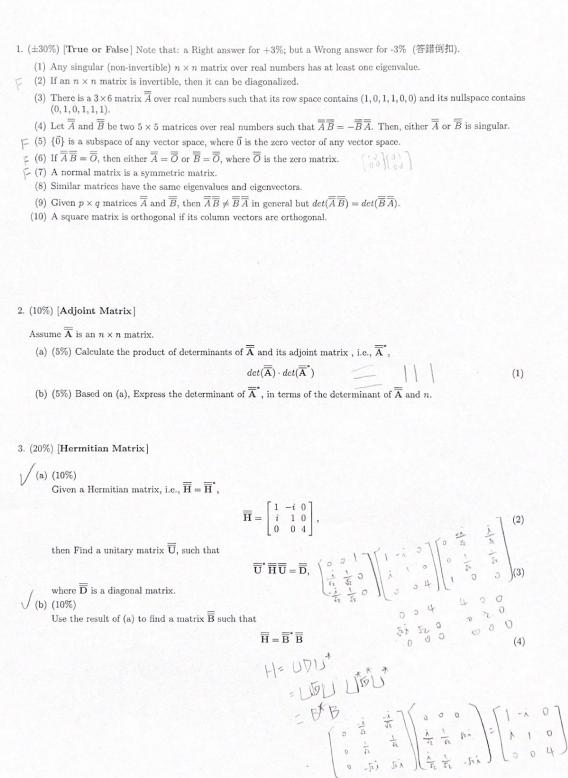
Linear Algebra, EE 10810EECS205004

Final Exam (Dated: Fall, 2020)

Total scores: 120%



4. (15%) [Spectral Theorem]

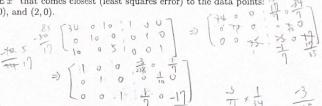
(4) (19%) Find an orthogonal matrix $\overline{\overline{P}}$ that diagonalizes

$$\overline{\overline{S}} = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}, \tag{5}$$

(b) 15%) Perform the spectral decomposition for the matrix \$\overline{\overline{S}}\$.

5/(15%) [Least Squares Approximation]

Find the parabola $y = C + Dx + Ex^2$ that comes closest (least squares error) to the data points: (x, y) = (-2, 0), (-1, 0), (0, 1), (1, 0), and (2, 0).



6. (15%) [SVD]

Consider the matrix:

$$\overline{\overline{\mathbf{A}}}_{1} = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix},\tag{6}$$

Find the corresponding Singular Value Decomposition, i.e.,

$$\overline{\overline{A}}_1 = \overline{\overline{U}} \, \overline{\overline{\Sigma}} \, \overline{\overline{V}}$$

7. (15%) [Jordan Canonical Form]

Given

$$\overline{\overline{A}}_2 = \begin{bmatrix} 0 & -1 & -1 \\ -3 & -1 & -2 \\ 7 & 5 & 6 \end{bmatrix}.$$

$$0 & -7 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & -3 & 0 & 0 & 0 \end{bmatrix}$$

$$\overline{\overline{J}} \text{ with Jordan form, i.e.,}$$

then, Express $\overline{\overline{A}}_2$ being similar to the matrix $\overline{\overline{J}}$ with Jordan form, i.e., $\overline{\overline{J}} = \overline{\overline{M}}^{-1} \, \overline{\overline{A}}_2 \, \overline{\overline{M}}$

$$\overline{\overline{J}} = \overline{\overline{\overline{M}}}^{-1} \, \overline{\overline{\overline{A}}}_2 \, \overline{\overline{\overline{M}}}$$

(8)