EE2030Linear Algebra

Homework#2

Due: 03/22/2023 10:10(Wed)

1. Which of the following subsets of $\mathbf{R}^3$ are actually subspaces?
   (a) The plane of vectors (b_1,b_2,b_3) with $b_1 = b_2$.
   (b) The plane of vectors with $b_2 = 1$.
   (c) The vectors with $b_1b_2b_3=0$.
   (d) All linear combination of $\boldsymbol{\nu} = (1,4,0)$ and $\boldsymbol{\omega} = (2,2,2)$.
   (e) All vectors that satisfy $b_1 + b_2 + b_3 = 0$.
   (f) All vectors with $b_1 \leq b_2 \leq b_3$.

2. If we add an extra column $\mathbf{b}$ to a matrix A, then the column space gets larger unless _____. Give an example where the column space gets larger and an example where it doesn't. Why is $A\boldsymbol{x}=\boldsymbol{b}$ solvable exactly when the column space doesn't get larger — it is the same for A and $[A\ \mathbf{b}]$?

3. Construct a 3 by 3 matrix whose column space contains (1,1,0) and (1,0,1) but not (1,1,1). Construct a 3 by 3 matrix whose column space is only a line.

4. The equation $x - 3y - z = 0$ determines plane in $\mathbf{R}^3$.What is the matrix A in this equation? Which are the free variables? The special solutions are (3,1,0) and _____.

5. The plane $x - 3y - z = 12$ is parallel to the plane $x - 3y - z = 0$ in former problem. One particular point on this plane is (12,0,0). All points on the plane have the form (fill in the first components)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} \\ 0 \\ 1 \end{bmatrix}$$

6. Suppose column 1 +column 3 +column 5 = $\mathbf{0}$ in a 4 by 5 matrix with four pivots. Which column is sure to have no pivot (and which variable is free)?What is the special solution? What is the nullspace?

7. Construct a matrix whose column space contains (1,1,0) and (0,1,1) and whose nullspace contains (1,0,1) and(0,0,1).

8. Fill out these matrices so that they have rank 1:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & & \\ 4 & & \end{bmatrix} and B = \begin{bmatrix} & 9 & \\ 1 & & \\ 2 & 6 & -3 \end{bmatrix} and M = \begin{bmatrix} a & b \\ c & \end{bmatrix}$$

9. Choose vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ so that $A = \boldsymbol{u}\boldsymbol{v}^T$ = column times row:

$$A = \begin{bmatrix} 3 & 6 & 6 \\ 1 & 2 & 2 \\ 4 & 8 & 8 \end{bmatrix} andA = \begin{bmatrix} 2 & 2 & 6 & 4 \\ -1 & -1 & -3 & -2 \end{bmatrix}$$

$A = \boldsymbol{u}\boldsymbol{v}^T$ is the natural form for every matrix that has rank r=1.

10. What is the nullspace matrix N (containing the special solutions) for A,B,C ?

$$A = \begin{bmatrix} I & I \end{bmatrix} andB = \begin{bmatrix} I & I \\ 0 & 0 \end{bmatrix} andC = \begin{bmatrix} I & I & I \end{bmatrix}$$