1. a) $d=4, c=0$

b) $C(A)$ basis: $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}-4\\2\\4\end{bmatrix}$

c) $C(A^T)$ basis: $[1\,4\,3\,0\,1], [0\,2\,0\,5\,3]$

2. a) $b_3+b_2-3b_1=0$

b) $N(A)=\text{span}\left\{\begin{pmatrix}-2\\-1\\1\\0\end{pmatrix}, \begin{pmatrix}-3\\2\\0\\1\end{pmatrix}\right\}$

c) $N(A^T)=\text{span}\left\{\begin{pmatrix}-3\\1\\1\end{pmatrix}\right\}$

3. (a),(b),(c),(d)皆不是subspace

4. a) $c=8$

b) $C(A^{-1})=R^3$

$N(A^{-1})=\left\{\begin{pmatrix}0\\0\\0\end{pmatrix}\right\}$

c) $L_1=\begin{bmatrix}1&0&0\\1&1&0\\2&\frac{2}{3}&1\end{bmatrix}$

$U_1=\begin{bmatrix}1&2&3\\0&3&3\\0&0&12\end{bmatrix}$

$L_2=\begin{bmatrix}1&0&0\\1&1&0\\2&\frac{2}{3}&1\end{bmatrix}$

$D_2=\begin{bmatrix}1&0&0\\0&3&0\\0&0&12\end{bmatrix}$

$U_2=\begin{bmatrix}1&2&3\\0&1&1\\0&0&1\end{bmatrix}$

5. a) $\underline{x}=x_2\begin{pmatrix}-2\\1\\0\end{pmatrix}, x_2\in R$

b)
$\underline{x}=x_2\begin{pmatrix}-2\\1\\0\end{pmatrix}+\begin{pmatrix}9\\0\\-2\end{pmatrix}$

complete solution: $\underline{x}$

particular solution: $\begin{pmatrix}9\\0\\-2\end{pmatrix}$

6. Yes,
Matrices like $A=\begin{pmatrix}1&0&-1\\0&1&-1\\-1&-1&2\end{pmatrix}$ or $\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}$ can make

vector $[1\,1\,1]^T$ become $N(A)$ and $N(A^T)$

7. (a)

rank of $A = 2$

(b)

$\dim C(A) = 2$

$\dim C(A^T) = 2$

$\dim N(A) = 3$

$\dim N(A^T) = 1$

(c)

$C(A)$ basis : $(1,1,-1)^T$, $(1,2,0)^T$

$C(A^T)$ basis : $(1,2,1,0,0)^T$, $(1,2,2,2,3)^T$

$N(A)$ basis : $(-2,1,0,0,0)^T$, $(2,0,-2,1,0)^T$, $(3,0,-3,0,1)^T$

$N(A^T)$ basis : $(2,-1,1)^T$

8. (a)

True, by row exchange can prove it.

(b)

False,

counterexample : $S = \left\{ \begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\\0\end{bmatrix} \right\}$, but vectors in S are indep.

(c)

True,

if there is a zero vector in S, e.g. $x_1 \begin{bmatrix}0\\0\end{bmatrix} + x_2 \underline{v_2} + \cdots + x_n \underline{v_n} = \underline{0}$

$\Rightarrow S$ is linearly dependent

(d)

False,

counterexample : $A = \begin{bmatrix}1 & 2\\0 & 0\end{bmatrix}$, $C(A^T) = \text{span}\left\{\begin{bmatrix}1\\2\end{bmatrix}\right\} \neq C(A) = \text{span}\left\{\begin{bmatrix}1\\0\end{bmatrix}\right\}$

(e)

False,

cunterexample : $A = \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$ $\quad C(A) = \text{span}\left\{\begin{bmatrix}1\\1\end{bmatrix}\right\}$

$R = \begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}$ $\quad C(R) = \text{span}\left\{\begin{bmatrix}1\\0\end{bmatrix}\right\}$

$\Rightarrow C(A) \neq C(R)$