EE 361002 Signal and System HW13 Answer

9.21

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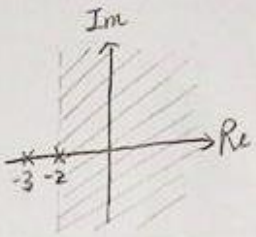
(a) $x(t)=e^{-2t}u(t)+e^{-3t}u(t)$

$e^{-2t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+2}$, $Re\{s\}>-2$

$e^{-3t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+3}$, $Re\{s\}>-3$

$\Rightarrow X(s)=\frac{1}{s+2}+\frac{1}{s+3}=\frac{2s+5}{s^2+5s+6}$



ROC: $Re\{s\}>-2$ #

(d) $x(t)=te^{-2|t|}=te^{-2t}u(t)+te^{2t}u(-t)$

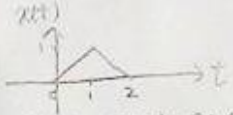
$e^{-2t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+2}$, $Re\{s\}>-2$

$e^{2t}u(-t) \xleftrightarrow{\mathcal{L}} \frac{-1}{s-2}$, $Re\{s\}<2$

$x_1(t)=e^{-2t}u(t)+e^{2t}u(-t) \xleftrightarrow{\mathcal{L}} X_1(s)=\frac{1}{s+2}+\frac{-1}{s-2}=\frac{-4}{s^2-4}$, ROC: $-2<Re\{s\}<2$

$x(t)=tx_1(t) \xleftrightarrow{\mathcal{L}} X(s)=-\frac{d}{ds}X_1(s)=\frac{-8s}{(s^2-4)^2}$, ROC: $-2<Re\{s\}<2$ #

(h) $x(t)=\begin{cases} t, & 0\le t\le 1 \\ 2-t, & 1\le t\le 2 \end{cases}$



Let $x_1(t)=t[u(t)-u(t-1)]$, then $x(t)=x_1(t)+x_1(-t+2)$

$u(t)-u(t-1) \xleftrightarrow{\mathcal{L}} \frac{1-e^{-s}}{s}$

$x_1(t) \xleftrightarrow{\mathcal{L}} -\frac{d}{ds}\left[\frac{1-e^{-s}}{s}\right]=\frac{-se^{-s}+1-e^{-s}}{s^2}$

$x_1(-t+2) \xleftrightarrow{\mathcal{L}} e^{-2s}\frac{se^{s}+1-e^{s}}{s^2}$

$\Rightarrow X(s)=\frac{-se^{-s}+1-e^{-s}}{s^2}+e^{-2s}\frac{se^{s}+1-e^{s}}{s^2}=\frac{1-2e^{-s}+e^{-2s}}{s^2}$

$x(t)$ is of finite duration and is absolutely integrable $\Rightarrow$ ROC: All s #

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(a) $X(s) = \frac{1}{s^2+9}$ $\quad \mathrm{Re}\{s\} > 0$

$= \frac{3}{s^2+3^2} \times \frac{1}{3}$

$\Leftrightarrow \frac{1}{3}\sin(3t)\,u(t)$

(d) $X(s) = \frac{s}{s^2+7s+12}$, $\quad -4 < \mathrm{Re}\{s\} < -3$

$= \frac{-1}{s+3} + \frac{2}{s+4}$

$\alpha = 3$, $\mathrm{Re}\{s\} < -3$ $\qquad$ $\alpha = 4$, $\mathrm{Re}\{s\} > -4$

$= e^{-3t}u(t) + 2e^{-4t}u(t)$

(f) $X(s) = \frac{(s+1)^2}{s^2-s+1}$, $\quad \mathrm{Re}\{s\} > \frac{1}{2}$

$= \frac{s^2-s+1 \;+3s}{s^2-s+1}$

$= 1 + \frac{3s}{s^2-s+\frac{1}{4}+\frac{3}{4}}$

$= 1 + 3\left[\frac{s-\frac{1}{2}}{(s-\frac{1}{2})^2+(\frac{\sqrt{3}}{4})^2} + \frac{\frac{1}{2}}{(s-\frac{1}{2})^2+(\frac{\sqrt{3}}{4})^2}\right]$

$= 1 + 3\times\frac{(s-\frac{1}{2})}{(s-\frac{1}{2})^2+(\frac{\sqrt{3}}{4})^2} + 3\times\frac{\frac{\sqrt{3}}{2}}{(s-\frac{1}{2})^2+(\frac{\sqrt{3}}{4})^2}\times\frac{1}{\sqrt{3}}$

$\alpha = -\frac{1}{2}$, $\mathrm{Re}\{s\} > -\alpha$.

$\Leftrightarrow \delta(t) + 3e^{\frac{1}{2}t}\cos(\frac{\sqrt{3}}{2}t)\,u(t) + \sqrt{3}e^{\frac{1}{2}t}\sin(\frac{\sqrt{3}}{2}t)\,u(t)$

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Possible ROCs

Plot (a): $\mathrm{Re}\{s\} < -2$ or $-2 < \mathrm{Re}\{s\} < 2$ or $\mathrm{Re}\{s\} > 2$

Plot (b): $\mathrm{Re}\{s\} < -2$ or $\mathrm{Re}\{s\} > -2$

Plot (c): $\mathrm{Re}\{s\} < 2$ or $\mathrm{Re}\{s\} > 2$

Plot (d): entire s-plane

1. $x(t)e^{-3t} \xleftrightarrow{\mathcal{L}} X(s+3)$

⇒ R_1 is R shifted by 3 to the left and includes the jω-axis.

(a) $\mathrm{Re}\{s\} > 2$

(b) $\mathrm{Re}\{s\} > -2$

(c) $\mathrm{Re}\{s\} > 2$

(d) entire s-plane

2. $e^{-t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+1}$ $\mathrm{Re}\{s\} > -1$

⇒ R_2 is the intersection of R and $\mathrm{Re}\{s\} > -1$ and includes the jω-axis.

(a) $-2 < \mathrm{Re}\{s\} < 2$

(b) $\mathrm{Re}\{s\} > -2$

(c) $\mathrm{Re}\{s\} < 2$

(d) entire s-plane

3. R_3 is left-sided.

(a) $\mathrm{Re}\{s\} < -2$

(b) $\mathrm{Re}\{s\} < -2$

(c) $\mathrm{Re}\{s\} < 2$

(d) entire s-plane

4. R_4 is right-sided.

(a) $\mathrm{Re}\{s\} > 2$

(b) $\mathrm{Re}\{s\} > -2$

(c) $\mathrm{Re}\{s\} > 2$

(d) entire s-plane

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$$x_1(t) = e^{-2t}u(t) \xleftrightarrow{\mathcal{L}} X_1(s) = \frac{1}{s+2}, \quad \mathrm{Re}\{s\} > -2$$

$$x_2(t) = e^{-3t}u(t) \xleftrightarrow{\mathcal{L}} X_2(s) = \frac{1}{s+3}, \quad \mathrm{Re}\{s\} > -3$$

$$\Rightarrow x_1(t-2) \xleftrightarrow{\mathcal{L}} e^{-2s}X_1(s) = \frac{e^{-2s}}{s+2}, \quad \mathrm{Re}\{s\} > -2$$

$$x_2(-t+3) \xleftrightarrow{\mathcal{L}} e^{-3s}X_2(-s) = \frac{e^{-3s}}{3-s}, \quad \mathrm{Re}\{s\} < 3$$

$$y(t) = x_1(t-2) \cdot x_2(-t+3) \xleftrightarrow{\mathcal{L}} Y(s) = \left[\frac{e^{-2s}}{s+2}\right] \cdot \left[\frac{e^{-3s}}{3-s}\right], \quad -2 < \mathrm{Re}\{s\} < 3$$