

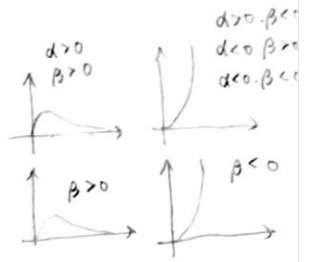
# EE 361002 Signal and System HW3 Answer

2.22 (a)  $x(t) = e^{-\alpha t} u(t)$

$h(t) = e^{-\beta t} u(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = \int_0^t e^{-\alpha \tau} e^{-\beta(t-\tau)} d\tau = \int_0^t e^{-\beta t} e^{(\beta-\alpha)\tau} d\tau \quad (t \geq 0)$$

$$= \begin{cases} e^{-\beta t} \frac{e^{(\beta-\alpha)t} - 1}{\beta - \alpha} u(t), & \text{if } \alpha \neq \beta \\ -\beta t e^{-\beta t} u(t), & \text{if } \alpha = \beta \end{cases}$$



(b)  $x(t) = u(t) - 2u(t-2) + u(t-5)$

$h(t) = e^{2t} u(1-t)$

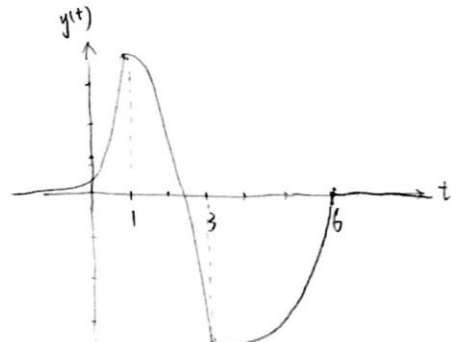
$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = \int_0^2 h(t-\tau) d\tau - \int_2^5 h(t-\tau) d\tau$$

$$= \begin{cases} \int_0^t e^{2(t-\tau)} d\tau, & t \leq 1 \\ \int_{t-1}^2 e^{2(t-\tau)} d\tau, & 1 \leq t \leq 3 \\ -\int_{t-1}^5 e^{2(t-\tau)} d\tau, & 3 \leq t \leq 6 \\ 0, & t \geq 6 \end{cases}$$

$$= \begin{cases} e^{2t} \frac{1 - e^{-2t}}{2}, & t \leq 1 \\ e^{2t} \frac{e^{-2(t-1)} - e^{-2(t-2)}}{2}, & 1 \leq t \leq 3 \\ -e^{2t} \frac{e^{-2(t-1)} - e^{-2(t-5)}}{2}, & 3 \leq t \leq 6 \\ 0, & t \geq 6 \end{cases}$$



$$= \begin{cases} \frac{1}{2} [e^{2t} - 2e^{2(t-2)} + e^{2(t-5)}], & t \leq 1 \\ \frac{1}{2} [e^{2t} - 2e^{2(t-2)} + e^{2(t-5)}], & 1 \leq t \leq 3 \\ \frac{1}{2} [e^{2(t-5)} - e^{2t}], & 3 \leq t \leq 6 \\ 0, & t \geq 6 \end{cases}$$



(d) Let  $h(t) = h_1(t) - \frac{1}{3} \delta(t-2)$

$h_1(t) = \begin{cases} \frac{4}{3}, & 0 \leq t \leq 1 \\ 0, & \text{o.w.} \end{cases}$

$y(t) = h(t) * x(t) = h_1(t) * x(t) - \frac{1}{3} x(t-2)$

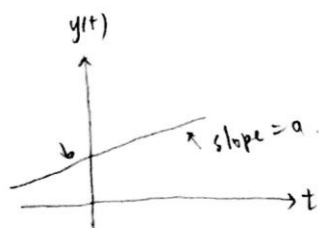
$h_1(t) * x(t) = \int_0^1 \frac{4}{3} [a(t-\tau) + b] d\tau = \frac{4}{3} [a\tau - \frac{a}{2}\tau^2 + b\tau]_0^1 = \frac{4}{3} [a - \frac{a}{2} + b]$

$y(t) = \frac{4}{3} [a - \frac{a}{2} + b] - \frac{1}{3} [a(t-2) + b]$

$= \frac{4}{3} at - \frac{2}{3} a + \frac{4}{3} b - \frac{1}{3} at + \frac{2}{3} a - \frac{1}{3} b$

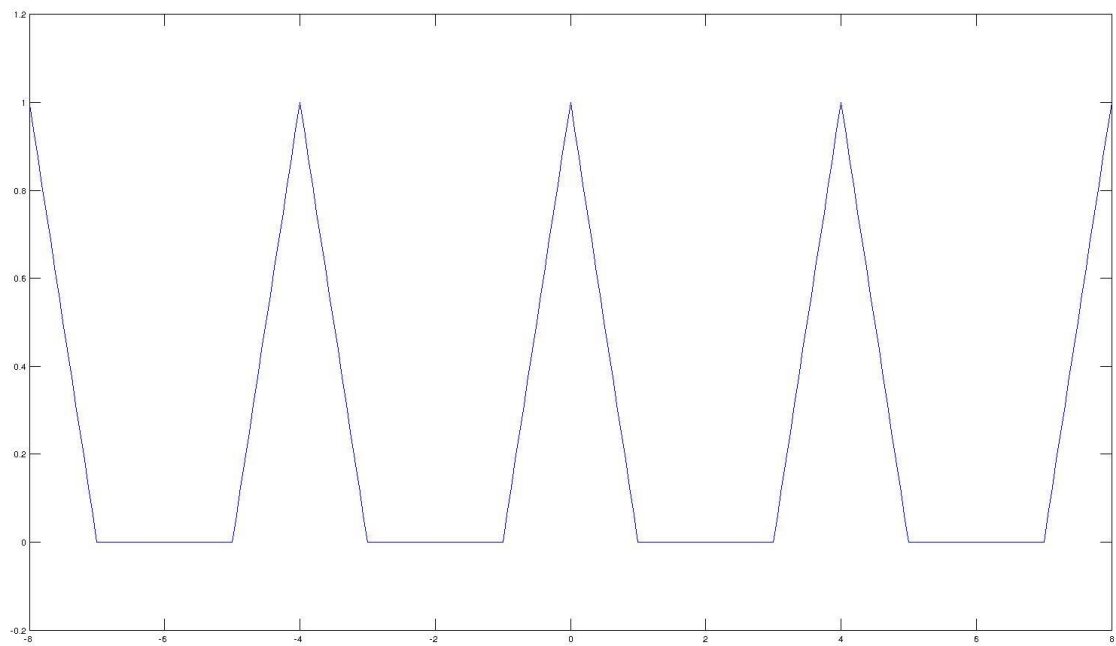
$= at + b$

$= x(t)$

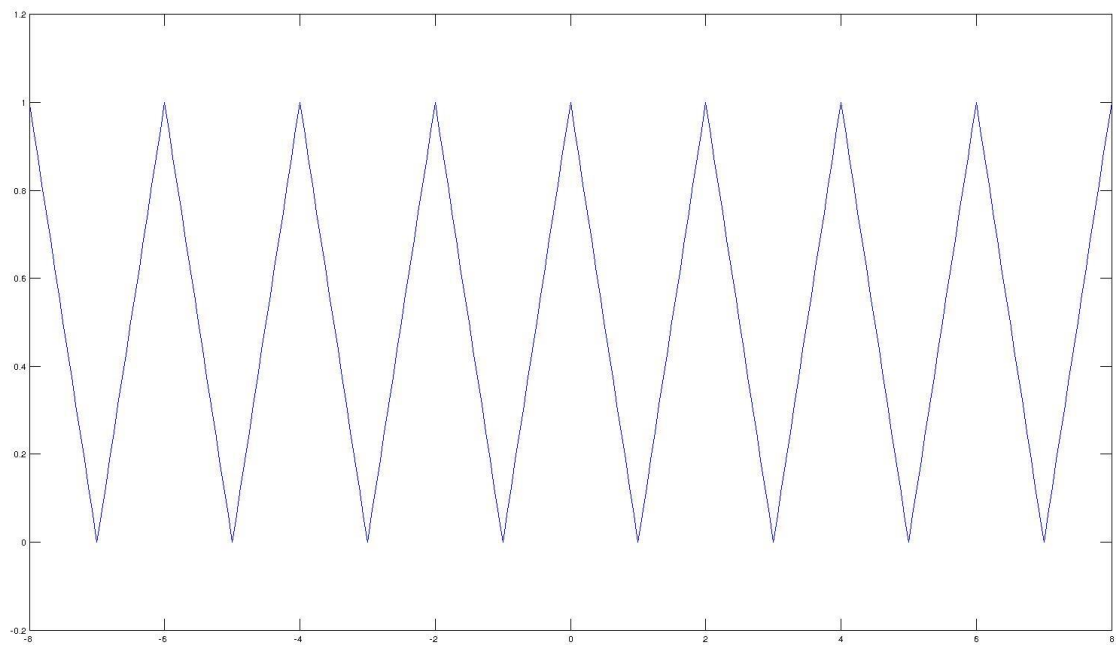


2.23

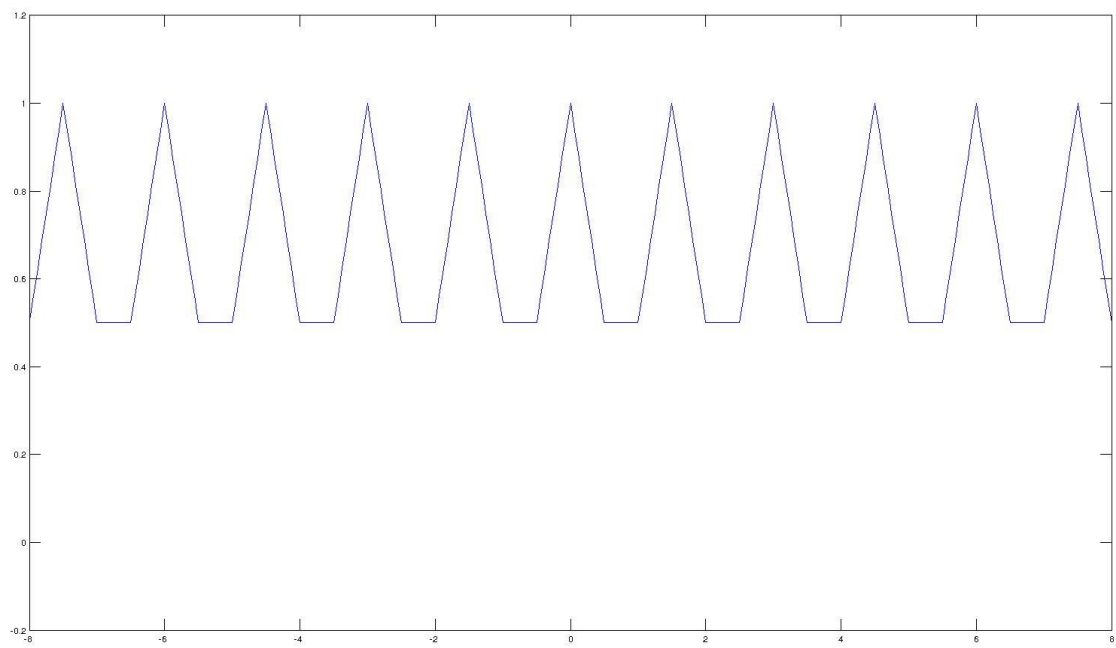
(a)  $T = 4$



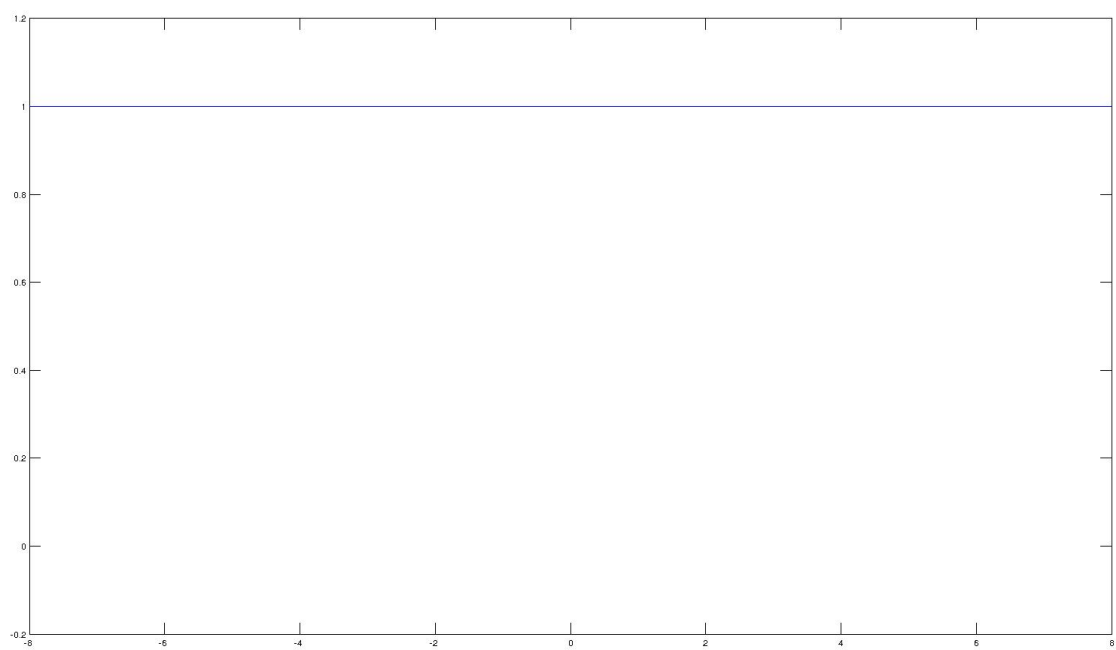
(b)  $T = 2$



(c)  $T = 3/2$



(d)  $T = 1$



2.24

(a)  $x[n] \rightarrow h_1 \rightarrow h_2 \rightarrow h_2 \rightarrow y[n]$

$$h[n] = h_1[n] * h_2[n] * h_2[n]$$

$$h_2[n] * h_2[n] = (u[n] - u[n-2]) * (u[n] - u[n-2])$$

$$= (\delta[n] + \delta[n-1]) * (\delta[n] + \delta[n-1])$$

$$= \delta[n] + 2\delta[n-1] + \delta[n-2]$$

$$\Rightarrow h[n] = h_1[n] * (\delta[n] + 2\delta[n-1] + \delta[n-2])$$

$$= h_1[n] + 2h_1[n-1] + h_1[n-2]$$

$$h[0] = h_1[0] + 2h_1[-1] + h_1[-2] = 1 \Rightarrow h_1[0] = 1$$

$$h[1] = h_1[1] + 2h_1[0] + h_1[-1] = 5 \Rightarrow h_1[1] = 5 - 2 = 3$$

$$h[2] = h_1[2] + 2h_1[1] + h_1[0] = 10 \Rightarrow h_1[2] = 10 - 1 - 6 = 3$$

$$h[3] = h_1[3] + 2h_1[2] + h_1[1] = 11 \Rightarrow h_1[3] = 11 - 3 - 6 = 2$$

$$h[4] = h_1[4] + 2h_1[3] + h_1[2] = 8 \Rightarrow h_1[4] = 8 - 3 - 4 = 1$$

$$h[5] = h_1[5] + 2h_1[4] + h_1[3] = 4 \Rightarrow h_1[5] = 4 - 2 - 2 = 0$$

$$h[6] = h_1[6] + 2h_1[5] + h_1[4] = 1 \Rightarrow h_1[6] = 1 - 1 - 0 = 0$$

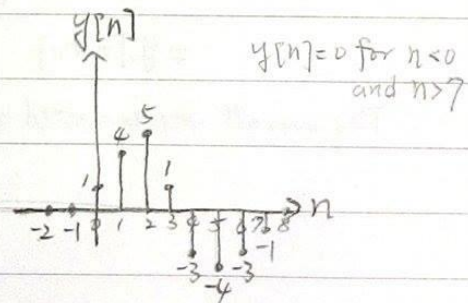
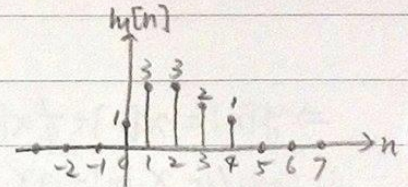
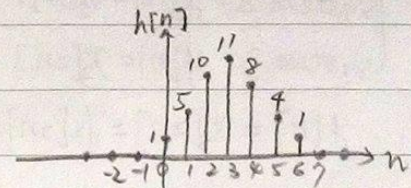
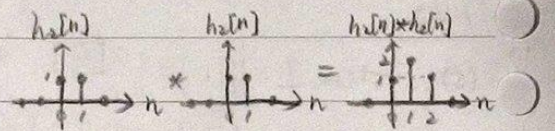
$$\Rightarrow h_1[n] = 0 \text{ for } n < 0 \text{ and } n > 4 \quad \#$$

(b)  $x[n] = \delta[n] - \delta[n-1]$

$$y[n] = x[n] * h[n]$$

$$= (\delta[n] - \delta[n-1]) * h[n]$$

$$= h[n] - h[n-1] \quad \#$$





2-32

$$(a) y_h[n] = A\left(\frac{1}{2}\right)^n$$

$$y_h[n] - \frac{1}{2}y_h[n-1]$$
$$= A\left(\frac{1}{2}\right)^n - \frac{1}{2}A\left(\frac{1}{2}\right)^{n-1}$$

$$= A\left(\frac{1}{2}\right)^n - A\left(\frac{1}{2}\right)^n$$

$$= 0, \text{ is true that } y_h[n] = A\left(\frac{1}{2}\right)^n$$

(b) for  $n \geq 0$

$$y_p[n] = B\left(\frac{1}{3}\right)^n$$

$$y_p[n] - \frac{1}{2}y_p[n-1] = \left(\frac{1}{3}\right)^n$$

$$\Rightarrow B\left(\frac{1}{3}\right)^n - \frac{1}{2}B\left(\frac{1}{3}\right)^{n-1} = \left(\frac{1}{3}\right)^n$$

$$\Rightarrow B - \frac{3}{2}B = 1$$

$$\Rightarrow B = -2$$

$$(c) y[n] = A\left(\frac{1}{2}\right)^n + B\left(\frac{1}{3}\right)^n \text{ for } n \geq 0$$

$$\Rightarrow y[0] = A + B$$

$$\text{and } y[0] - \frac{1}{2}y[-1] = x[0] = 1$$

$$\text{for } x[n] = \left(\frac{1}{3}\right)^n u[n]$$

$$\Rightarrow y[0] = A + B = 1$$

$$\Rightarrow A = 3$$