EE2030Linear Algebra

Homework#1

Due: 03/10/2023 10:10(Fri)

1. For which three number a will elimination fail to give three pivots?

$$A = \begin{bmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{bmatrix} \text{ is singular for three values of a.}$$

2. Suppose elimination takes A to U without row exchanges.Then row j of U is a combination of which rows of A?If $A\mathbf{x} = 0$,is $U\mathbf{x} = 0$?If $A\mathbf{x} =$ b,is $U\mathbf{x} =$ b?If A starts out lower triangular,what is the upper triangular U?

3. Choose the numbers a,b,c,d in this augmented matrix so that there is (a) no solution (b) infinitely many solutions.

$$\begin{bmatrix} A & b \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & a \\ 0 & 4 & 5 & b \\ 0 & 0 & d & c \end{bmatrix}$$

   Which of the numbers a,b,c or d have no effect on the solvability?

4. Find elimination matrices E_{21} then E_{32} then E_{43} to change K into U:

$$E_{43}E_{32}E_{21} \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & 0 & 5/4 \end{bmatrix}$$

   Apply those three steps to the identity matrix I, to multiply $E_{43}E_{32}E_{21}$.

5. True or false:
   (a) If A^2 is defined then A is necessarily square.
   (b) If AB and BA are defined then A and B are square.
   (c) If AB and BA are defined then AB and BA are square.
   (d) If $AB = B$ then $A = I$.

6. Which matrices E_{21} and E_{31} produce zeros in the (2,1) and (3,1) positions of $E_{21}A$ and $E_{31}A$?

$$A = \begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 1 \\ 8 & 5 & 3 \end{bmatrix}$$

   Find the single matrix E=$E_{31}E_{21}$ that produces both zeros at once. Multiply EA.

7. For which three numbers c is this matrix not invertible, and why not?

$$A = \begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}$$

8. A is a 4 by 4 matrix with 1's on the diagonal and -a, -b, -c on the diagonal above. Find A^{-1} for this bidiagonal matrix.

9. Compute L and U for the symmetric matrix A :

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

   Find the four conditions on a,b,c,d to get $A{=}LU$ with four pivots.

10. If $A{=}LDU$ and also $A{=}L_1D_1U_1$ with all factors invertible, then $L{=}L_1$ and $D{=}D_1$ and $U{=}U_1$. "***The three factors are unique.*** "

    Derive the equation $L_1^{-1}LD = D_1U_1U^{-1}$. Are the two sides triangular or diagonal? Deduce $L{=}L_1$ and $U{=}U_1$ (they all have diagonal 1's). Then $D{=}D_1$.

11. Which permutation makes PA upper triangular? Which permutations make P_1AP_2 lower triangular? ***Multiplying*** A ***on the*** *right* ***by*** P_2 ***exchanges the*** ____ ***of*** A.

$$A = \begin{bmatrix} 0 & 0 & 6 \\ 1 & 2 & 3 \\ 0 & 4 & 5 \end{bmatrix}$$

12. Factor the following matrix into $PA{=}LU$. Factor it also into $A{=}L_1P_1U_1$ (hold the exchange of row 3 until 3 times row 1 is subtracted from row 2):

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 3 & 8 \\ 2 & 1 & 1 \end{bmatrix}$$