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1.

If a subset of a vector space is a vector space, it is a subspace

(a) True

(b) False. The plane doesn't have a zero vector

(c) False. The sum of two vectors doesn't necessarily locate on the subset

(d) True

(e) True

(f) True

2.

If b is in C(A), it doesn't get larger

For example, if $A=\begin{bmatrix}1&2\\3&4\end{bmatrix}$ and $b=\begin{bmatrix}1\\4\end{bmatrix}$, the column space gets larger from

$\left\{\begin{bmatrix}i\\j\end{bmatrix} \middle| i=x+2y, j=3x+4y, x \text{ and } y \in R\right\}$ to $\left\{\begin{bmatrix}i\\j\end{bmatrix} \middle| i=x+2y+z, j=3x+4y+4z, x,y,z\in R\right\}$

If $A=\begin{bmatrix}1&2\\3&4\end{bmatrix}$ and $b=\begin{bmatrix}1\\2\end{bmatrix}$ in C(A), the column space doesn't get larger, for

$\left\{\begin{bmatrix}i\\j\end{bmatrix} \middle| i=x+2y, j=3x+4y, x \text{ and } y \in R\right\}$ equals to $\left\{\begin{bmatrix}i\\j\end{bmatrix} \middle| i=X+2Y+Z, j=3X+4Y+2Z, X,Y,Z\in R\right\}$

if we let $x=X$ and $y=Y+\frac{1}{2}Z$

So we have know that if the column space doesn't get larger, $b\in C(A)$

and $Ax=b$ means $b=\sum x_i A_i$ for $x_i \in X$ and A_i is the column of A, which means $b\in C(A)$

So $Ax=b$ is solvable exactly because there must be $b\in C(A)$ if the column space doesn't get larger

And Because the column space of $[A\ b]$ is the same as C(A), the result is the same, too.

3.

The column space of $\begin{bmatrix}1&1&1\\1&0&1\\0&1&0\end{bmatrix}$ contains (1, 1, 0) and (1, 0, 1) but not (1, 1, 1)

The column space of $\begin{bmatrix}1&1&1\\0&0&0\\0&0&0\end{bmatrix}$ is only a line

4.

By "$x-3y-z=0$" and "determines plane in R^3", we know $A=\begin{bmatrix}1&-3&-1\\0&0&0\\0&0&0\end{bmatrix}$

The free variables are y and z

To get the special solution, set (y, z) = (1, 0) or (0, 1), and the special solutions are (3, 1, 0) and (1, 0, 1)

5.

$\because x-3y-z=12 \quad \therefore x=12+3y+z \qquad \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}12\\0\\0\end{bmatrix}+y\begin{bmatrix}3\\1\\0\end{bmatrix}+z\begin{bmatrix}1\\0\\1\end{bmatrix}$

6.

Assume a 4 by 5 matrix A with 5 columns $A_1 \sim A_5$

$\because A_1+A_3+A_5=0 \quad \therefore$ After elimination, $R_1+R_3+R_5=0$

$\because$ If R_5 is a pivot column, $R_{41}+R_{43}+R_{45}\neq 0$ *

$\therefore R_5$ has no pivot, which means R_5 has the free variable

$N(A)=\left\{x=x_5\begin{bmatrix}a\\b\\c\\d\\1\end{bmatrix}, a,b,c,d \text{ are constant}\right\}$, the special solution is $\begin{bmatrix}a\\b\\c\\d\\1\end{bmatrix}$

7.

If the matrix exist, it must be a 3 by 3 matrix, for the column space and nullspace contain 3 Dimentional matrix

If the matrix A exist, $A_1+A_3=0$ and $A_3=0$ because N(A) contains (1, 0, 1) and (0, 0, 1)

$\therefore A_1=A_3=0$, $C(A)=\{k[A_2], k\in R\}$ doesn't contain (1, 1, 0) and (0, 1, 1) at the same time

8.

$A=\begin{bmatrix}1&2&4\\2&a&c\\4&b&d\end{bmatrix}\rightarrow\begin{bmatrix}1&2&4\\0&a-4&c-8\\0&b-8&d-16\end{bmatrix} \quad \because \text{rank}=1 \quad \therefore \begin{matrix}a-4=0\\b-8=0\\c-8=0\\d-16=0\end{matrix} \rightarrow \begin{matrix}a=4\\b=8\\c=8\\d=16\end{matrix}$

$A=\begin{bmatrix}1&2&4\\2&4&8\\4&8&16\end{bmatrix}$

$B=\begin{bmatrix}a&9&c\\1&b&d\\2&6&-3\end{bmatrix}\rightarrow\begin{bmatrix}0&9-3a&c+\frac{3}{2}a\\0&b-3&d+\frac{3}{2}\\2&6&-3\end{bmatrix} \quad \because \text{rank} \quad \therefore \begin{matrix}9-3a=0\\b-3=0\\c+\frac{3}{2}a=0\\d+\frac{3}{2}=0\end{matrix} \rightarrow \begin{matrix}a=3\\b=3\\c=-\frac{9}{2}\\d=-\frac{3}{2}\end{matrix}$

$B=\begin{bmatrix}3&9&-\frac{9}{2}\\1&3&-\frac{3}{2}\\2&6&-3\end{bmatrix}$

$M=\begin{bmatrix}a&b\\c&d\end{bmatrix}=\begin{bmatrix}a&b\\0&d-\frac{bc}{a}\end{bmatrix} \because \text{rank}=1 \quad \therefore d-\frac{bc}{a}=0 \rightarrow d=\frac{bc}{a}$

$M=\begin{bmatrix}a&b\\c&\frac{bc}{a}\end{bmatrix}$

9.

$A=\begin{bmatrix}3&6&6\\1&2&2\\4&8&8\end{bmatrix} \xrightarrow[\text{By column elimination}]{\text{By row elimination}} \begin{bmatrix}1&2&2\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}3&0&0\\1&0&0\\4&0&0\end{bmatrix} \Rightarrow \begin{bmatrix}3&6&6\\1&2&2\\4&8&8\end{bmatrix}=\begin{bmatrix}3\\1\\4\end{bmatrix}[1\ 2\ 2]$

$A=\begin{bmatrix}2&2&6&4\\-1&-1&-3&-2\end{bmatrix} \xrightarrow[\text{By column elimination}]{\text{By row elimination}} \begin{bmatrix}1&1&3&2\\0&0&0&0\end{bmatrix}, \begin{bmatrix}2&0&0&0\\-1&0&0&0\end{bmatrix} \Rightarrow \begin{bmatrix}2&2&6&4\\-1&-1&-3&-2\end{bmatrix}=\begin{bmatrix}2\\-1\end{bmatrix}[1\ 1\ 3\ 2]$

10.

Let I = n×n matrix

The pivot column of A is $A_1 \sim A_n$, and $N(A)=\left\{x=x_{n+1}\begin{bmatrix}-1\\0\\\vdots\\1\\0\\\vdots\end{bmatrix}+x_{n+2}\begin{bmatrix}0\\-1\\0\\\vdots\\1\\0\\\vdots\end{bmatrix}+\cdots\right\}$

$N=\begin{bmatrix}-I\\I\end{bmatrix}$

The extra zero rows don't matter, so N(B) = N(A), $N=\begin{bmatrix}-I\\I\end{bmatrix}$

For C, $x_i+x_{n+i}+x_{2n+i}=0$, $N(C)=\left\{x=x_{n+1}\begin{bmatrix}-1\\0\\\vdots\\1\\0\\\vdots\end{bmatrix}+\cdots+x_{2n+1}\begin{bmatrix}-1\\0\\\vdots\\\vdots\\1\\0\\\vdots\end{bmatrix}\right.$

$N=\begin{bmatrix}-I&-I\\I&0\\0&I\end{bmatrix}$

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