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1.

$= 101 \det \begin{bmatrix} 202 & 302 \\ 203 & 303 \end{bmatrix} - 201 \begin{bmatrix} 102 & 302 \\ 103 & 303 \end{bmatrix} + 301 \begin{bmatrix} 102 & 202 \\ 103 & 203 \end{bmatrix}$

$= 101 \times (-100) - 201 \times (-200) + 301 \times (-100)$

$= -80400$

2.

$\det L = 1 \times 1 \times 1 = 1$

$\det U = 3 \times 2 \times (-1) = -6$

$\det A = \det U = -6$ (Because there isn't row exchanging)

$\det U^{-1} = 1/\det U = -\frac{1}{6}$

$\det L^{-1} = 1/\det L = 1$

$U^{-1}L^{-1}A = U^{-1}L^{-1}LU = I$, $\det U^{-1}L^{-1}A = \det I = 1$

3.(a)

True. If A isn't invertible, $\det A = 0$, $\det AB = \det A \det B = 0$, so AB isn't invertible

3.(b)

False. If A need odd times of row exchange, $\det A = (-1) \times$ the product of pivots

e.g. $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\det A \neq 1 \times 1$

3.(c)

False. e.g. $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $\det(A-B) = -1 \neq 0 = \det A - \det B$

3.(d)

False. e.g. $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\det(AB) = \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} \neq \det(BA) = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$

4.(a)

$\det C_1 = 0$

$\det C_2 = (-1) \times |1| = -1$

$\det C_3 = 0$

$\det C_4 = (-1)^2 \times 1 \times 1 \times 1 \times 1 = 1$

4.(b)

$C_4 = (-1) \times 1 \times \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$

$= (-1) \times 1 \times 1 \times \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$

$= -C_2$

$C_n = -C_{n-2}$

$C_{10} = -C_8 = C_6 = -C_4 = -1$

5.

$G_2 = -1$

$G_3 = -1\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + 1 \times \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1 + 1 = 2$

$G_4 = (-1) \times 1 \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} + 1 \times \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} + (-1) \times 1 \times \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix}$

$= -\left(\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}\right) + \left(\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}\right) - \left(\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}\right)$

$= -1 - 1 - 1$

$= -3$

$G_3 = -\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$

$G_2 + G_3 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -1 + 2 = 1$

$G_3 + G_4 = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -(G_2 + G_3)$

$G_4 + G_5 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix} = -\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -(G_3 + G_4)$

$G_{k-1} + G_k = -(G_{k-2} + G_{k-1})$, $G_k = -G_{k-2} - 2G_{k-1}$

$\because$ When $k = 2 \sim 3$, $G_k = (-1)^{k-1}(k-1)$, $G_{k+2} = (-1)^{k-1}[-(k-1) - 2 \times (-1) \times k] = (-1)^{k+1}(k+1)$

$\therefore \forall k, G_k = (-1)^{k+1}(k+1)$

6.

$P = -\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = -\det I = -1$

If the ith number isn't i, exchange it with i, so it needs 3 exchanges to reorder 4, 1, 2, 3 into 1, 2, 3, 4

$|P^2| = |P|^2 = (-1)^2 = 1$

7.

The cofactors of A is $\begin{vmatrix} 2 & 2 \\ 2 & 5 \end{vmatrix}$, $\begin{vmatrix} 1 & 2 \\ 1 & 5 \end{vmatrix}$, $\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix}$, $\begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix}$, $\begin{vmatrix} 1 & 4 \\ 1 & 5 \end{vmatrix}$, $\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$, $\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix}$, $\begin{vmatrix} 1 & 4 \\ 1 & 2 \end{vmatrix}$,

Because $|A| = a_{11}C_{11} - a_{12}C_{12} + a_{13}C_{13}$ and $C_{13} = \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0$, so the value of a_{13} doesn't affect $\det A$

8.

The area of the first parellelogram is $\left|\begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix}\right| = |6-2| = 4$

The area of the second one is $\left|\begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix}\right| = |6-2| = 4$

So they are the same

9.

$\text{Height} = \frac{\text{volume}}{\text{area}} = \frac{|i\ j\ w|}{|i \times j|} = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 4 \end{vmatrix}}{1} = 4$

The volume is 4 from $\begin{vmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 4 \end{vmatrix}$

$i \times j = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = k\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = k$

$k \cdot w = k^T w = [0\ 0\ 1]\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = 4$

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