EE2030Linear Algebra

Homework#5

Due: 05/05/2023 10:10(Fri)

1. Use row operations to simplify and compute these determinants:

$$\det\begin{bmatrix}101 & 201 & 301\\102 & 202 & 302\\103 & 203 & 303\end{bmatrix} \qquad \text{and} \qquad \det\begin{bmatrix}1 & t & t^2\\t & 1 & t\\t^2 & t & 1\end{bmatrix}.$$

2. Elimination reduces A to U. Then $A = LU$.

$$A = \begin{bmatrix}3 & 3 & 4\\6 & 8 & 7\\-3 & 5 & -9\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\2 & 1 & 0\\-1 & 4 & 1\end{bmatrix}\begin{bmatrix}3 & 3 & 4\\0 & 2 & -1\\0 & 0 & -1\end{bmatrix} = LU.$$

   Find the determinants of L, U, A, U^{-1}, L^{-1}, and $U^{-1}L^{-1}A$.

3. True or False(give a reason if true or a 2 by 2 example if false):
   (a)If A is not invertible then AB is not invertible.
   (b)The determinant of A is always the product of its pivots.
   (c)The determinant of A-B equals detA - detB
   (d)AB and BA have the same determinant.

4. The n by n determinant C_n has 1's above and below the main diagonal:

$$C_1 = \begin{vmatrix}0\end{vmatrix}, C_2 = \begin{vmatrix}0 & 1\\1 & 0\end{vmatrix}, C_3 = \begin{vmatrix}0 & 1 & 0\\1 & 0 & 1\\0 & 1 & 0\end{vmatrix}, C_4 = \begin{vmatrix}0 & 1 & 0 & 0\\1 & 0 & 1 & 0\\0 & 1 & 0 & 1\\0 & 0 & 1 & 0\end{vmatrix}.$$

   (a) What are these determinants C_1, C_2, C_3, C_4?
   (b) By cofactors find the relation between C_n and C_{n-1} and C_{n-2}. Find C_{10}.

5. Find G_2 and G_3 and then by row operations G_4.Can you predicet G_n?

$$G_2 = \begin{vmatrix}0 & 1\\1 & 0\end{vmatrix} \qquad G_3 = \begin{vmatrix}0 & 1 & 1\\1 & 0 & 1\\1 & 1 & 0\end{vmatrix} \qquad G_4 = \begin{vmatrix}0 & 1 & 1 & 1\\1 & 0 & 1 & 1\\1 & 1 & 0 & 1\\1 & 1 & 1 & 0\end{vmatrix}$$

6. Find the determinant of this cyclic P by cofactors of row 1 and then the "big formula". How many exchanges reorder 4, 1, 2, 3 into 1, 2, 3, 4? Is $|P^2| = 1$ or -1?

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad P^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}.$$

7. Find the cofactors of A and multiply AC^T to find $\det A$:

$$A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix} \qquad \text{and} \qquad C = \begin{bmatrix} 6 & -3 & 0 \\ . & . & . \\ . & . & . \end{bmatrix} \qquad \text{and} \quad AC^T = \_\_\_\_.$$

If you change that 4 to 100, why is $\det A$ unchanged?

8. The parellelogram with sides (2, 1) and (2, 3) has the same area as the parallelogram with sides (2, 2) and (1, 3). Find those areas from 2 by 2 determinants and say why they must be equal. (I can't see why from a picture. Please write to me if you do.)

9. The box with edges i and j and w=2i+3i+4k has height ____.What is the volume?What is the matrix with determinant?What is $i \times j$ and what is its dot product with w?