s + 2}.$$

Since the poles of $H(s)$ are at $-1 \pm j$ and since $h(t)$ is causal and stable, the ROC of $H(s)$ is $\operatorname{Re}\{s\} > -1$.

9.36

(a) We know that $Y_1(s)$ and $Y(s)$ are related by

$$Y(s) = (2s^2 + 4s - 6)Y_1(s).$$

Taking the inverse Laplace transform, we get

$$y(t) = 2 \frac{d^2 y_1(t)}{dt^2} + 4 \frac{dy_1(t)}{dt} - 6y_1(t).$$

(b) Since $Y_1(s) = F(s)/s$, $f(t) = dy_1(t)/dt$.

(c) Since $F(s) = E(s)/s$, $e(t) = df(t)/dt = d^2 y_1(t)/dt^2$.

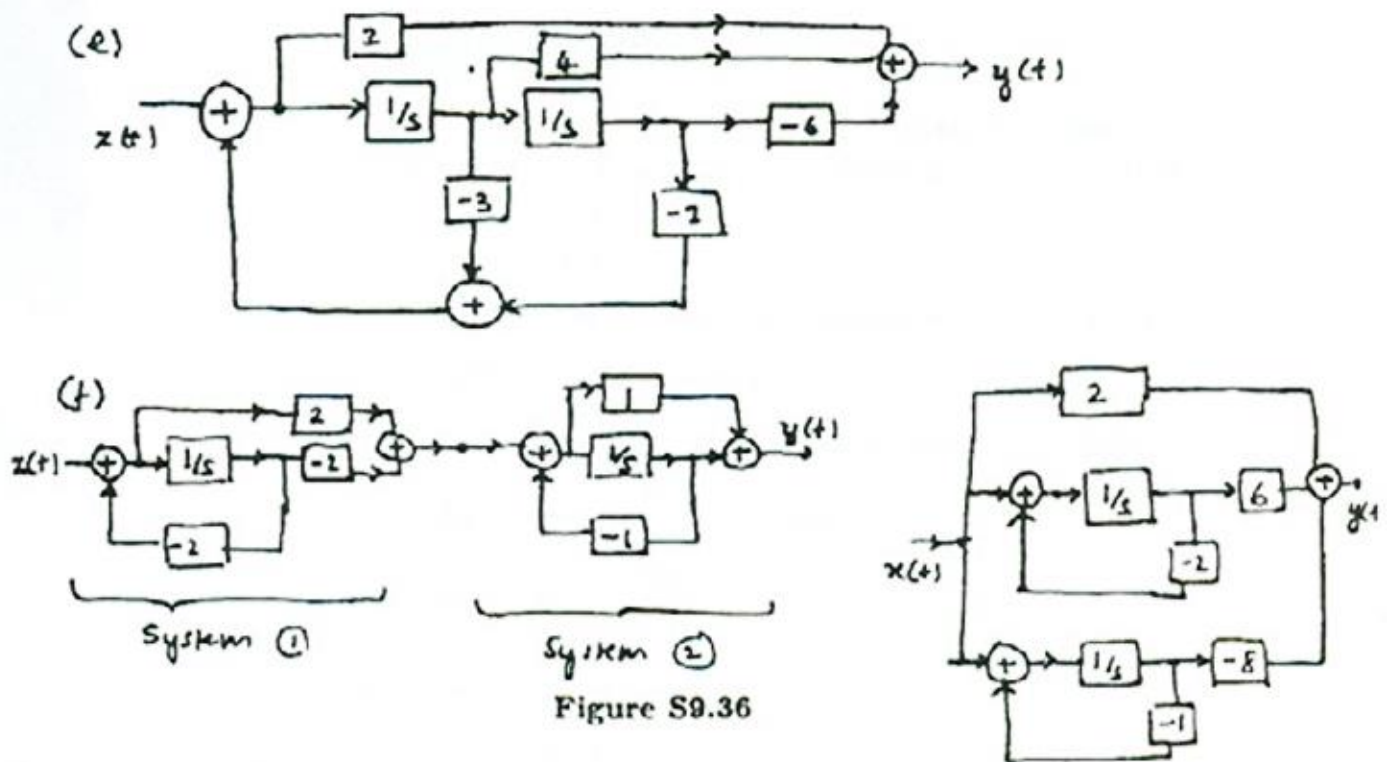
(d) From part (a), $y(t) = 2e(t) + 4f(t) - 6y_1(t)$.

(e) The extended block diagram is as shown in Figure S9.36.

(f) The block diagram is as shown in Figure S9.36.

(g) The block diagram is as shown in Figure S9.36.

The three subsystems may be connected in parallel as shown in the figure above to obtain the overall system



(a) Taking the Laplace transform of the signal $x(t)$, we get

$$Y(s) = \frac{2/3}{s-2} + \frac{1/3}{s+1} = \frac{s}{(s-2)(s-1)}.$$

The ROC is $-1 < \operatorname{Re}\{s\} < 2$. Also, note that since $x(t)$ is a left-sided signal, the ROC for $X(s)$ is $\operatorname{Re}\{s\} < 2$.

Now,

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s}{(s+2)(s+1)}.$$

We know that the ROC of $Y(s)$ has to be the intersection of the ROCs of $X(s)$ and $H(s)$. This leads us to conclude that the ROC of $H(s)$ is $\operatorname{Re}\{s\} > -1$.

(b) The partial fraction expansion of $H(s)$ is

$$H(s) = \frac{2}{s+2} - \frac{1}{s+1}.$$

Therefore,

$$h(t) = 2e^{-2t}u(t) - e^{-t}u(t).$$

(c) e^{3t} is an Eigen function of the LTI system. Therefore,

$$y(t) = H(3)e^{3t} = \frac{3}{20}e^{3t}.$$