EE2030Linear Algebra

Homework#4

Due: 04/26/2023 10:10(Wed)

1. The floor $\mathbf{V}$ and the wall $\mathbf{W}$ are not orthogonal subspaces,because they share a nonzero vector (along the line where they meet).No planes $\mathbf{V}$ and $\mathbf{W}$ in $\mathbf{R}^3$ can be orthogonal!Find a vector in the column spaces of both matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & 4 \\ 6 & 3 \\ 5 & 1 \end{bmatrix}$$

   This will be a vector $A\mathbf{x}$ and also $B\,\widehat{\mathbf{x}}$. Think 3 by 4 with the matrix [A B].

2. If $\mathbf{S}$ is the subspace of $\mathbf{R}^3$ containing only the zero vector, what is $\mathbf{S}^\perp$? If $\mathbf{S}$ is spanned by (1,1,1), what is $\mathbf{S}^\perp$? If $\mathbf{S}$ is spanned by (1,1,1) and (1,1,-1), what is a basis for $\mathbf{S}^\perp$?

3. Why is each of these statements false:
   (a) (1,1,1) is perpendicular to (1,1,-2) so the planes $x+y+z=0$ and $x+y-2z=0$ are orthogonal spaces.
   (b) The subspace spanned by (1,1,0,0,0) and (0,0,1,1,1) is the orthogonal complement of the subspace spanned by (1,-1,0,0,0) and (2,-2,3,4,-4).
   (c) Two subspaces that meet only in the zero vector are orthogonal.

4. (Quick and Recommend) Suppose A is the 4 by 4 identity matrix with its last column removed. A is 4 by 3. Project $\mathbf{b}$ = (1,2,3,4) onto the column space of A.What shape is the projection matrix P and what is P?

5. If $P^2 = P$ show that $(I-P)^2 = I-P$. When P projects onto the column space of A,$I-P$ projects onto the ____ .

6. Use $P^T = P$ and $P^2 = P$ to prove that the length squared of column 2 always equals the diagonal entry P_{22}.This number is $\frac{2}{6} = \frac{4}{36}+\frac{4}{36}+\frac{4}{36}$ for

$$P = \frac{1}{6}\begin{bmatrix} 5 & 2 & -1 \\ 2 & 2 & 2 \\ -1 & 2 & 5 \end{bmatrix}$$

7. The average of the four times is $\widehat{t} = \frac{1}{4}(0+1+3+4) = 2$. The average of the four b's is $\widehat{b} = \frac{1}{4}(0+8+8+20) = 9$.
   (a) Verify that the best line goes through the center point $(\widehat{t},\widehat{b})$=(2,9).
   (b) Explain why $C+D\widehat{t} = \widehat{b}$ comes from the first equation in $A^TA\widehat{\mathbf{x}} = A^T\mathbf{b}$.

8. This problem projects $\mathbf{b} = (b_1, \ldots, b_m)$ onto the line through $\mathbf{a}$ = (1,...,1). We solve m equations $\mathbf{a}x$=$\mathbf{b}$ in 1 unknown (by least squares).
(a) Solve $a^T a\widehat{x} = a^T\mathbf{b}$ to show that $\widehat{x}$ is the mean (the average) of the b's.
(b) Find $\mathbf{e} = \mathbf{b}$ - $\mathbf{a}\widehat{x}$ and the variance $\|\mathbf{e}\|^2$ and the standard deviation $\|\mathbf{e}\|$.
(c) The horizontal line $\widehat{\mathbf{b}} = 3$ is closest to $\mathbf{b}$ = (1,2,6). Check that $\mathbf{p}$ = (3,3,3) is perpendicular to $\mathbf{e}$ and find the 3 by 3 projection matrix P.

9. Find the best line $C+Dt$ to fit b = 4,2,-1,0,0 at times t = -2,-1,0,1,2.

10. Find orthogonal vectors **A**,**B**,**C** by Gram-Schmidt from **a**, **b**,**c**:

$$\mathbf{a} = (1, -1, 0, 0)\ \mathbf{b} = (0, 1, -1, 0)\ \mathbf{c} = (0, 0, 1, -1)$$

A,**B**,**C** and **a**,**b**,**c** are bases for the vectors perpendicular to $\mathbf{d}$ = (1,1,1,1).

11. (a) Find a basis for the subspaces $\mathbf{S}$ in $\mathbf{R}^4$ spanned by all solutions of

$$x_1 + x_2 + x_3 - x_4 = 0$$

(b) Find a basis for the orthogonal complement $\mathbf{S}^\perp$.
(c) Find $\mathbf{b}_1$ in $\mathbf{S}$ and $\mathbf{b}_2$ in $\mathbf{S}^\perp$ so that $\mathbf{b}_1 + \mathbf{b}_2 = \mathbf{b} = (1, 1, 1, 1)$

12. (a) Choose c so that Q is an orthogonal matrix:

$$Q = c\begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$$

Projects $\mathbf{b}$ = (1,1,1,1) onto the first column. Then project $\mathbf{b}$ onto the plane of the first two columns.